Modelling Stokes parameter spectra of CP stars

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Abstract. I review recent progress in the techniques of reconstructing stellar magnetic topologies. Availability of the new high-quality Stokes parameter observations and development of the magnetic Doppler mapping techniques led to a major breakthrough in our ability to study the structure of magnetic fields in CP stars and to reveal the relations between the magnetic field and chemical nonuniformities. In particular, we carried out the first simultaneous and self-consistent reconstructions of stellar magnetic field distributions and accompanying abundance inhomogeneities. Moreover, magnetic Doppler imaging method was successfully applied to the reconstruction of stellar magnetic topology using spectra in all four Stokes parameters. These magnetic maps of CP stars were derived for the first time without making *a priori* assumptions about field geometry. Magnetic inversions demonstrate that some well-known CP stars possess magnetic fields of considerably more complicated topology than can be represented by a multipolar expansion of low degree.

Key words: stars: chemically peculiar – stars: magnetic fields – polarization – stars: individual: 53 Cam, α^2 CVn, CS Vir

1 Introduction

Magnetic fields play a crucial role for various stages of stellar formation and evolution. According to generally accepted theoretical and observational frameworks, magnetic fields are involved in many prominent episodes of the stellar life, such as emergence of young stars from the protostellar gas clouds, interaction between accretion disks and young stellar objects, dynamo driven activity cycles in solar-type stars and different types of stellar variability. As stars age, magnetic fields are again introduced to explain details of the mass loss from the giant stars and the origin of magnetic fields in stellar remnants, such as white dwarfs and neutron stars. But despite this apparent ubiquity of stellar magnetism, we have very little knowledge about actual strengths and geometries of magnetic fields in stellar atmospheres. The specific mechanisms and the extent of the interaction between magnetic fields and related surface structures (inhomogeneities of temperature and chemical composition) are also highly uncertain. Very often magnetic fields are postulated ad hoc or studied indirectly using different proxy methods. This unsatisfactory situation is partially due to very subtle observational signatures of stellar magnetic fields. Nevertheless, modern high-resolution spectroscopy and spectopolarimetry have attained sufficient precision to allow direct detection and analysis of magnetic structures in a few classes of main sequence stars (Landstreet 1992). In particular, several types of *chemically peculiar stars* were discovered to host strong, global magnetic fields and, thus became arguably the most interesting and undoubtedly the most numerous objects suitable for in-depth analysis of a wide range of magnetohydrodynamical phenomena in stellar envelopes.

Magnetic fields in chemically peculiar (CP) and related upper main sequence magnetic stars are detected and measured using the Zeeman splitting of spectral lines (Babcock 1947). The surface magnetic geometries in these objects were believed to be not very far from a dipole and certainly dramatically different from complex patchy magnetic structures observed in the solar photosphere. The strength of global magnetic fields in CP stars ranges from a few hundred Gauss to tens of kGauss. Field strengths and orientations of the majority of CP stars were found to vary in a smooth regular manner, on time-scales from half a day to many decades. The period of magnetic variation coincides with the period of photometric variability and changes of stellar spectra. This behaviour is explained by the *oblique rotator model* (Babcock 1949; Stibbs 1950). In this phenomenological picture a roughly dipolar surface magnetic field is constant on time-scales of, at least, many decades and appears to be frozen into a rigidly rotating star with the axis of dipole tilted relative to the stellar rotation axis. The magnetic field induces variation of chemical composition over the stellar surface by altering the processes of chemical diffusion. Distribution of chemical abundances is thus also not axisymmetric with respect to the rotation axis and is expected to be closely related to the magnetic field geometry (Michaud et al. 1981).

The oblique rotator hypothesis has proved to be very successful for basic empirical interpretation of the hot star magnetic variability. However, it could not provide insights into many fundamental problems posed by the discovery of global stellar magnetic fields. For instance, the origin of magnetic fields in hot stars is still a matter of deep controversy. Most traditional theories suggest that fields in CP stars are essentially fossil remnants left from the epoch of stellar formation (Moss 1990). However, a few recent observational findings (Glagolevskij & Chountonov 2001; Hubrig et al. 2000) and theoretical works (Dorch & Bigot 2003) hinted that fields in hot stars can also be generated by dynamo processes operating in stellar convective cores. Furthermore, the role of global fields in stellar evolution is very uncertain owing to lack of any information about the field strengths and geometries in stellar interiors and our poor knowledge of the physical mechanisms responsible for the rapid loss of angular momentum by young CP stars and their separation from normal (non-magnetic and rapidly rotating) stars of similar masses. The relation between surface magnetic and abundance structures is also far from being clear because observations failed to unambiguously identify any chemical abundance patterns that would faithfully correlate with basic parameters of the stellar magnetic topologies.

Part of these discrepancies may arise from overly simplified and inconsistent analysis of magnetic fields and related surface structures. During half a century hundreds of magnetic CP stars were discovered. However, magnetic geometries in these objects were always studied under a number of very restrictive assumptions about the process of line formation in magnetized plasma and global geometry of the stellar magnetic field (very often assuming a pure dipolar magnetic topology). Moreover, the effects of inhomogeneous chemical composition on the magnetic observables were largely ignored and magnetic models have never been systematically verified by direct comparison with the observed shapes of Stokes parameter spectra in individual stellar spectral lines. As the quality of observations and amount of information available to constrain the magnetic models gradually improves, it becomes clear that most, if not all, magnetic CP stars have fields considerably more complicated than dipolar (Bagnulo et al. 2002) and in some stars even more complicated than any low-order multipolar geometry (Bagnulo et al. 2001; Kochukhov et al. 2004). These findings emphasize the necessity for a thorough reevaluation of the basic empirical facts as well as the theories of magnetic phenomena in the upper main sequence stars.

2 Reconstructing magnetic field geometries of CP stars

The evolution of methods applied to study magnetic fields in CP stars was traditionally determined by three factors: available data, sophistication of models and available computing resources. Early measurements of line splitting and circular polarization (Babcock 1958, 1960) together with a very successful oblique rotator model (Stibbs 1950) created a seemingly consistent picture of a magnetic CP star. The oblique rotator model predicts periodic changes in the observed intensity of spectral lines and explains smooth modulation of line splitting and circular polarization as a simple consequence of changes in geometrical aspect over the course of stellar rotation. The amplitude and the range of the magnetic variation are determined by the dipolar field strength $B_{\rm d}$, inclination of the rotational axis i and the tilt of the dipolar axis β.

Under a number of simplifying assumptions, such as formation of spectral lines in weak field and weak line regimes, the observed low-order moments of spectral line profiles measured in intensity and circular polarization can be directly related to the disk averages of the global magnetic field (e.g., Mathys 1989). In particular, the measurements of magnetic splitting or broadening of spectral lines and circular polarization signal, integrated over the line profiles, can be interpreted in terms of, respectively, the disk averaged field strength $\langle B \rangle$ and the net longitudinal (line-of-sight) field component $\langle B_z \rangle$ (these quantities are often collectively called *magnetic observables* or *field moments*). This correspondence between characteristics of stellar line profiles and integral properties of global stellar field facilitated simple modelling of the magnetic geometries of CP

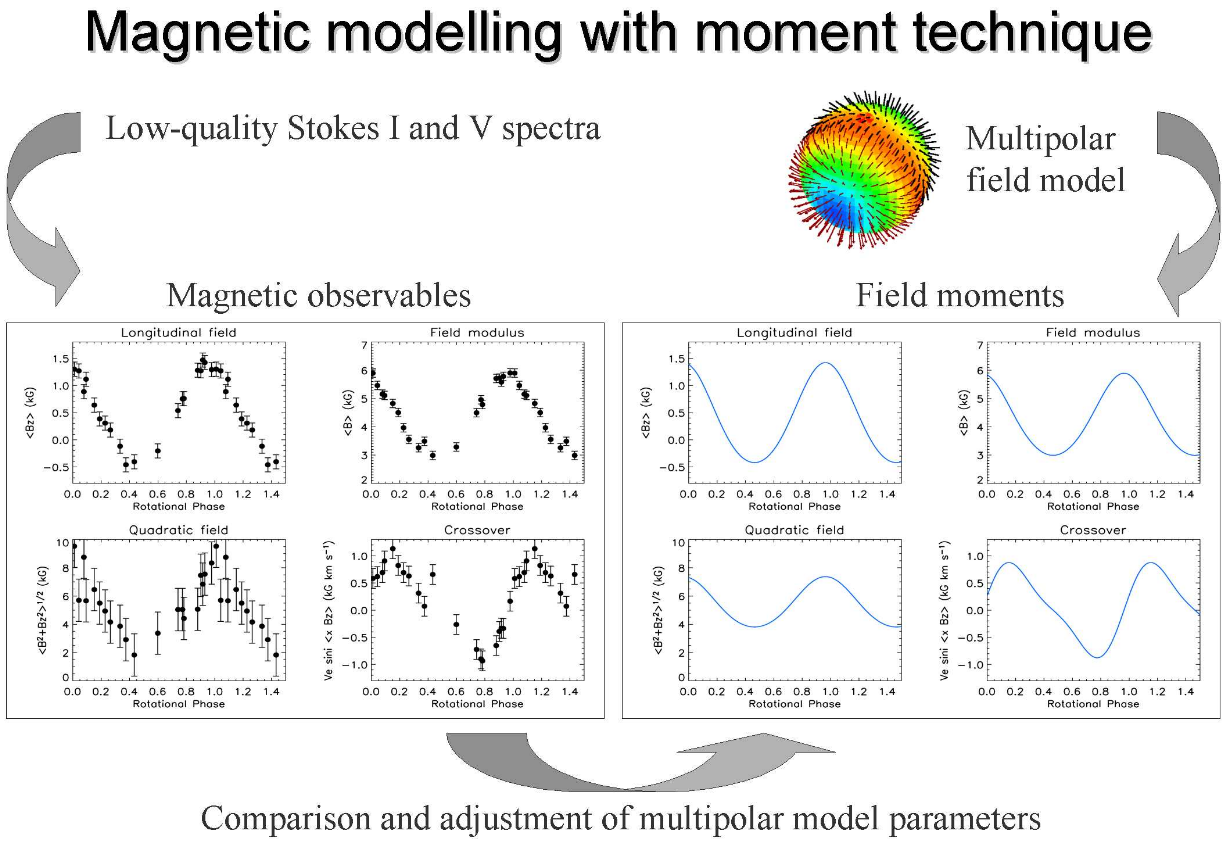

Figure 1: Schematical presentation of magnetic field modelling with the moment technique. Moderate or low-resolution circular polarization observations are used to derive magnetic observables, such as $\langle B_z \rangle$, $\langle B \rangle$, $\langle xB_z \rangle$ and $\langle B^2 + B_z^2 \rangle^{1/2}$. These observations are phased with the stellar rotational period and are compared with the disk-averaged characteristics of simple multipolar magnetic geometries. Multipolar model parameters are derived by adjusting the fit to the observed phase curves of magnetic measurements.

stars by straightforward comparison of observed and computed phase curves of the magnetic observables and derivation of the parameters of dipolar magnetic topologies. This procedure adopted in the so-called *moment technique* to infer the basic parameters of global stellar magnetic fields is illustrated in Fig. 1.

