Astrophysical Dynamics, VT 2010

Gravitational Collapse and Star Formation

Susanne Höfner

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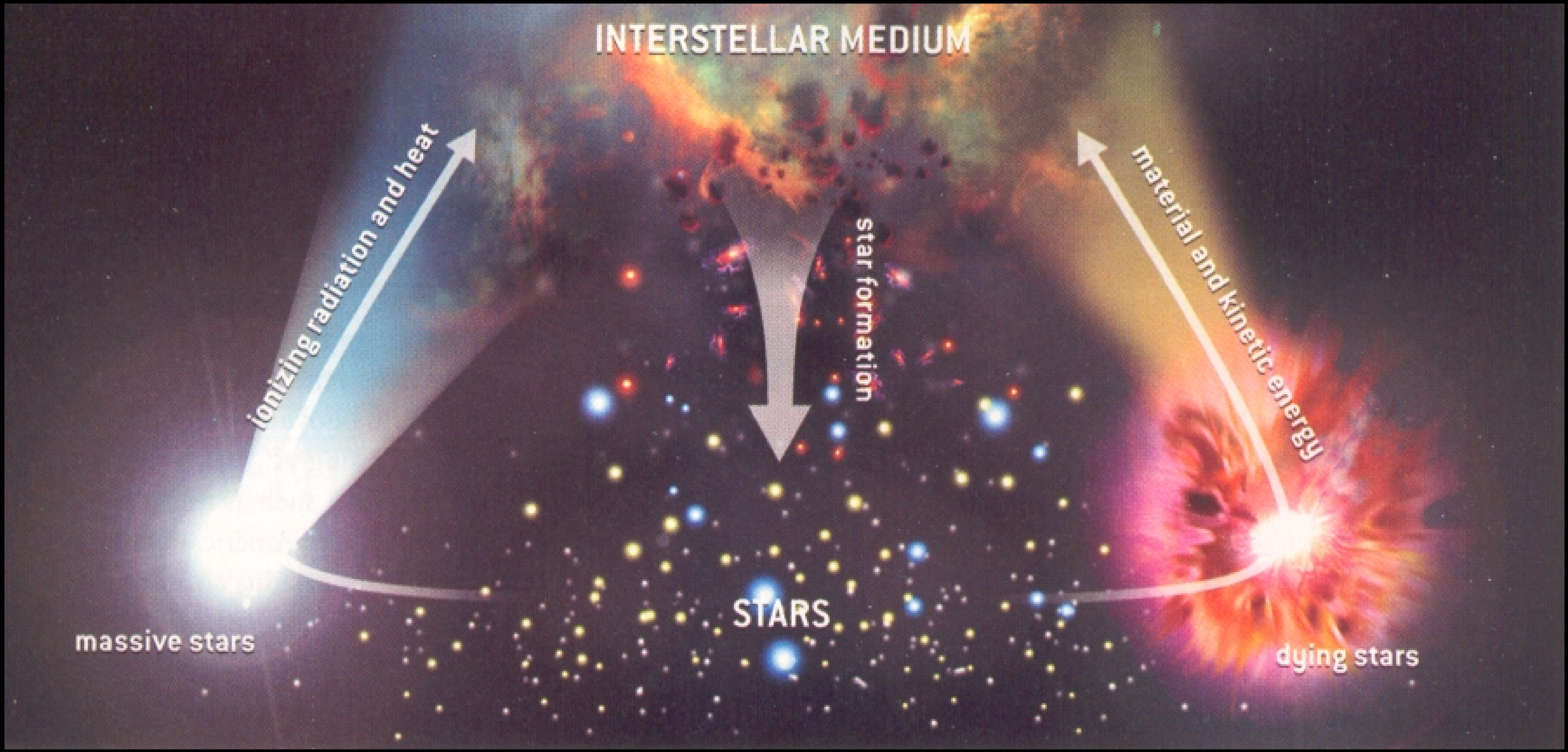
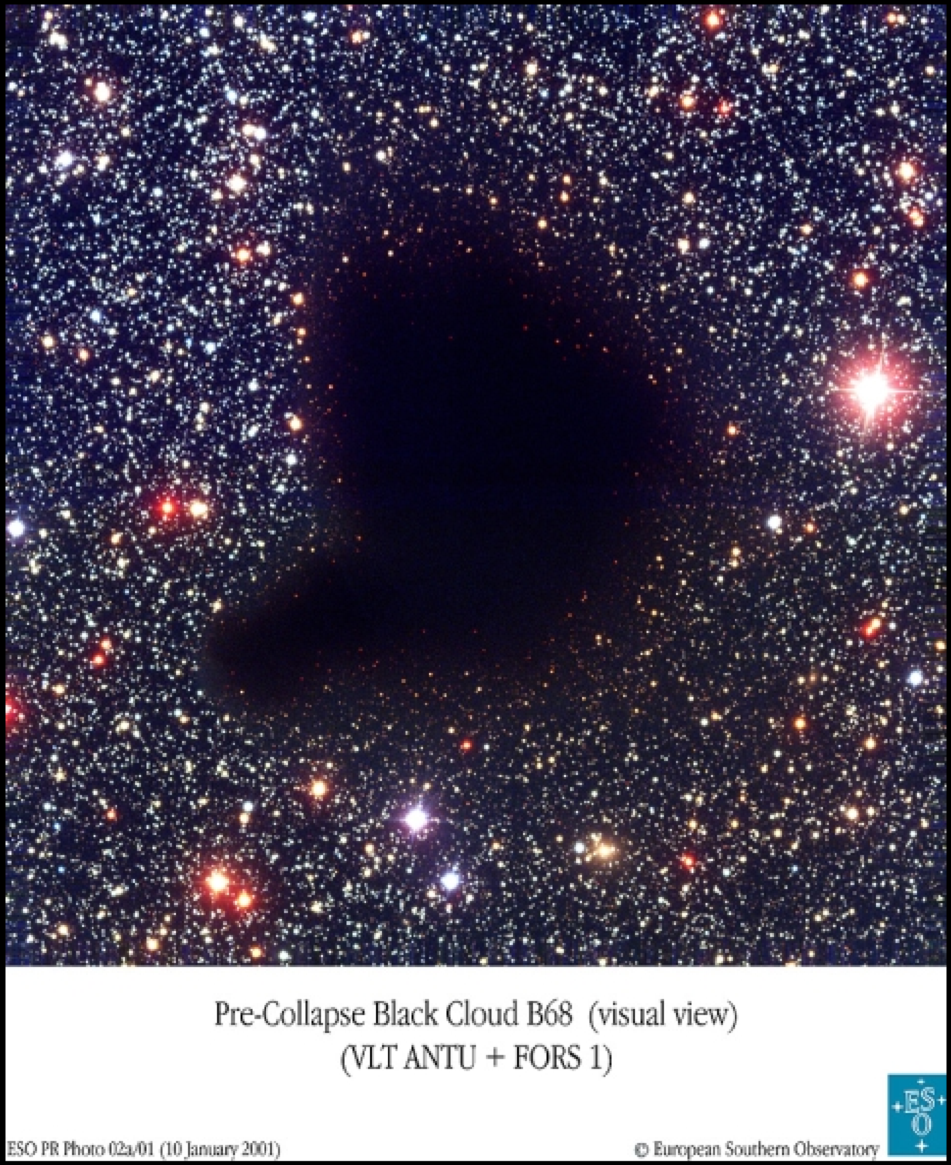
The Cosmic Matter Cycle