For a while, the puzzle of CP star magnetic field appeared to be solved since the quality of early observations was only good enough to reveal the general shape of modulation. Nevertheless, the success of simple magnetic model stimulated major observational and modelling efforts (Borra & Landstreet 1980), resulting in determination of dipolar field strength and orientation for a few dozens of stars. Subsequent improvements in the observational techniques led to sufficiently accurate polarization and field strength measurements which could not be explained by a pure dipolar field within the observational errors and thus prompted the introduction of more complex models, such as the dipole decentered along its axis (Landstreet 1970), superposition of dipolar and axisymmetric quadrupolar components (Pyper 1969) or the dipole displaced to an arbitrary point of the star (Stift 1975). Later, additional very useful information about the transverse field component was provided by broad-band linear polarization measurements (Leroy 1995), which opened a possibility to further constrain magnetic models and quantify deviations of the fields of CP stars from a pure dipolar geometry (Leroy et al. 1995). In a recent development, Mathys (1995a, 1995b) introduced two additional magnetic observables, the crossover $\langle xB_z \rangle$ and the mean quadratic field $\langle B^2 + B_z^2 \rangle^{1/2}$, which are derived from the circularly polarized and intensity line profiles and are sensitive to higher moments of the stellar magnetic field. These moments, together with the longitudinal field, were measured for a large sample of magnetic CP stars using multi-line analysis of low-resolution moderate S/N Stokes I and V spectra. A parallel major survey of slowly rotating CP stars (Mathys et al. 1997) has also considerably increased the number of stars

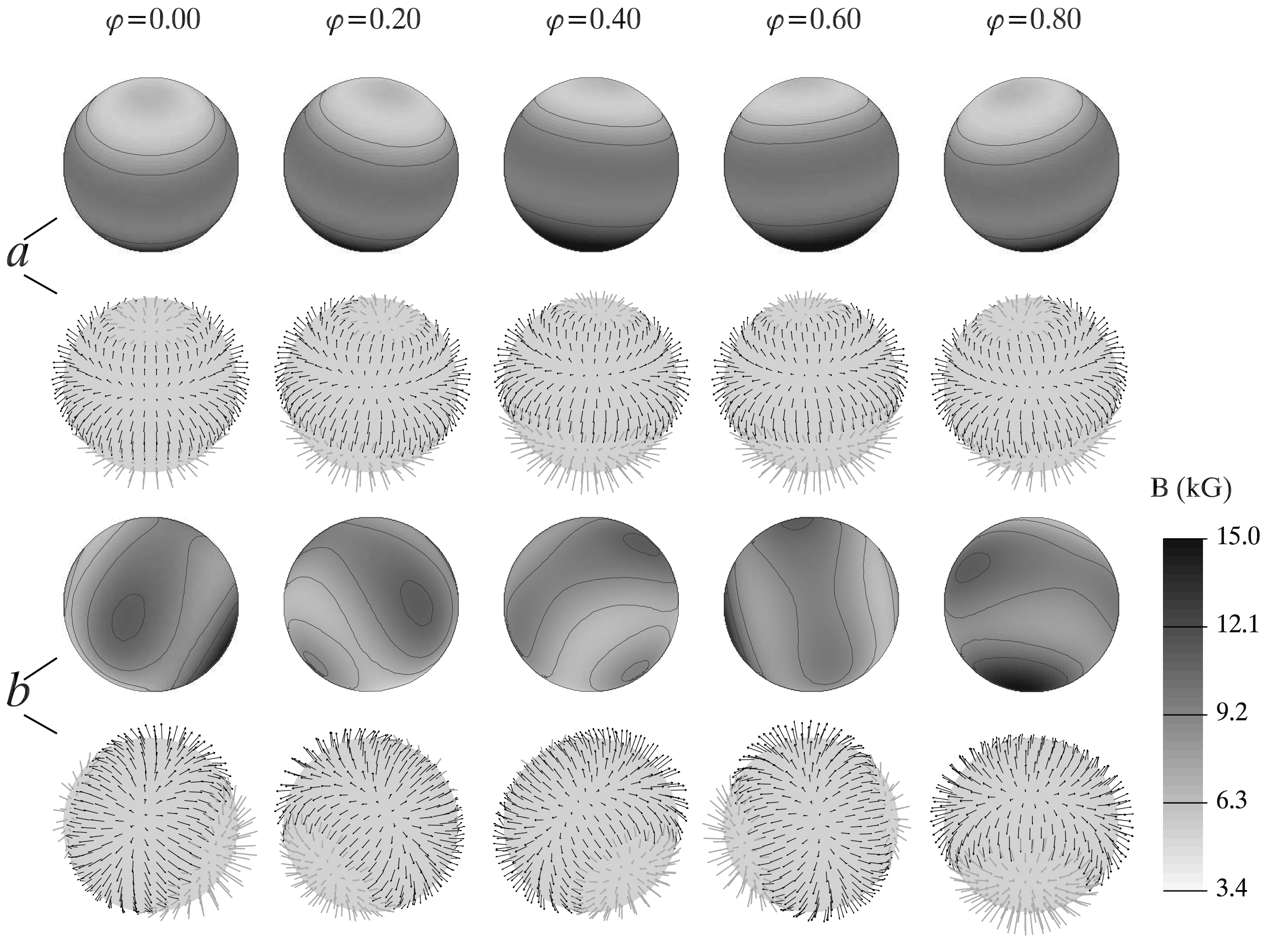

Figure 2: The illustration of the discrepancy between global magnetic field geometry of the magnetic CP star HD 144897 obtained by **a**) Landstreet & Mathys (2000) and **b**) Bagnulo et al. (2002) using *identical* set of magnetic observables but different parameterization of low-order multipolar models. In this and similar spherical plots of magnetic field topology the star is shown at five equidistant rotational phases as indicated at the top of the figure. The aspect corresponds to the inclination angles derived fo HD 144897 in two studies and vertically oriented rotational axis. In each pair of rows the greyscale plot (the upper panel) visualizes the distribution of field strength, while the lower panel shows the orientation of the magnetic vectors. In these vector maps the black arrows show field vectors pointing outside the stellar surface and the grey arrows correspond to the vectors pointing inwards. The arrow length is proportional to the field strength.

with measured $\langle B\rangle$ curves, enabling application of a more sophisticated multipolar modelling to statistically significant stellar samples. Availability of the new observational material prompted Bagnulo and collaborators (e.g., Bagnulo et al. 1999) to generalize the multipolar fitting technique to determine parameters of fairly complex non-axisymmetric superpositions of dipolar and non-linear quadrupolar fields.

Although multipolar modelling based on magnetic observables has established itself as a powerful tool for initial estimates of basic parameters of global magnetic field geometries, a number of important limitations of the method have been identified in the past decades. In particular,

- the field moments are related to observed quantities under a set of assumptions (weak line, weak field) which may bias magnetic measurements in some yet poorly determined way;
- magnetic observables ignore the effects of chemical spots and therefore cannot be applied to stars with appreciable surface variations of chemical abundances;
- magnetic observables are supposed to be derived from a set of unblended spectral lines, but very often it is extremely difficult to find sufficient number of such clean diagnostic features in complex spectra of CP stars;

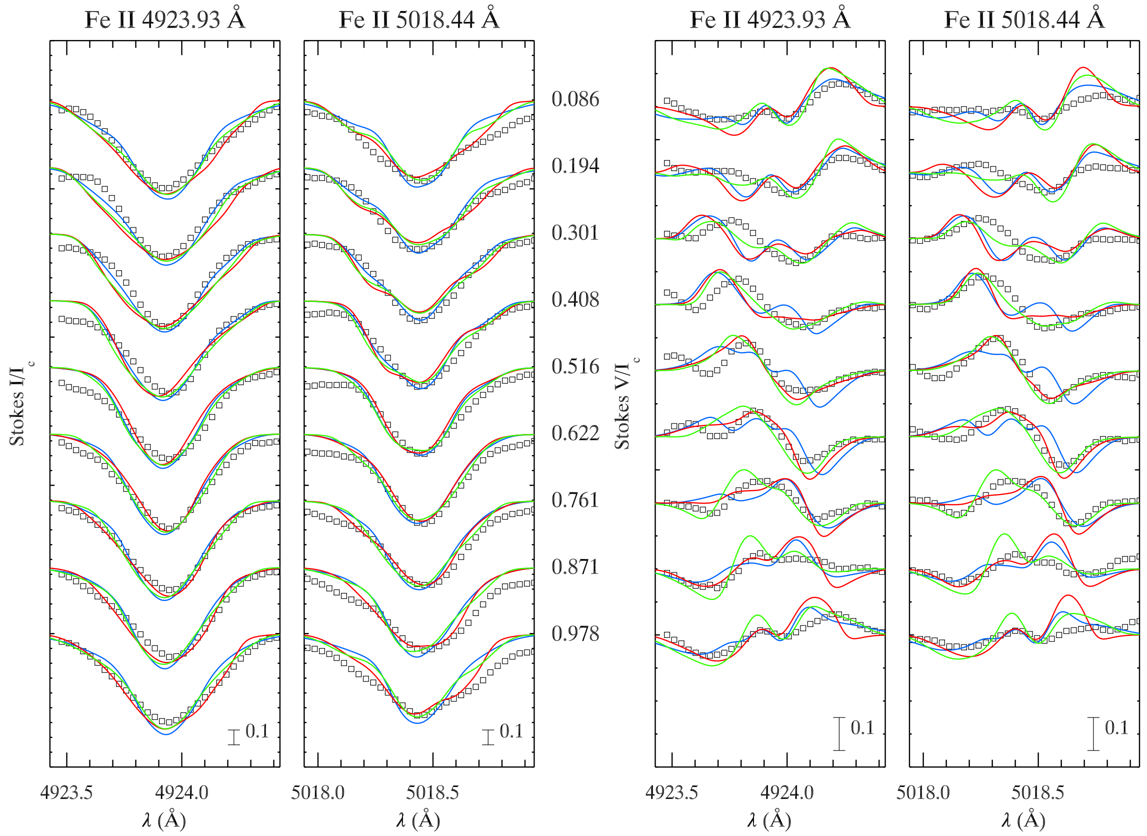

Figure 3: Comparison of the high-resolution Stokes I and V spectra of the magnetic CP star CS Vir (symbols) and predictions of the three multipolar models derived by Bagnulo et al. (2002) using the moment technique (solid lines). Although all three multipolar geometries give reasonably good representation of the magnetic observables, they fail to reproduce the shape of Stokes V profiles of individual Fe II spectral lines. In this figure the spectra for the consecutive rotational phases are shifted in the vertical direction. Phases are indicated in the column to the right of the Stokes I panel. The bars at the lower left of each panel show the vertical scale (10% of the Stokes I continuum intensity).

- magnetic observables, in particular $\langle B\rangle$ and $\langle B_z\rangle$ available for most of the stars, are poorly suited to constrain higher-order terms in multipolar expansion of the stellar magnetic fields;
- multipolar fitting is intrinsically non-unique and requires extensive analysis of the whole space of model parameters in order to find best-fit models. In some situations substantially different magnetic models give similar curves of magnetic observables and thus cannot be distinguished on the basis of the measurements of field moments;
- the choice of multipolar parameterization appears somewhat arbitrary as different researchers tend to characterize magnetic geometries with dissimilar sets of parameters, precluding direct intercomparison of the results of different studies.

The last two issues sometimes lead to striking discrepancies between results of multipolar modelling of magnetic observables. For example, in Fig. 2 I show spherical maps of the surface magnetic field derived for HD 144897 by Landstreet & Mathys (2000) and Bagnulo et al. (2002), who used *identical* set of observational data, but employed somewhat different multipolar parameterization of the global field. Clearly, resulting field geometry and even stellar inclination differ considerably, despite the fact that the two multipolar models give similar curves of $\langle B_z\rangle$, $\langle B\rangle$, $\langle xB_z\rangle$ and $\langle B^2+B_z^2\rangle^{1/2}$.

And last but not least, it should be emphasized that the moment technique is, in principle, fundamentally limited because it is based on the interpretation of secondary quantities derived from stellar line profiles. The method gives absolutely no guarantee that resulting magnetic models would also provide an adequate description of the actual stellar Stokes parameter spectra. Indeed, very often predictions of multipolar models which successfully fit several magnetic observables disagree with direct observations of stellar spectra. Fig. 3 shows an example of such disagreement for the well-known magnetic star CS Vir. Evidently, none of the three multipolar models derived for this star by Bagnulo et al. (2002) fits high-resolution Stokes I and V profiles of the Fe II lines.

Thus, more detailed studies of the magnetic stars based on direct analysis of Stokes profiles are not only essential to complement and verify field models derived via application of the moment technique, but (in at least some cases) provide *the only possibility* to obtain reliable information about the stellar magnetic structures.

The analysis of spectral variability of CP stars has led to the discovery of chemical inhomogeneities on their surfaces, but it took many years, until the introduction of inverse problem techniques (Goncharskij et al. 1982), before it was possible to map the chemical spots using Doppler imaging (DI) and compare resulting maps with the field models. The resemblance was rather poor, creating significant problems for the radiation driven diffusion theory developed to explain CP-star phenomenon. Furthermore, abundance Doppler imaging was typically applied to study the surface chemical structures on rapidly rotating magnetic CP stars, for which few determinations of magnetic observables have been obtained and the magnetic field geometry is thus especially poorly known.

A complex problem of simultaneous mapping of abundance distributions and reconstruction of the magnetic geometry was addressed by Landstreet in the studies of 53 Cam (Landstreet 1988) and Babcock's star (Landstreet et al. 1989). These investigations were based on the analysis of the rotational modulation of the Stokes I spectra and modelling field moments. Khokhlova et al. (2000) extended the method to both intensity and circular polarization line profiles. Both approaches still employed multipolar parameterizations to describe magnetic field, but tried to make use of all information contained in the variation of line profiles. However, the computational demands related to synthesizing the spectra of magnetic stars become prohibitively large even for studies of a few spectral lines. As a result, previous attempts of simultaneous analyses of magnetic and abundance structures had to either assume a very simple parameterization of abundance distributions, *a priori* symmetric with respect to the magnetic field axis (in Landstreet's modelling), or use simplified and probably unrealistic analytical representation of the local Stokes profiles (in investigations by Khokhlova and collaborators).

By the beginning of the new century studies of the stellar magnetic fields have reached a new phase with the introduction of high-resolution spectropolarimeters at a number of medium-sized telescopes, which allowed to achieve first definite and direct detection of polarization signatures in active late-type stars (Donati et al. 1997) and acquire first systematic measurements of the linear polarization in individual spectral lines of CP stars (Wade et al. 2000). At that point it became clear that

- in many CP stars the dipolar component is not dominating the field structure (Bagnulo et al. 2002);
- different chemical elements tend to form very different abundance patterns on the surfaces of CP stars (e.g., Kuschnig et al. 1999) and in general abundance spots are not symmetric with respect to the lowest order component of the field topology;
- polarization spectra are noticeably affected by the chemical inhomogeneities (see Kochukhov et al. 2002);
- the inverse problem approach is a very powerful tool and can be extended to constructing realistic models of stellar magnetic field and self-consistent mapping of chemical and magnetic structures.

Impressive advances in the quality of available polarization spectra have been matched by rapid increase in computing power and improvements in numerical techniques, making it possible to synthesize disk integrated stellar Stokes spectra in reasonable amount of time (Wade et al. 2001). These circumstances suggest that a magnetic Doppler imaging reconstruction of the chemical and magnetic structures based on direct analysis of high-resolution line profiles in Stokes parameters is a very promising tool for studies of CP stars. It allows us to circumvent many limitations of traditional modelling of magnetic observables. Most importantly, magnetic Doppler imaging generalized to the analysis of all four Stokes parameters does not require an *a priori* parameterization of the global magnetic field structure and thus appears to be the only method capable of addressing a fundamental question: what is the actual structure of magnetic fields in CP and related stars?

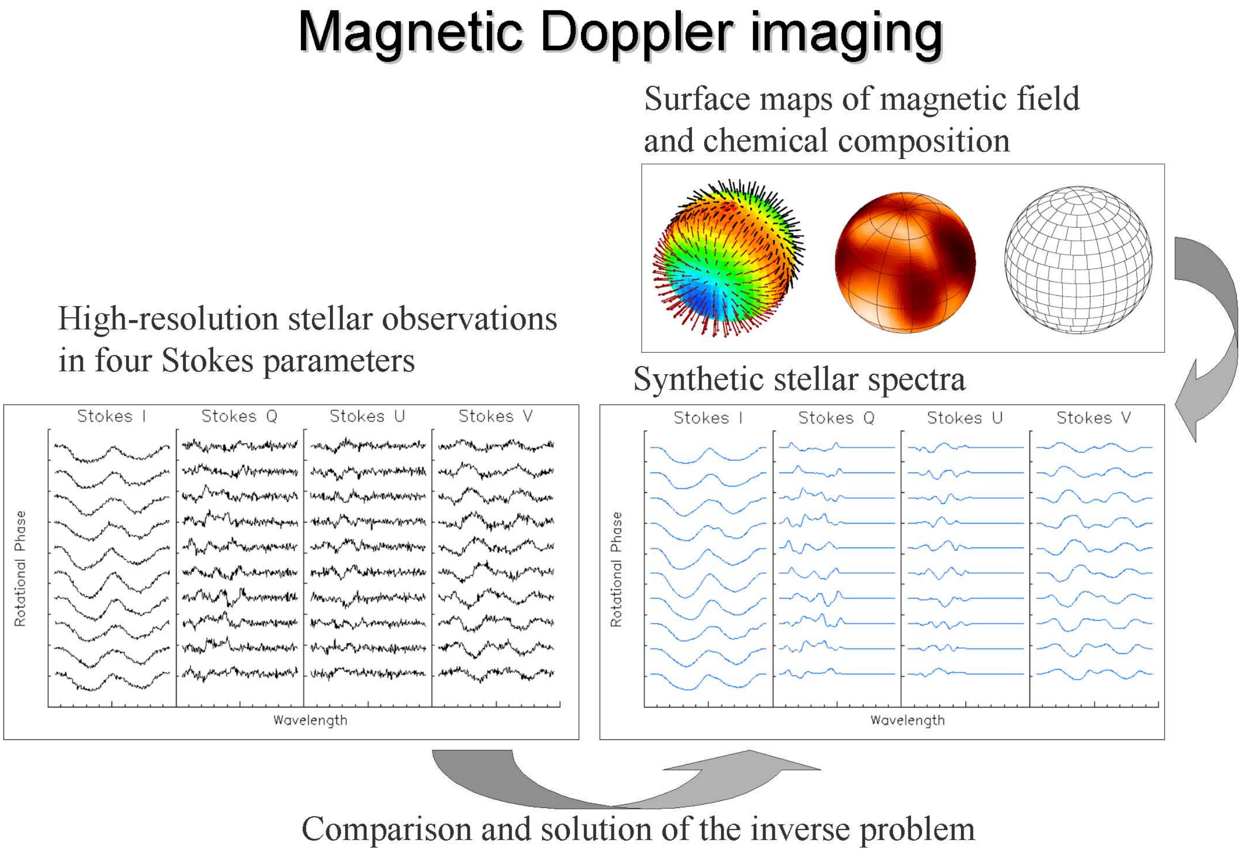

Figure 4: Schematical presentation of modelling stellar magnetic topologies with magnetic Doppler imaging. Initial surface maps of chemical composition and magnetic field are used to synthesize variation of line profiles in all four Stokes parameters. Theoretical spectrum synthesis is directly compared with high-resolution stellar spectropolarimetric observations. Surface maps are adjusted to fit observed Stokes spectra without imposing any *a priori* constraints on the possible geometry of magnetic field and chemical inhomogeneities.

3 Magnetic Doppler imaging

Doppler imaging (DI) has proved to be a very successful technique for obtaining in-depth knowledge about stellar surface structures. In general DI is built around complex mathematical procedure that inverts time-series of high-resolution observations of stellar spectra into maps of stellar surface parameters, like temperature, abundance and magnetic field. In its conventional form, Doppler imaging has been successfully applied to mapping inhomogeneities of chemical composition and temperature spots on, respectively, chemically peculiar and active stars (see Rice 2002 for a recent review of DI methods and applications). However, for both types of stars magnetic fields were typically not studied directly, but were only hypothesized to be responsible for creating and supporting stellar surface structures.

In order to understand the role of stellar magnetism and make use of new spectropolarimetric observations, we have generalized the Doppler imaging method to map stellar magnetic fields and reconstruct chemical composition and temperature maps directly from rotational modulation of line profiles recorded in the Stokes $IQUV$ parameters, as illustrated in Fig. 4. The basic numerical techniques and the concept of magnetic Doppler imaging (MDI) modelling of high-resolution spectropolarimetric observations was discussed in detail by Piskunov & Kochukhov (2002). In this paper we introduced the new magnetic Doppler imaging code `Invers10` and described essential numerical techniques, such as the solution of the polarized radiative transfer equation, the integration of the emergent Stokes profiles over the stellar surface and optimization procedure, and parallel execution of the code on multi-processor supercomputers.

The mathematical procedure of magnetic Doppler imaging consists of constructing a discrepancy function

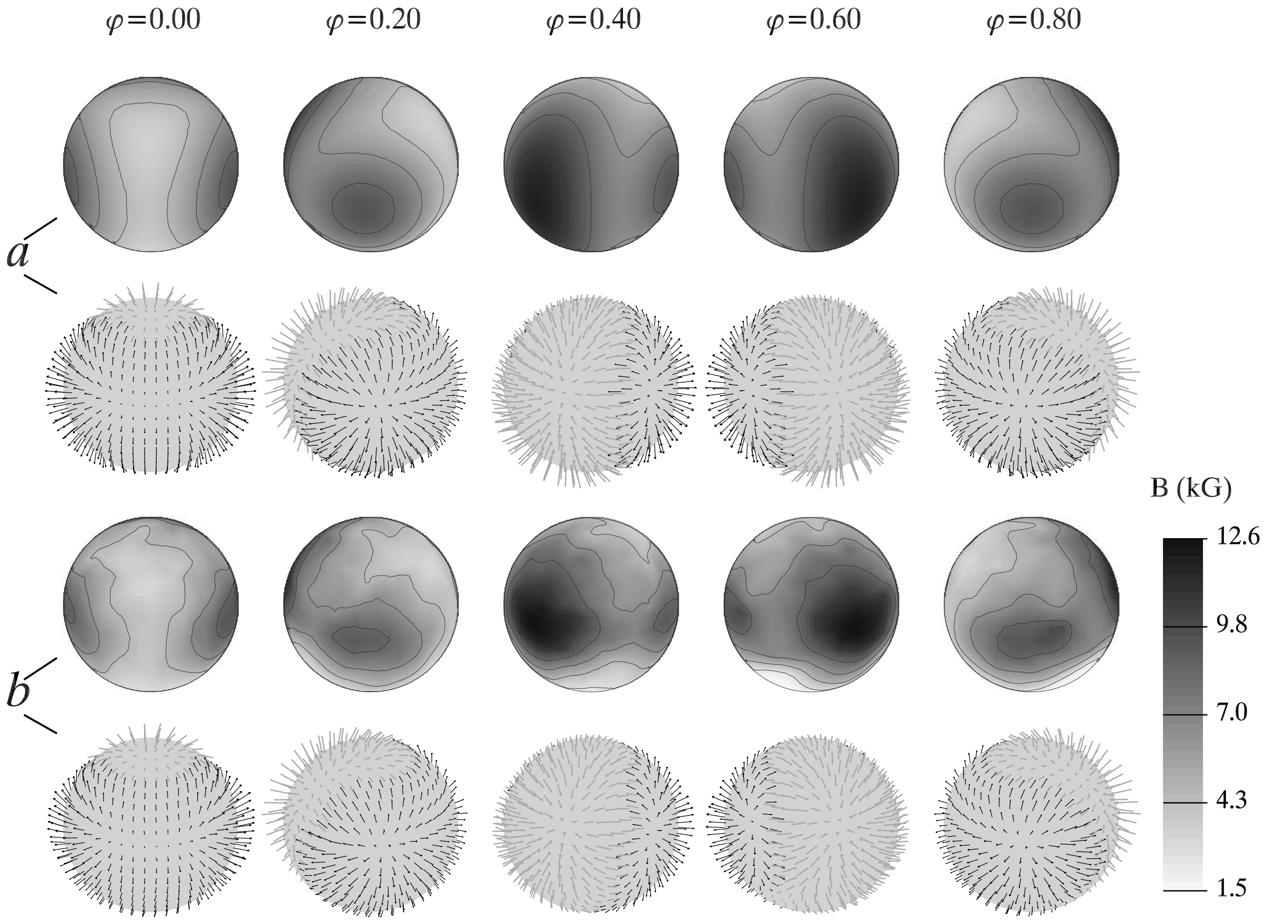

Figure 5: Comparison between true (**a**) and reconstructed (**b**) geometry of complex multipolar magnetic field recovered in numerical experiment with the MDI code `Invers10`. The input magnetic distribution consisted of a superposition of non-aligned dipole and quadrupole. Vector magnetic map is successfully recovered from observations in all four Stokes parameters.