Dense Clouds in the ISM



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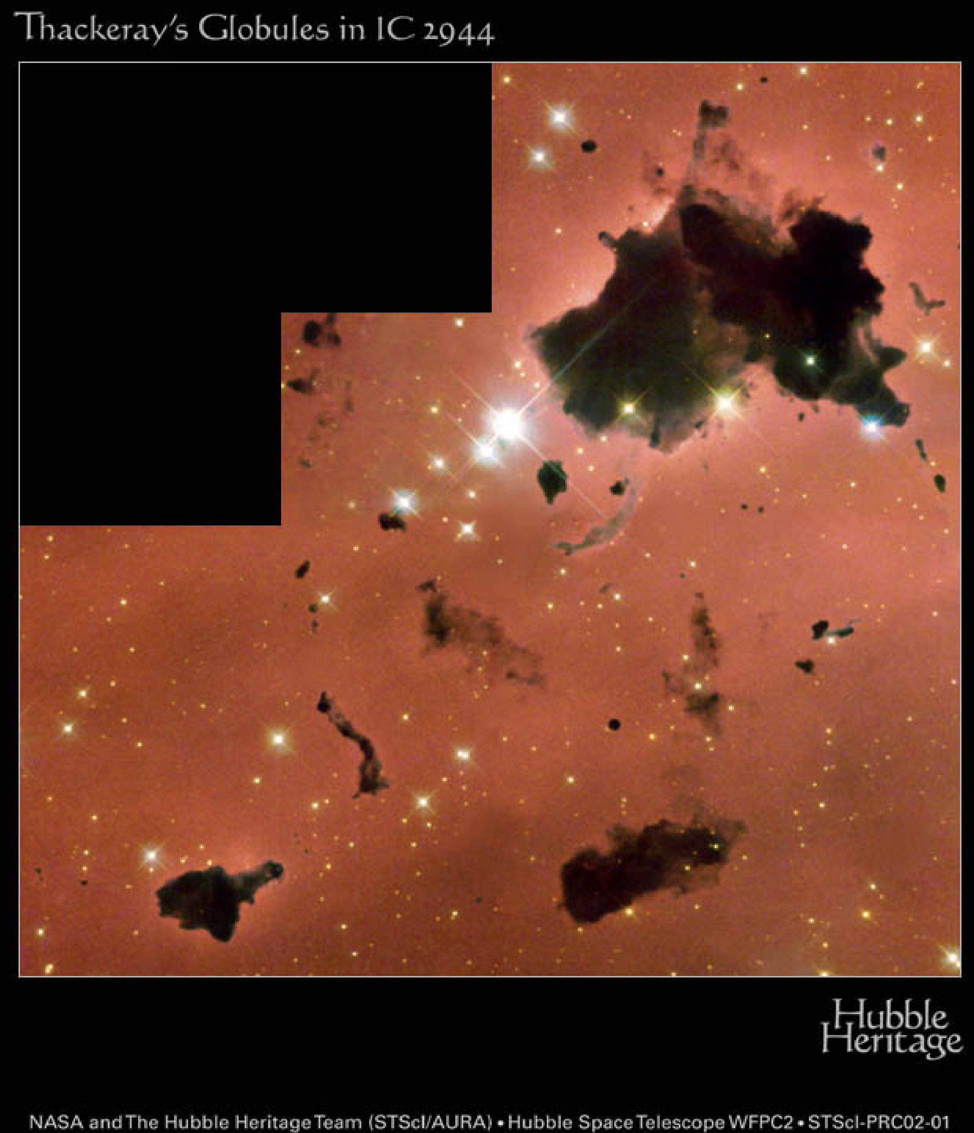

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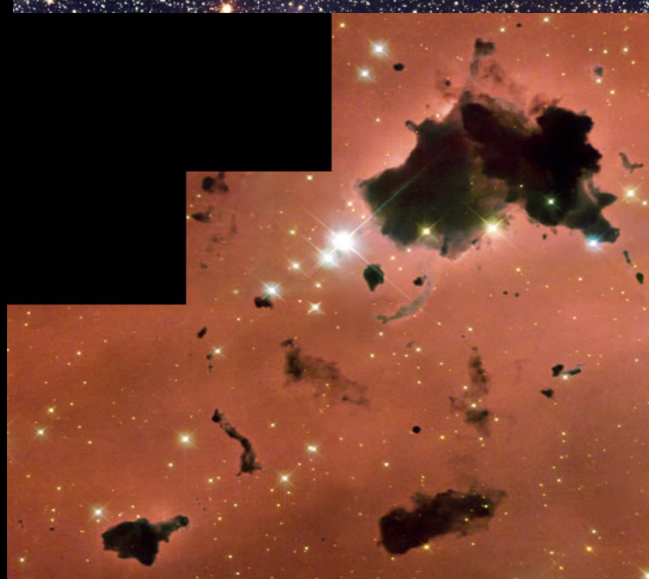

Is a cloud stable or
will it tend to collapse due to
gravitational forces ?

Dense Clouds: Stability or Collapse ?

1D, gravity included:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) = - \frac{\partial P}{\partial x} - \rho \frac{\partial \Phi}{\partial x}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = 4 \pi G \rho$$

isothermal gas

$$P = c_s^2 \rho$$

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isothermal gas

$$P = c_s^2 \rho$$

small-amplitude disturbances
in gas which initially is at rest
with constant pressure and density

$$P = P_0 + P_1(x,t)$$
$$\rho = \rho_0 + \rho_1(x,t)$$
$$u = u_1(x,t)$$
$$\Phi = \Phi_0 + \Phi_1(x,t)$$

Dense Clouds: Stability or Collapse ?

linearised equations:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial u_1}{\partial x} = 0$$

$$\frac{\partial u_1}{\partial t} = - \frac{c_s^2}{\rho_0} \frac{\partial \rho_1}{\partial x} - \frac{\partial \Phi_1}{\partial x}$$

$$\frac{\partial^2 \Phi_1}{\partial x^2} = 4\pi G \rho_1$$

isothermal perturbation

$$P_1 = c_s^2 \rho_1$$

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Dense Clouds: Stability or Collapse ?