that characterizes the deviation between the theoretical spectra computed from a current surface map and the observed spectra. A penalty function representing the smoothness or amount of information contained in a current image is added to the discrepancy function to form a total error function:

$$E = \sum_{\varphi} \sum_{\lambda} \left(\boldsymbol{I}^{obs}_{\lambda\varphi} - \boldsymbol{I}^{calc}_{\lambda\varphi}(x, \boldsymbol{B}) \right)^2 / \boldsymbol{\sigma}^2_{\lambda\varphi} + \Lambda F(x, \boldsymbol{B}), \tag{1}$$

where $\boldsymbol{I} = \{I, Q, U, V\}$, $\boldsymbol{I}^{obs}_{\lambda\varphi}$ and $\boldsymbol{\sigma}_{\lambda\varphi}$ represent the observed Stokes spectra and their error bars respectively. The $\boldsymbol{I}^{calc}_{\lambda\varphi}$ corresponds to the line profiles predicted for a given surface distribution of scalar parameter x (chemical composition or temperature) and vector magnetic field $\boldsymbol{B}$. The summation of the squared difference between the observations and theoretical spectra is carried over all rotational phases φ and wavelengths λ. The regularization function is represented by the $F(x, \boldsymbol{B})$ and Λ denotes the regularization parameter. The role of regularization is essential to ensure that the Doppler imaging algorithm will fit observations with the simplest possible surface map. The Levenberg-Marquardt optimization algorithm (Press et al. 1992) is used to iteratively reduce the total error function until the discrepancy between the observed and predicted line profiles does not exceed the errors of observations.

Among characteristics distinguishing our modelling approach the following are the most important and unique features implemented in the `Invers10` code:

- The new inversion method is based on a detailed spectrum synthesis using stellar model atmospheres and takes into account all relevant physics of the transfer of polarized radiation through the magnetized

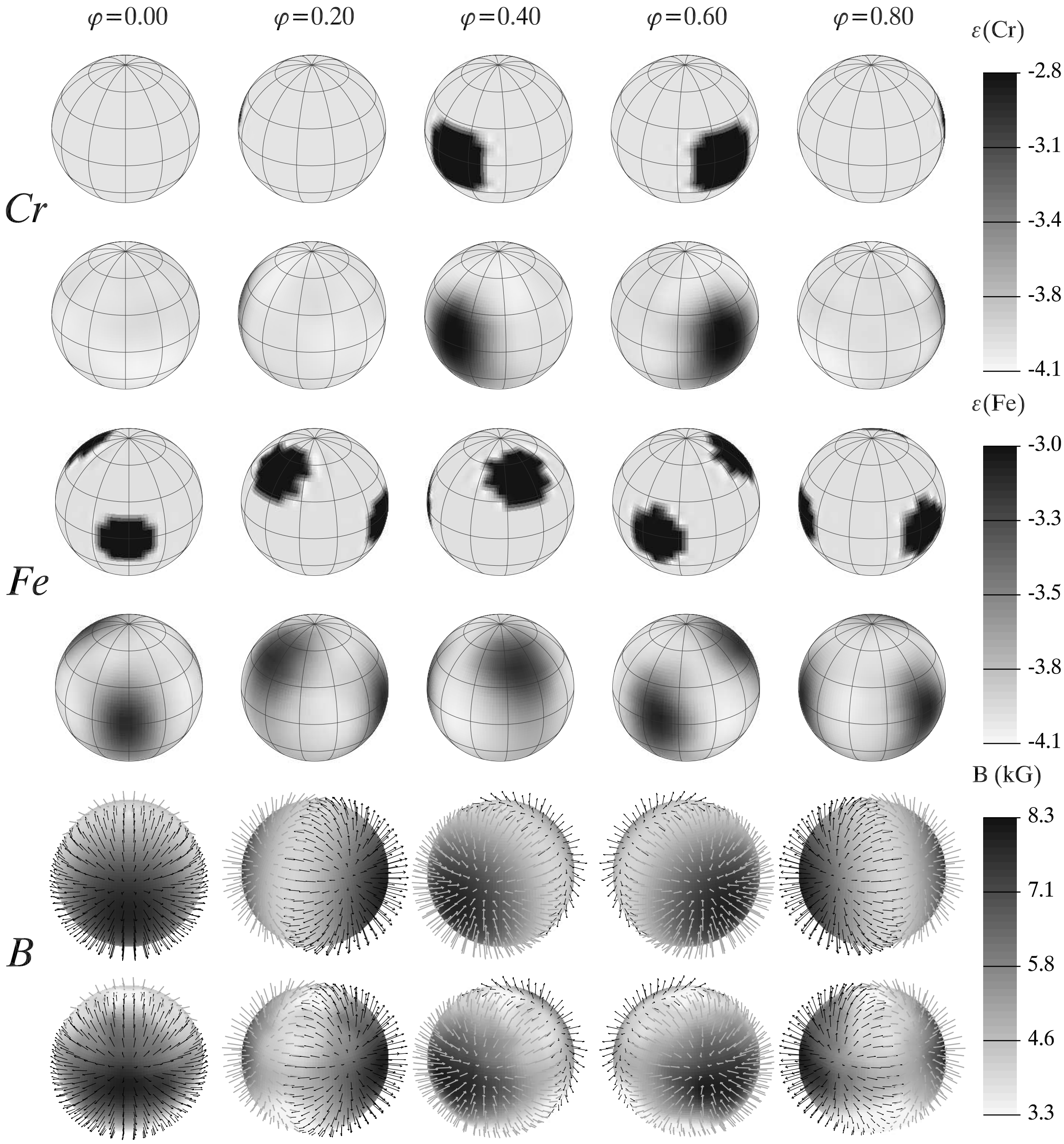

Figure 6: Multi-element magnetic DI with the `Invers10` code. In this numerical experiment dipolar field topology and surface distributions of Fe and Cr were reconstructed simultaneously from the simulated dataset in four Stokes parameters shown in Fig. 7. In each pair of rows the upper sequence of spherical plots shows the true magnetic or abundance distribution, while the lower panel shows corresponding surface map recovered by `Invers10`. In the magnetic field maps (the two lower panels) the spherical vector plots show distribution of the field orientation, while underlying greyscale images illustrate the field strength maps.

stellar plasma. In contrast to all previous attempts to model Stokes profiles of magnetic stars, we do not resort to any simplifying approximations (e.g., weak field or Milne-Eddington model) in solving the polarized RT equation. Our forward radiative transfer calculations were shown to be compatible with the synthetic spectra generated by the two other independent polarized RT codes (Wade et al. 2001).

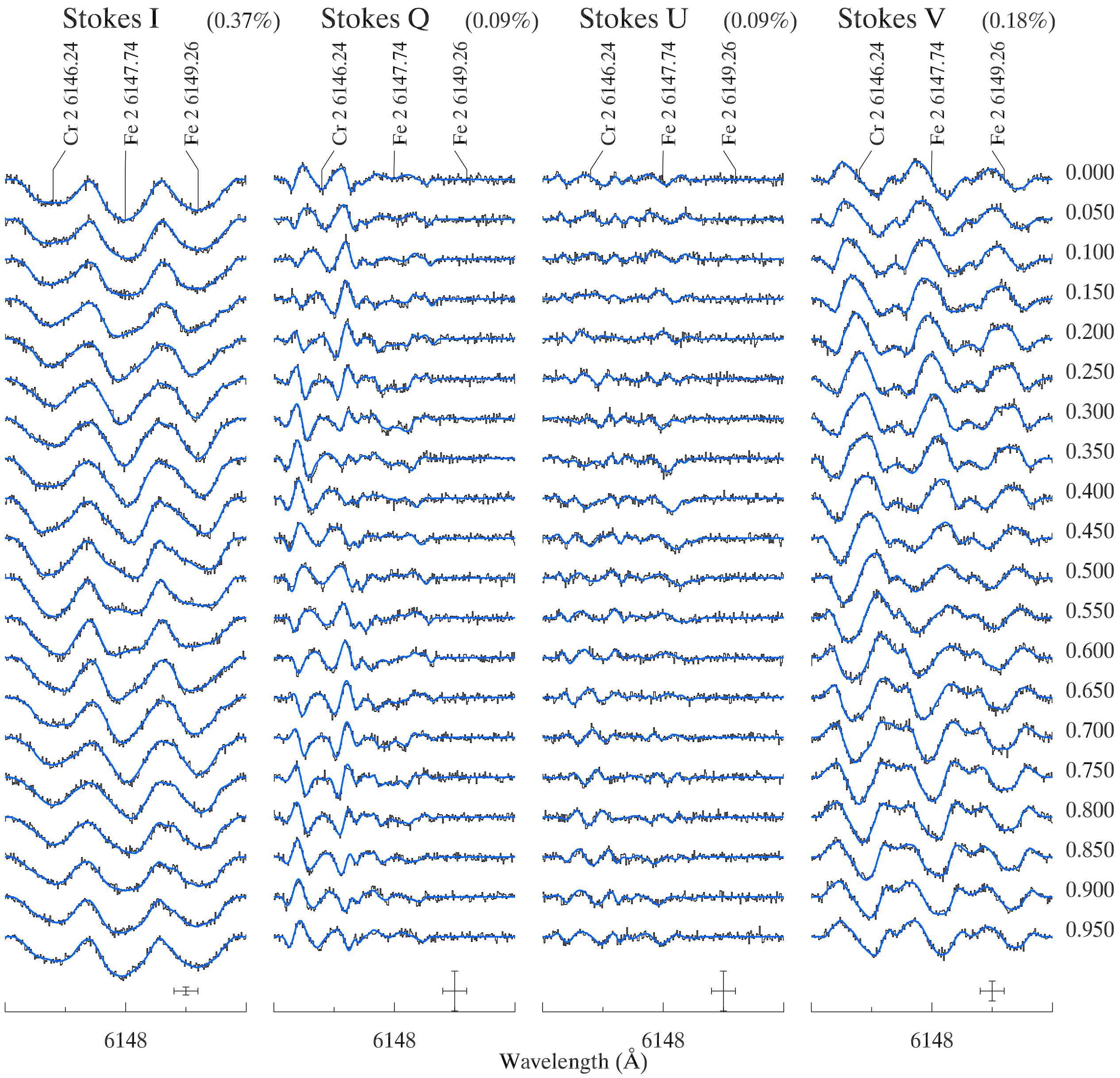

Figure 7: Simulated four Stokes parameter dataset (histogram) employed in the numerical experiment presented in Fig. 6. The thick solid line shows the final fit achieved by `Invers10`. In this figure spectra for the consecutive rotational phases are shifted in the vertical direction. Rotational phases are indicated in the column to the right of the Stokes V panel. The bars at the lower left of each panel show the horizontal and vertical scale (1 Å and 5% of the Stokes I continuum intensity respectively). The number in brackets at the top of each panel quotes mean deviation between observations and spectrum synthesis.

- `Invers10` is the first DI code designed for simultaneous and self-consistent reconstruction of the three vector components of the stellar magnetic field and accompanying inhomogeneities of chemical composition or temperature.

- In mapping magnetic topologies of CP stars with spectra in all four Stokes parameters we do not assume a low-order multipolar field expansion. High-resolution Stokes spectra contain sufficient information to allow direct mapping of the field and thus obtaining a totally new view of magnetic structures in the atmospheres of upper main sequence stars.

Development of the `Invers10` code included careful reassessment of the role of regularization in magnetic DI. In ordinary scalar abundance or temperature mapping two alternative regularization functionals, Tikhonov regularization and Maximum Entropy, usually lead to compatible maps when high-quality data is used in Doppler imaging (Strassmeier et al. 1991). In contrast, the choice of regularization function is crucial for the vector magnetic mapping of CP stars. According to the results of our in-depth analysis of various regularization strategies presented in Kochukhov & Piskunov (2002), the Tikhonov regularization functional that we use in `Invers10` is more appropriate for mapping global magnetic structures, expected in the atmospheres of CP stars and magnetic white dwarfs. On the other hand, the Maximum Entropy regularization method is typically used in Zeeman-Doppler imaging of the complex patchy magnetic fields of active late-type stars (e.g., Hussain et al. 2000), but shows poor performance when applied to reconstruction of the more uniform magnetic topologies of CP stars. The main reason for this failure of the Maximum Entropy method becomes obvious when we recall that entropy functional tries to suppress large-scale deviations from the average value of surface distribution in favour of small localized spots. Clearly the latter physical picture may be applied to structures on the surfaces of late-type stars but is certainly inadequate when modelling magnetic fields in upper main sequences stars and magnetic white dwarfs.

Apart from recent pioneering observations by Wade et al. (2000), who detected linear polarization signatures in the line profiles of a few brightest CP stars, no systematic full Stokes vector spectropolarimetric observations have been carried out for main sequence stars. On the other hand, fairly extensive high-quality Stokes I and V datasets have already been acquired for a number of CP stars or can be readily obtained using many of the existing échelle spectrographs equipped with Zeeman analysers. Looking into the problem of reconstructing stellar magnetic fields from partial spectropolarimetric datasets, we found that the Stokes I and V spectra do not contain enough information for a stable and unique magnetic inversion. However, useful information can still be extracted from such observations by introducing external constraints on the possible magnetic geometries. For this purpose we use *multipolar regularization* (see Kochukhov & Piskunov 2002) which encourages the code to search for a solution close to general second-order multipolar expansion (identical to the multipolar field parameterization introduced by Bagnulo et al. 1999). This restriction of the parameter space of magnetic distributions is sufficient for reliable recovery of magnetic fields and abundance distributions in stars with quasi-dipolar or quadrupolar magnetic topologies.

We used numerical experiments (Kochukhov & Piskunov 2002) to assess performance and reveal possible intrinsic limitations of the magnetic mapping with `Invers10`. In these tests the magnetic Doppler imaging code was used in the forward mode to calculate Stokes spectra for a given magnetic field and abundance geometry. The random noise component was added to these simulated observations in order to represent the imperfections of the real observational data. Subsequently, the magnetic inversion was carried out using the simulated Stokes $IQUV$ spectra and the resulting surface images were compared with the initial maps.

Fig. 5 shows an example of the reconstruction of a complex global magnetic field. In this test all four Stokes parameters were used and the MDI code successfully recovered magnetic topology formed by a superposition of non-aligned dipole and quadrupole. Evidently, the agreement between the true and reconstructed magnetic maps is good over most of the visible part of the stellar surface, except for the lowest latitude belts where the field strength is systematically underestimated by `Invers10`. This is to be expected since the low-latitude regions make very small contribution to the disk-integrated Stokes spectra and therefore retain some imprint of the initial guess which was zero field everywhere. In other regions on the stellar surface magnetic inversion recovers the true field strength within 10–15% of the magnitude of each magnetic vector component. The global field characteristics (i.e., best-fit dipole and quadrupole strengths and orientations derived from the final magnetic map) are typically recovered within a few degrees and 5–10% of the polar field strengths.

Another numerical experiment with `Invers10` is shown in Fig. 6 and illustrates new multi-element mapping capability of our MDI code. We used four Stokes parameters of the two Fe lines and one Cr line to recover magnetic field geometry and simultaneously reconstruct two substantially different chemical abundance distributions. The comparison between simulated observations used in this test and the final fit to the Stokes profiles achieved by `Invers10` is shown in Fig. 7. Possibility to obtain several abundance maps using blended spectral features and multiple wavelength regions considerably improves the performance of `Invers10` and reduces quality requirements to the spectral lines selected for modelling.

Summarizing the outcome of the numerical tests of magnetic mapping presented by Kochukhov & Piskunov (2002), we convincingly demonstrated that unique and reliable recovery of a global stellar magnetic map is possible with high-quality four Stokes parameter spectropolarimetric observational data. Our results suggest that the information contained in Stokes $IQUV$ profiles allows us to carry out the magnetic inversion without introducing additional assumptions about the large-scale magnetic field structure. Furthermore, compared to regular scalar DI of temperature or chemical composition, additional information content of Stokes

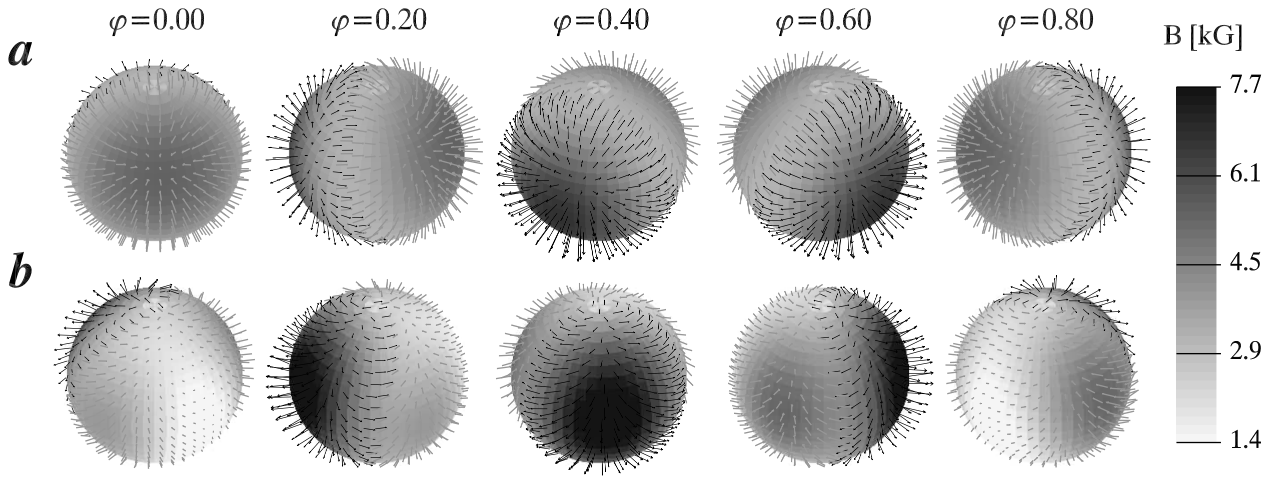

Figure 8: Magnetic field topology of α^2 CVn (**a**) reconstructed using the Stokes I and V spectra of the 13 different lines of singly ionized Cr, Si and Fe. The lower panel (**b**) shows distribution of magnetic field on the surface of CS Vir. This magnetic image is based on modelling the Stokes I and V observations of nine Fe II lines. In this figure the spherical vector plots show distribution of the field orientation, while underlying greyscale images illustrate the field strength maps.

parameters, in particular sensitivity of the Stokes Q and U profiles to the transverse field component significantly extended the group of stars suitable for application of MDI. Most importantly, by utilizing rotational modulation of the polarization signatures in individual lines magnetic imaging makes it possible to recover reasonably accurate surface maps for very slowly rotating CP stars as well as for pole-on or equator-on stars, unaccessible with scalar DI methods.

4 Stokes IV mapping

The first application of the `Invers10` code to real spectropolarimetric stellar observations was presented by Kochukhov et al. (2002). In this paper we discussed acquisition and analysis of the Stokes I and V spectra obtained for the bright chemically peculiar star α^2 CVn using the SOFIN spectrograph at the Nordic Optical Telescope. This dataset is characterized by the resolving power of $\lambda/\Delta\lambda = 70\,000$, $S/N = 200$–300 and covers about 30 rotational phases of the star, thus probably representing the best quality circular polarization spectra ever obtained for a magnetic CP star.

Magnetic mapping of α^2 CVn was carried out with `Invers10` using multipolar regularization technique and resulted in derivation the first self-consistent magnetic field and abundance maps for a chemically peculiar star. The field topology was derived from 13 lines of Si II, Cr II and Fe II. The average magnetic structure is shown in Fig. 8a and appears to be rather close to an oblique dipole geometry.

I emphasize the large number of spectral lines that we used for modelling α^2 CVn. In the literature determination of the longitudinal, quadratic field and other magnetic observables is often referred to as "multi-line technique" (e.g., Mathys 1989) and is opposed to the direct fitting of Stokes spectra, which is believed to be limited to a few diagnostic lines. But, in fact, in many studies of magnetic CP stars (e.g., Mathys 1991) only between 10 and 20 lines were used to measure longitudinal field and other magnetic quantities. As we show, modelling this number of spectral lines is already within reach of the magnetic imaging technique based on detailed polarized spectrum synthesis.

A different example of the Stokes IV field mapping is provided by another magnetic star, CS Vir, that was also observed in two Stokes parameters with the SOFIN spectrograph. Analysis of this object is currently in progress. Preliminary mapping based on 9 iron lines revealed that magnetic field of CS Vir has substantial quadrupolar component (see Fig. 8b). Spectral coverage of the data obtained for CS Vir at NOT is much more extensive than for α^2 CVn, hence a large number of diagnostic lines is available. This opens possibility to employ lines formed at different optical depths and attempt to map 3D abundance distribution and field geometry of CS Vir.

The ultimate goals of detailed studies of abundance and magnetic field geometry of CP stars is to provide