linearised equations:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial u_1}{\partial x} = 0$$

linear homogeneous system
of differential equations
with constant coefficients
for ρ_1, u_1 and Φ_1

$$\frac{\partial u_1}{\partial t} = - \frac{c_s^2}{\rho_0} \frac{\partial \rho_1}{\partial x} - \frac{\partial \Phi_1}{\partial x}$$

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ρ_1, u_1, Φ_1 $\propto exp[\, i\,(\,kx + \omega t\,)\,]$

Dense Clouds: Stability or Collapse ?

linearised equations:

$$i\omega\rho_1 + \rho_0 i k u_1 = 0$$

$$i\omega u_1 = -\frac{c_s^2}{\rho_0} i k \rho_1 - i k \Phi_1$$

$$-k^2\Phi_1 = 4\pi G\rho_1$$

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Dense Clouds: Stability or Collapse ?

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Dense Clouds: Stability or Collapse ?

linearised equations:

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$$\frac{c_s^2}{\rho_0} k \rho_1 + \omega u_1 + k \Phi_1 = 0$$

$$4\pi G \rho_1 + k^2 \Phi_1 = 0$$

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Dense Clouds: Stability or Collapse ?

dispersion relation: $$\omega^2 = k^2 c_s^2 - 4\pi G \rho_0$$

$k^2 c_s^2 - 4\pi G \rho_0 > 0 \quad \Rightarrow$ ω real (periodic, stable)

$k^2 c_s^2 - 4\pi G \rho_0 < 0 \quad \Rightarrow$ ω imaginary (growing perturb.)

isothermal perturbation $$P_1 = c_s^2 \rho_1$$

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in gas which initially is at rest
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Dense Clouds: Stability or Collapse ?

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critical wavenumber: $$k_J = \left(\frac{4\pi \; G \; \rho_0}{c_s^2} \right)^{1/2}$$

or critical wavelength: $$\lambda_J = c_s \left(\frac{\pi}{G \; \rho_0} \right)^{1/2} \qquad (\lambda = 2\pi / k)$$

Dense Clouds: Stability or Collapse ?

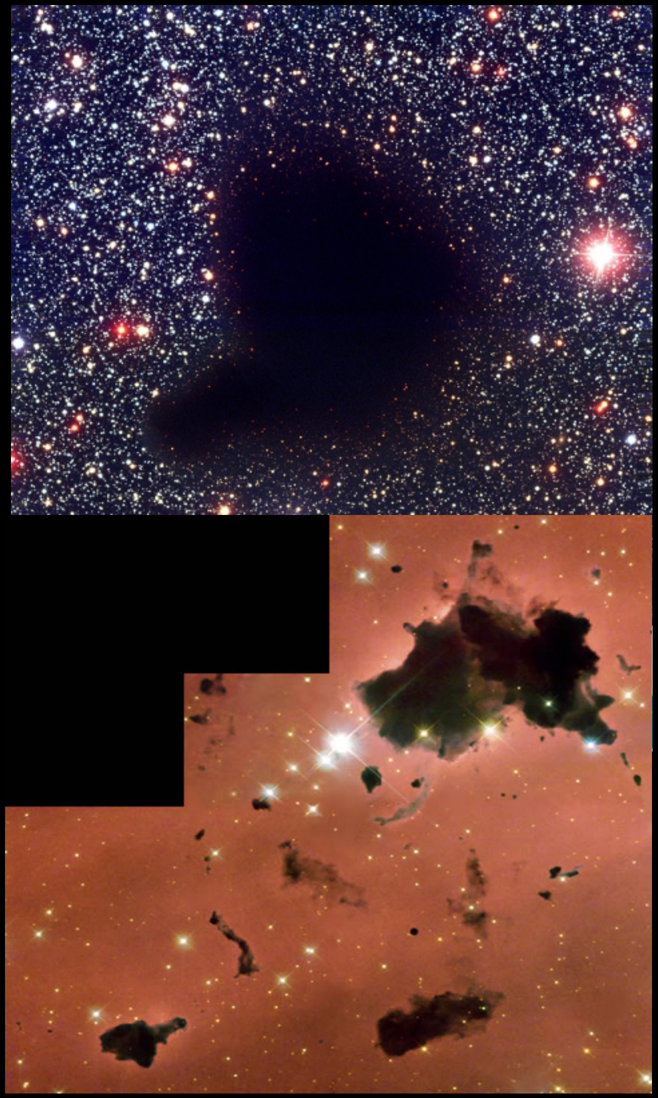
Jeans criterion: $$\lambda_J = c_s \left(\frac{\pi}{G \;\rho_0} \right)^{1/2} \qquad c_s = \left(\frac{P_0}{\rho_0} \right)^{1/2}$$

perturbations with $\lambda > \lambda_J$ are unstable (growing amplitudes) leading to collapse due to gravity