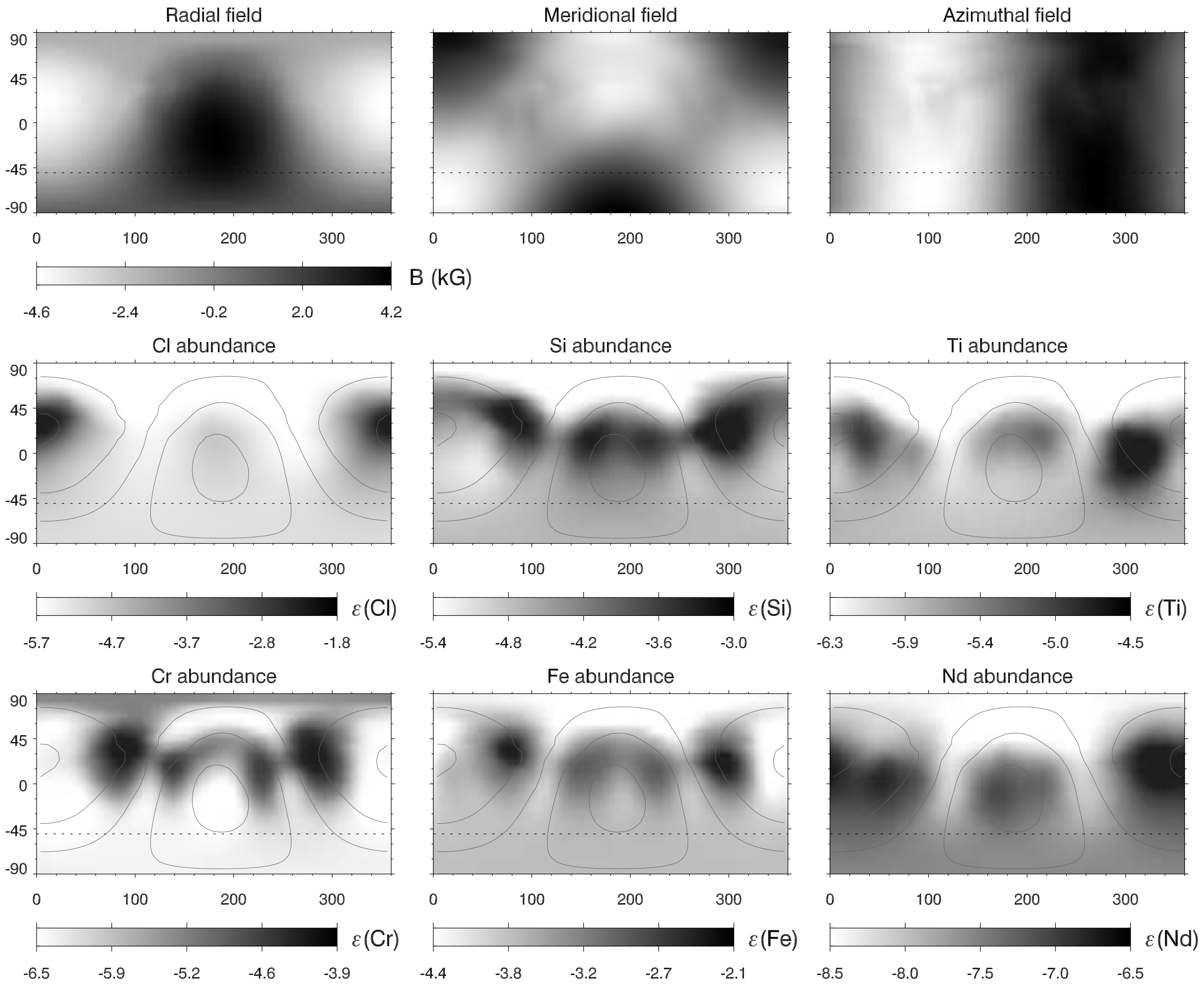

Figure 9: The rectangular projection of the surface maps of the radial, meridional and azimuthal components of the vector magnetic distribution derived for α^2 CVn using Stokes I and V spectra (the upper three panels). This magnetic map is compared with the surface abundance distributions of singly ionized Cl, Si, Ti, Cr, Fe and doubly ionized Nd (the lower six panels). The solid curves shown on top of the abundance maps are the contours of equal radial field, plotted for the field values between -4 and 4 kG with a 2 kG step. The horizontal dotted line in each panel indicates the lowest visible latitude for the inclination angle $i = 49\overset{\circ}{.}1$ adopted in the study of α^2 CVn.

observational constraints for the theoretical modelling of formation and evolution of global stellar magnetic fields and their interaction with chemical diffusion, which leads to the formation of surface abundance inhomogeneities. While modelling magnetic observables may, in principle, provide a rough idea of the field structure, there is no other way but to use magnetic DI to derive distribution of chemical elements on the surfaces of strongly magnetic chemically peculiar stars. We are moving in this direction as observations of the results of chemical diffusion in the presence of magnetic field with known geometry became available for α^2 CVn. Fig. 9 summarizes our abundance mapping of this star. Rectangular maps of the three vector magnetic components are compared with the surface abundance maps of six chemical elements. We confirm that, in general, the horizontal distributions of many chemical species exhibit some degree of correlation with the magnetic geometry. Our modelling reveals fine details of the complex interaction between magnetic field and stellar atmospheric plasma. For example, the Cr map is dominated by the ring of overabundance around the positive magnetic pole and two symmetric spots, which are located roughly at the intersection of the stellar rotational and magnetic equators. Another interesting case is chlorine. We reconstructed the first map of Cl distribution on the surface of a chemically peculiar star and unexpectedly discovered that in α^2 CVn

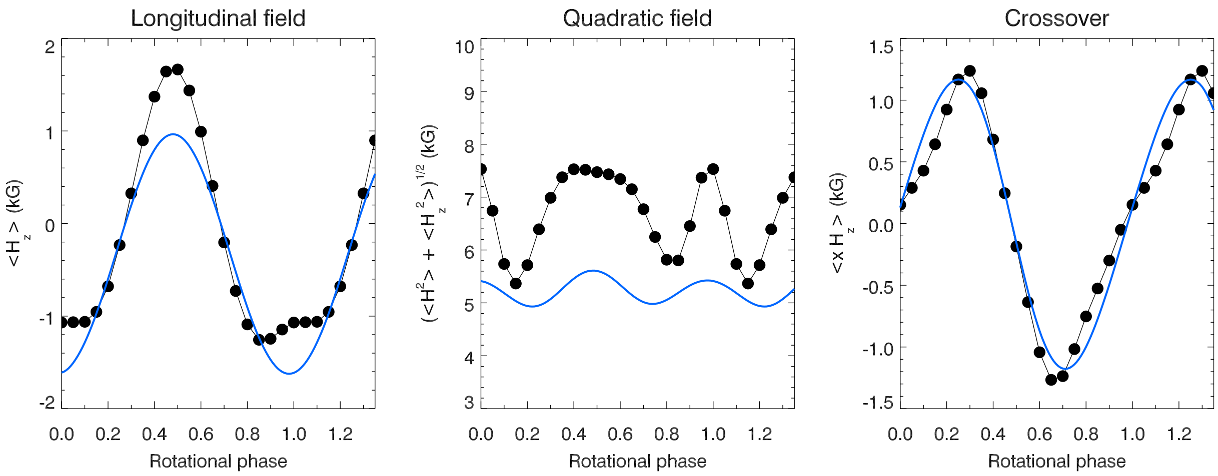

Figure 10: Phase variation of the magnetic observables (longitudinal field, quadratic field and crossover) determined for α^2 CVn from the synthetic Stokes I and V profiles of 25 iron lines computed with `Invers10` taking into account magnetic field and spotted iron surface distribution (symbols). Idealized variation of observables shown with solid line was computed by direct geometrical integration of the magnetic field distribution, neglecting abundance inhomogeneities, as it is practiced by the moment technique.

this element shares properties of the rare-earth elements. It is mostly concentrated at the negative magnetic pole.

At the same time, we found that, in contrast to a common assumption, intrinsic to some modelling approaches (such as those presented in the studies by Landstreet 1988 and Strasser et al. 2001), chemical elements do not always form structures strictly axisymmetric with respect to the field axis. This may be an indication that chemical diffusion is noticeably affected by phenomena other than magnetic field, such as stellar rotation or inhomogeneous magnetically confined mass loss.

Having derived detailed abundance and magnetic maps of α^2 CVn, it is interesting to verify the impact that abundance inhomogeneities have on classical magnetic observables. Fig. 10 shows a simple test: magnetic observables were first evaluated by direct surface integration of the magnetic map that we derived for α^2 CVn and then compared with the longitudinal field, quadratic field and crossover which would have been obtained by applying the moment technique to the actual profiles of a sample of 25 Fe II lines affected by both magnetic field *and* abundance spots. Clearly, chemical inhomogeneities have very noticeable effect, especially on the longitudinal and quadratic fields. Hence, neglecting abundance nonuniformities in the interpretation of the magnetic measurements can severely affect field mapping. The presence of abundance spots introduces an additional modulation in the phase curves of the magnetic observables obtained from the metallic lines. As a result, non-sinusoidal variations of, for example, the mean longitudinal field appear and are often erroneously interpreted as an evidence for a strong quadrupolar contribution to the topology of the stellar magnetic fields. Not surprisingly, magnetic models built on the basis of interpretation of the magnetic observables affected by the chemical surface inhomogeneities often fail when confronted with the high S/N observations in Stokes parameters (as discussed in Sect. 2). Thus, one has to be very careful when using parameters of individual magnetic models determined in the coarse statistical studies of field geometries in large samples of magnetic CP stars (e.g., Landstreet & Mathys 2000; Bagnulo et al. 2002). In particular, magnetic topologies obtained by fitting magnetic moments in stars where primary diagnostic features, such as lines of Cr and Fe, are affected by abundance spots are expected to be especially unreliable.

5 Magnetic mapping in all four Stokes parameters

The magnetic DI code `Invers10` was originally developed for mapping surface structures in magnetic stars using spectra in all four Stokes parameters. Such observational data are still exceedingly rare and have been acquired with sufficient phase coverage only for a few bright magnetic CP stars in a pioneering study by Wade et al. (2000), who used MuSiCoS spectropolarimeter mounted at the Cassegrain focus of the 2-m telescope at the Pic-du-Midi Observatory. Enormous diagnostic potential of these unique four Stokes parameter data

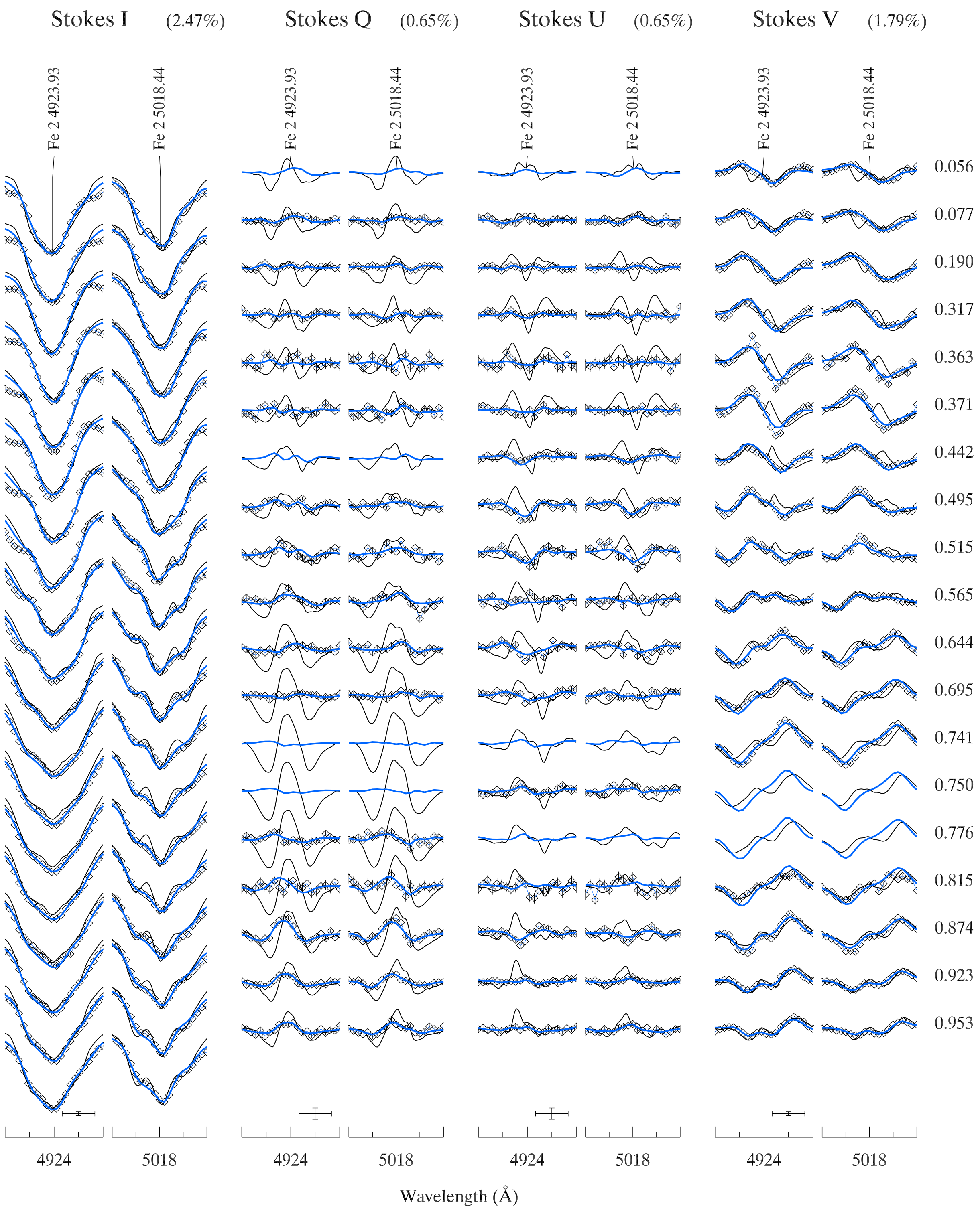

Figure 11: The comparison between observed (symbols) and synthetic (solid curves) four Stokes parameters of the multiplet 42 Fe II lines in the spectrum of 53 Cam. The thick grey curves show fits to the two Fe II lines obtained with the DI modelling of the magnetic field structure and iron abundance distribution. The thin black line shows the best fit to the Stokes profiles of the Fe II 4923.93 and 5018.44 Å lines obtained assuming that the field is given by the superposition of a dipole and an arbitrarily oriented non-linear quadrupole. The format of this figure is similar to Fig. 7.

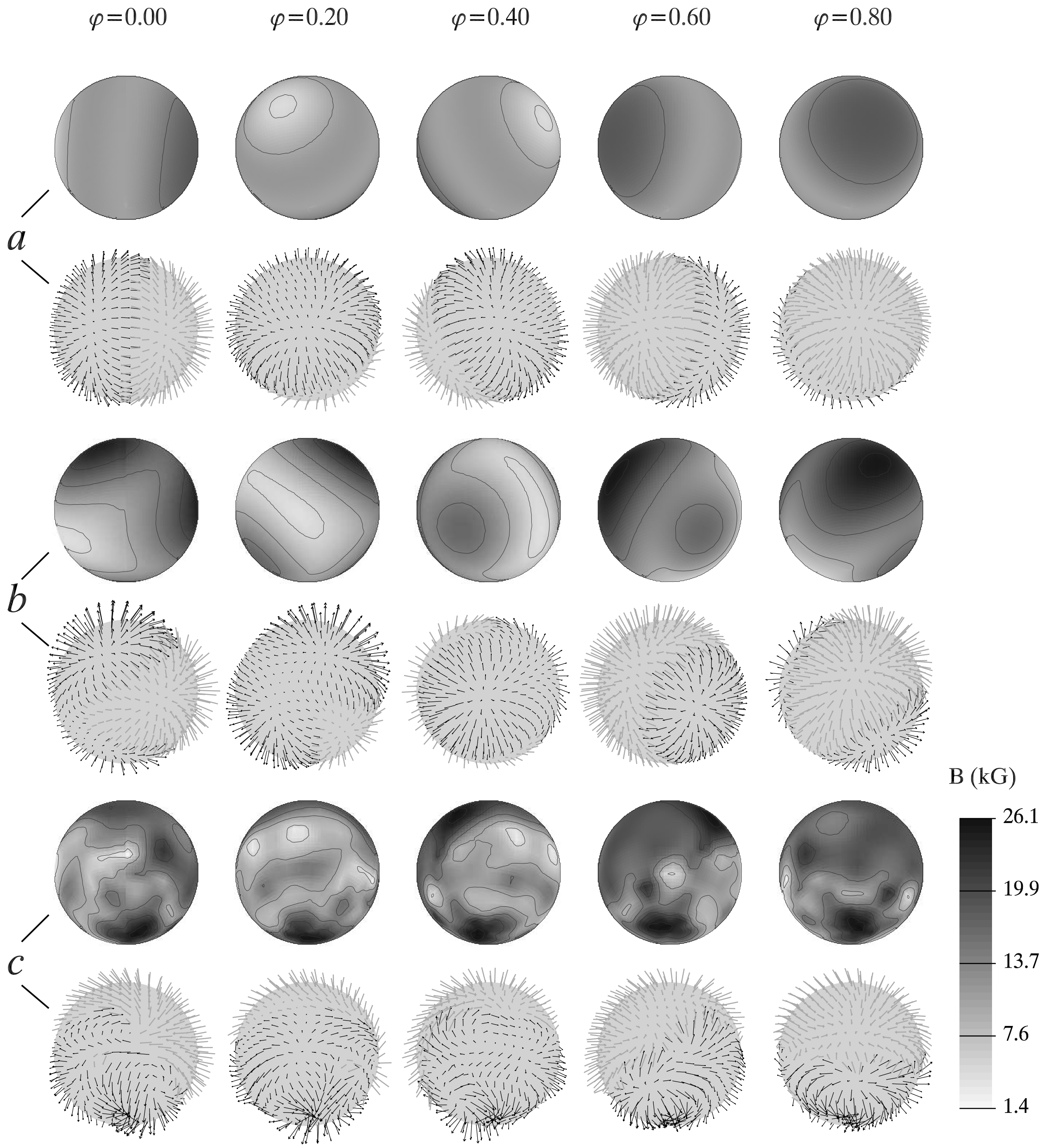

Figure 12: The comparison of multipolar magnetic models of 53 Cam found by Landstreet (1988) (**a**) and Bagnulo et al. (2001) (**b**) with the average magnetic distribution derived in the magnetic DI analysis with no *a priori* assumptions about field topology (Kochukhov et al. 2004) (**c**). This figure is analogous to Figs. 2 and 5. The structures in the greyscale maps of the field strength (the upper panel in each pair of rows) are highlighted with contour lines plotted for the field strengths between 5 and 25 kG with a step of 5 kG.

stimulated several researches to investigate to which extent observed linear polarization signatures in individual spectral lines agree with the predictions of simple multipolar magnetic models derived using exclusively Stokes I and V spectropolarimetry and, ocassionally, employing broad-band photopolarimetric measurements in Stokes Q and U.

The four Stokes parameter observations of 53 Cam were the first to be investigated in great detail. This

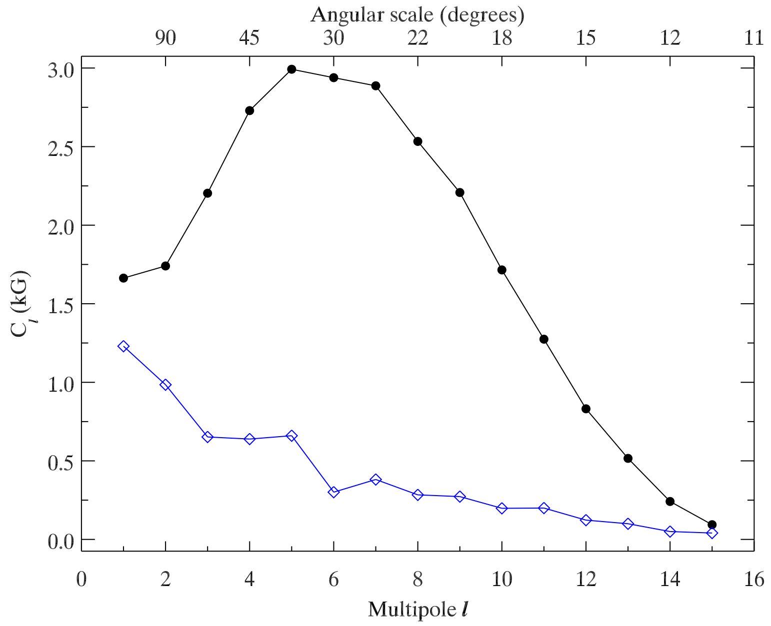

Figure 13: Results of the least-squares multipolar analysis of the magnetic DI map obtained for 53 Cam by Kochukhov et al. (2004). The poloidal (filled symbols) and toroidal (open symbols) expansion coefficients are averaged over azimuthal degree m and plotted as a function of ℓ and corresponding angular scale on the stellar surface.

star is often considered to have a well-studied magnetic geometry, since many investigators (e.g., Landstreet 1988) attempted to derive its field structure and abundance distributions by fitting various combinations of magnetic observables and using high-resolution unpolarized spectra. However, the unsatisfactory state of understanding the structure of the atmospheric magnetic field in this star became evident when Wade et al. (2000) and then Bagnulo et al. (2001) directly compared predictions of the two most sophisticated multipolar models with the MuSiCoS four Stokes parameter observations. Both models, which assumed multipolar parameterization of magnetic field, failed to reproduce details of the shape of the Stokes V spectra and displayed a major conflict with observations regarding the amplitude of the Stokes Q and U (see Fig. 8 in Bagnulo et al. 2001). This tentatively suggested that the magnetic topology of 53 Cam is more complicated or topologically very different from the magnetic models derived assuming multipolar magnetic geometry and using the moment technique.

This situation prompted us to attempt magnetic mapping with `Invers10`, which does not need to make assumptions about the global field geometry and derives magnetic field map directly from the Stokes line profiles. Modelling MuSiCoS four Stokes parameter observations allowed us to exploit full potential of the magnetic Doppler imaging method and to make our investigation the first attempt in the history of studies of stellar magnetic fields to map surface magnetic structures using spectra in all four Stokes parameters.

Fig. 11 shows the final fit achieved by `Invers10` to the Stokes $IQUV$ MuSiCoS observations of 53 Cam in the region of the Fe II 4923.93 and 5018.44 Å lines. Unfortunately, our choice of spectral lines used in magnetic inversions was necessarily restricted to these strong saturated spectral features because they are the only lines (together with the Fe II 5169.03 Å) showing conspicuous linear polarization signatures in the spectrum of 53 Cam. Despite all the difficulties (such as sensitivity to blending, continuum normalization errors and vertical abundance stratification) associated with the analysis of strong Fe II lines, the magnetic model of 53 Cam derived with the magnetic DI technique achieves a good fit to the observed intensity and the circular and linear polarization profiles of stellar spectral lines. This was not possible with earlier multipolar magnetic models based on modelling unpolarized spectra and fitting disk-integrated magnetic observables.

Also shown in Fig. 11 are the best-fit synthetic profiles corresponding to a multipolar model consisting of the superposition of a dipole and an arbitrarily oriented non-linear quadrupole magnetic components. This multipolar fit was obtained by S. Bagnulo using the code described by Bagnulo & Wade (2001). These model profiles fail to achieve acceptable description of the Stokes Q and U signatures of the Fe II lines, which once again emphasizes that the standard low-order multipolar axiomatic assumption is generally inadequate in modelling magnetic structures in CP stars.

Fig. 12 presents the spherical map of the average magnetic field topology of 53 Cam derived with `Invers10` using four Stokes parameter profiles of the three strong magnetically sensitive Fe II lines. The new DI magnetic model of 53 Cam appears to be strikingly different from the common notion of simple global magnetic morphology expected in the atmospheres of early-type stars. Our magnetic DI map is compared with the two most recent multipolar magnetic topologies of 53 Cam published by Landstreet (1988) and Bagnulo et al. (2001). None of these multipolar models is able to fit polarization spectra obtained with MuSiCoS. An adequate level of agreement between observations and spectrum synthesis can only be achieved by discarding traditional multipolar assumption and allowing arbitrary small-scale magnetic structures. Therefore, complexity of the magnetic map of 53 Cam inferred using the `Invers10` code is not a result of overinterpretation of the data, but a necessary requirement to fit linear polarization line profiles. The complexity of the magnetic distribution of 53 Cam is in agreement with the general trend of magnetic topologies of CP stars to deviate strongly from the pure dipole configurations (e.g., Bagnulo et al. 2002).

It is also important to note that despite fairly complex appearance of the field strength map in the DI image of 53 Cam, integration of this magnetic distribution over the stellar surface results in smooth variation of the mean longitudinal field and field modulus, in good agreement with published measurements of these quantities. Therefore, a simple form of the variation of magnetic observables does not invariably imply a low-order multipolar magnetic topology, as was assumed in many previous investigations of the magnetic field geometries of early-type stars.

The complexity of the field in 53 Cam can be qualitatively assessed with the expansion in the series of real spherical harmonics using the least-squares multipolar expansion technique introduced by Piskunov & Kochukhov (2002). Fig. 13 shows the amplitudes of the average toroidal and poloidal expansion coefficients as a function of angular degree. Apparently, the most important contribution to magnetic topology of 53 Cam comes from the harmonic components with $\ell \approx 5$–6, corresponding to structures at the angular scales 20–45°. Notably, contribution of low-order mutipoles is small and comparable to the strength of components with $\ell \approx 10$.

A detailed account of the magnetic Doppler imaging of 53 Cam in all four Stokes parameters can be found in the paper by Kochukhov et al. (2004). Our pioneering investigation of this star became an important step towards better understanding of the magnetic phenomena in peculiar A- and B-type stars. At the same time the magnetic DI map reconstructed with the MuSiCoS spectra is certainly not the last word in the 53 Cam story. It would be important to verify magnetic structures inferred with the application of the `Invers10` code by modelling four Stokes parameter data of better quality. In particular, observations with resolving power $R \geq 60\,000$–$70\,000$ (i.e., approximately twice that of MuSiCoS spectra) and $S/N \geq 300$ will help to investigate possible temporal variation of small-scale magnetic formations on the surface of 53 Cam and to extend magnetic modelling to a larger number of medium strength spectral lines.

Another fundamental question that we want to ask ourselves: does 53 Cam represent a typical example of the level of field complexity in chemically peculiar stars or a very special case. As it was shown in Sect. 4 and in the study by Kochukhov et al. (2002), magnetic DI analysis of the Stokes I and V spectra of α^2 CVn resulted in nearly dipolar magnetic field. On the other hand, substantially more complex field is revealed with the four Stokes parameter mapping of 53 Cam. One may suspect that the Stokes IV imaging of α^2 CVn is not able to reveal the true complexity of the field and hence is not much more trustworthy than previous multipolar models derived without detailed Stokes spectra modelling.

To verify the structure of magnetic field in α^2 CVn we analysed MuSiCoS four Stokes parameter spectra of this star. Fig. 14 shows comparison between observations and spectrum synthesis. Similar to the study of 53 Cam, in order to model meaningful linear polarization signatures we had to limit ourselves to very strong Fe II lines, which may be affected by vertical abundance stratification. Preliminary magnetic model derived from four Stokes parameters is presented in Fig. 15. This magnetic structure is also compared with the revised magnetic distribution obtained from the Stokes I and V SOFIN spectra. In the latter case multipolar regularization was used to constrain magnetic inversion, while in the former magnetic reconstruction no *a priori* assumptions were made, similar to mapping 53 Cam described above. The two magnetic maps differ in details, but, in general, the results of the two reconstructions are fairly consistent. Clearly, we don't see a striking evidence for very small-scale non-dipolar magnetic structures as in the case of 53 Cam. Thus, we come to the tentative conclusion that magnetic field of α^2 CVn is indeed close to dipolar and is radically different from the field geometry of 53 Cam. It seems that morphologies of magnetic fields in chemically peculiar stars span much larger range of complexity than thought before.

The four Stokes parameter mapping of α^2 CVn also confirms reliability of the magnetic DI based on

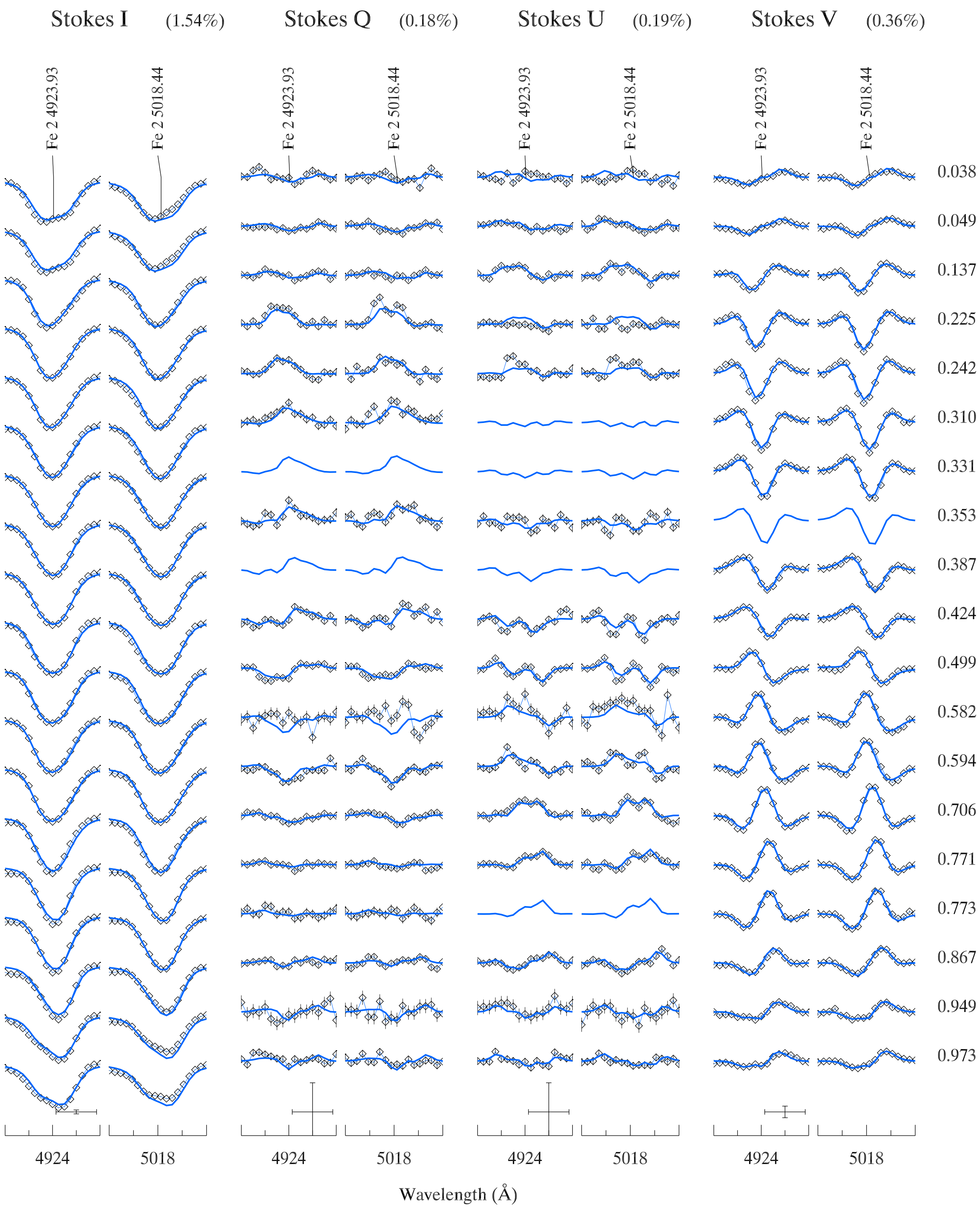

Figure 14: The comparison between four Stokes parameter MuSiCoS observations (symbols) and synthetic (solid curves) profiles of the Fe II 4923.93 and 5018.44 Å lines in the spectrum of α^2 CVn. The theoretical fit was obtained using magnetic DI code `Invers10` and making no *a priori* assumptions about field structure of α^2 CVn. The format of this figure is similar to Fig. 7.

high-resolution circular polarization spectra and using multipolar regularization – the method developed by Piskunov & Kochukhov (2002) and Kochukhov et al. (2002). Although not as efficient and spectacular as full four Stokes parameter inversion, *IV* magnetic mapping represents substantial step forward compared to the

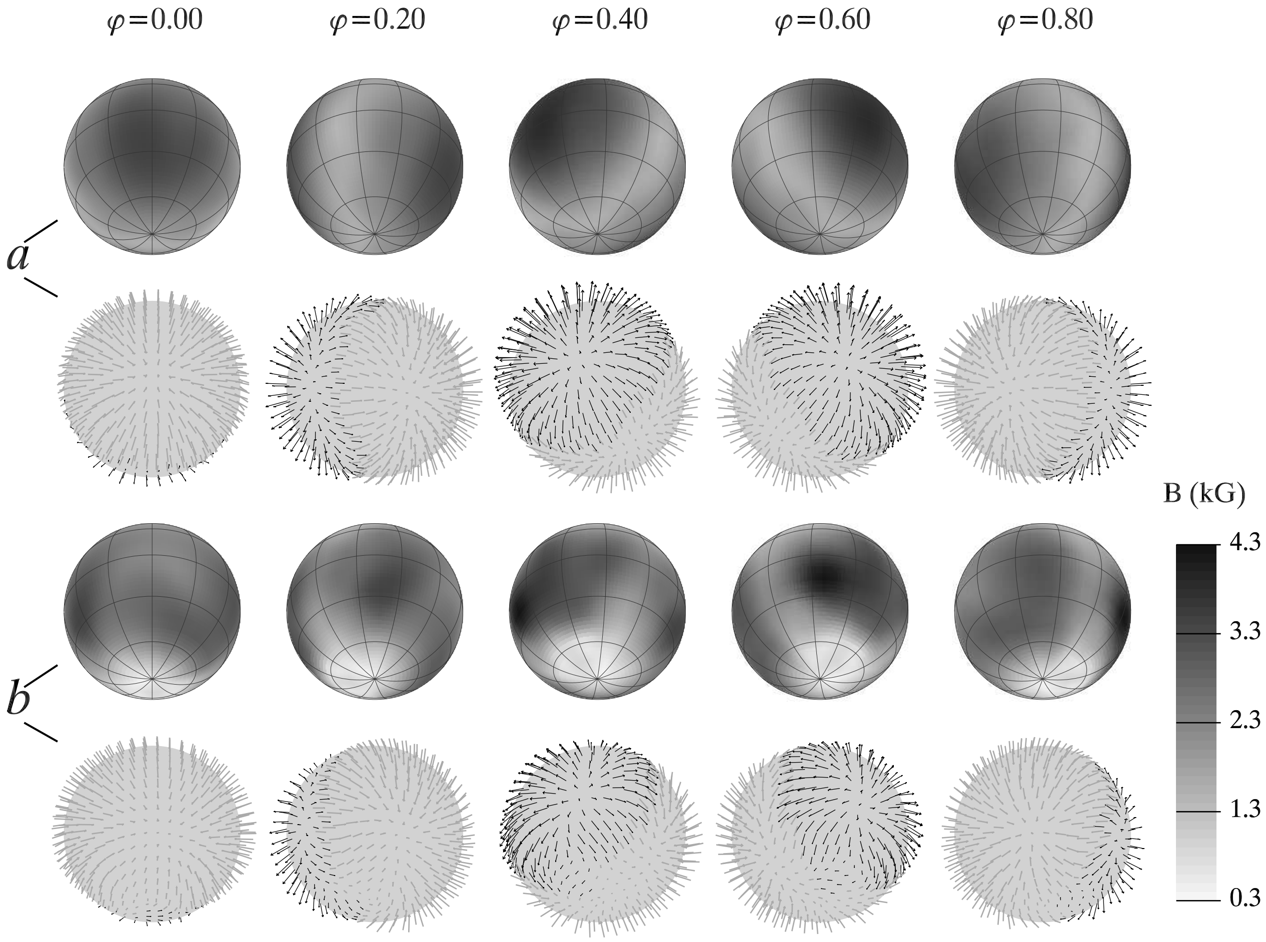

Figure 15: The comparison between magnetic field distributions of α^2 CVn obtained from the modelling of Stokes IV spectra (**a**) and all four Stokes parameters (**b**). The two upper panels show magnetic map derived by Kochukhov et al. (2002) from the high-resolution circular polarization SOFIN/NOT spectra and using magnetic Doppler mapping with multipolar regularization. The two lower panels illustrate results of the full four Stokes parameter inversion with `Invers10` based on Stokes $IQUV$ MuSiCoS spectra and without using external constraints on the possible field geometry.

practices of the moment technique and is certainly very useful in establishing magnetic geometry of CP stars with inhomogeneous abundance distribution.

6 Conclusions

The combination of unique spectropolarimetric observational data and magnetic Doppler imaging technique allowed us to obtain first detailed and self-consistent maps of magnetic fields and abundance inhomogeneities in CP stars, reveal unexpected complexity of magnetic structures in some CP stars and demonstrate the large diversity in the level of complexity of magnetic topologies in different magnetic stars. Indisputable success of our first attempts to obtain an in-depth knowledge about surface magnetic topologies in the upper main sequence stars ensures that extension of this modelling to a larger stellar sample, including other magnetic CP stars, active late-type stars and magnetic white dwarfs, will lead to a major breakthrough in understanding the stellar magnetism.

Taking into account the first very successful applications of magnetic Doppler imaging, we suggest, as a first step, to extend magnetic mapping to a larger sample of CP stars. This would allow us to perform a detailed study of the magnetic phenomena in hot stars and fill the missing link between observed stellar variability and underlying magnetic phenomena. With the help of magnetic Doppler imaging we plan to address a number of scientific problems including the most general and important question: what is the typical structure and the origin of stellar magnetic fields?

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