Jeans mass: $$M_J = \rho_0 \left(\frac{\lambda_J}{2} \right)^3$$

or $$M_J = \rho_0 \left(\frac{\pi \;k\; T_0}{4 \;\mu\; m_u \;G\; \rho_0} \right)^{3/2} \propto T_0^{3/2} \;\rho_0^{-1/2}$$

Dense Clouds in the ISM



Is a cloud stable or
will it tend to collapse due to
gravitational forces ?

Jeans criterion:

given density, temperature

$\Downarrow$

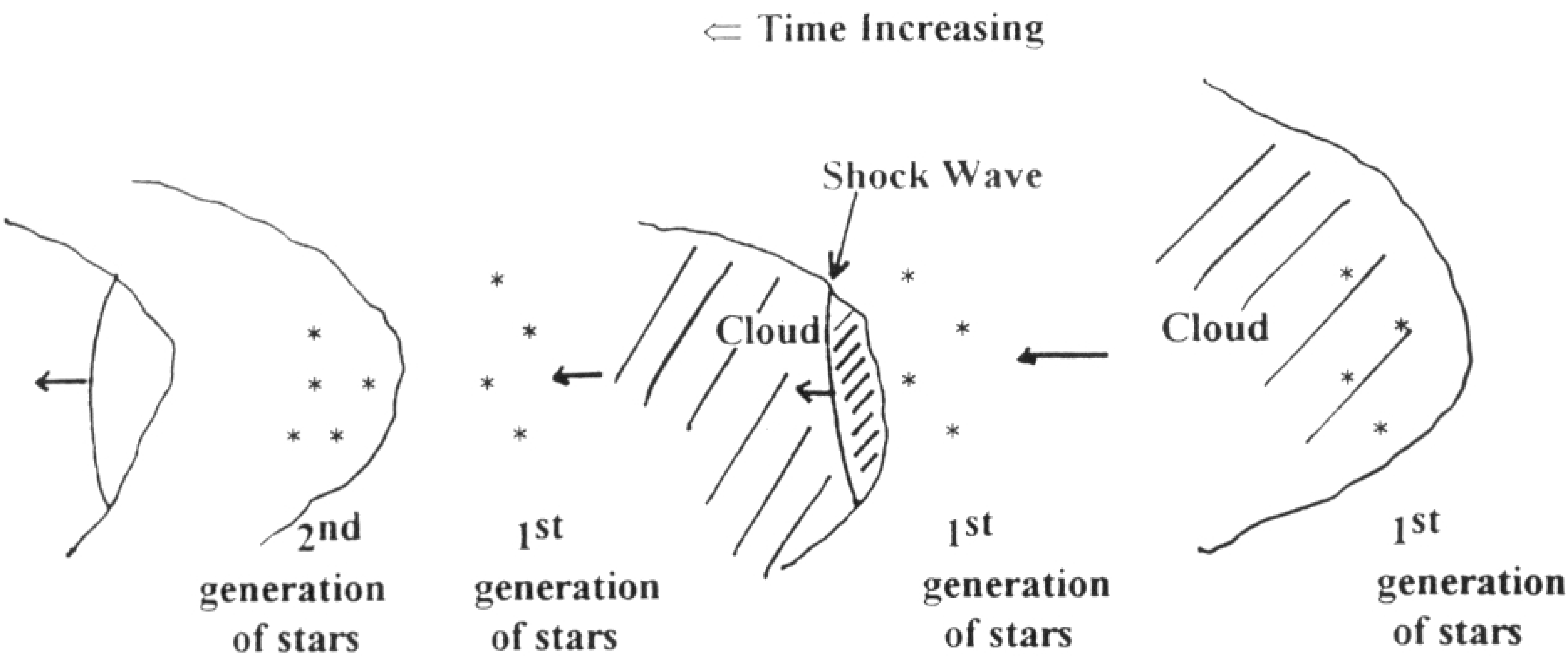
critical mass M_J
associated with a critical
length scale for a disturbance

Dense Clouds in the ISM

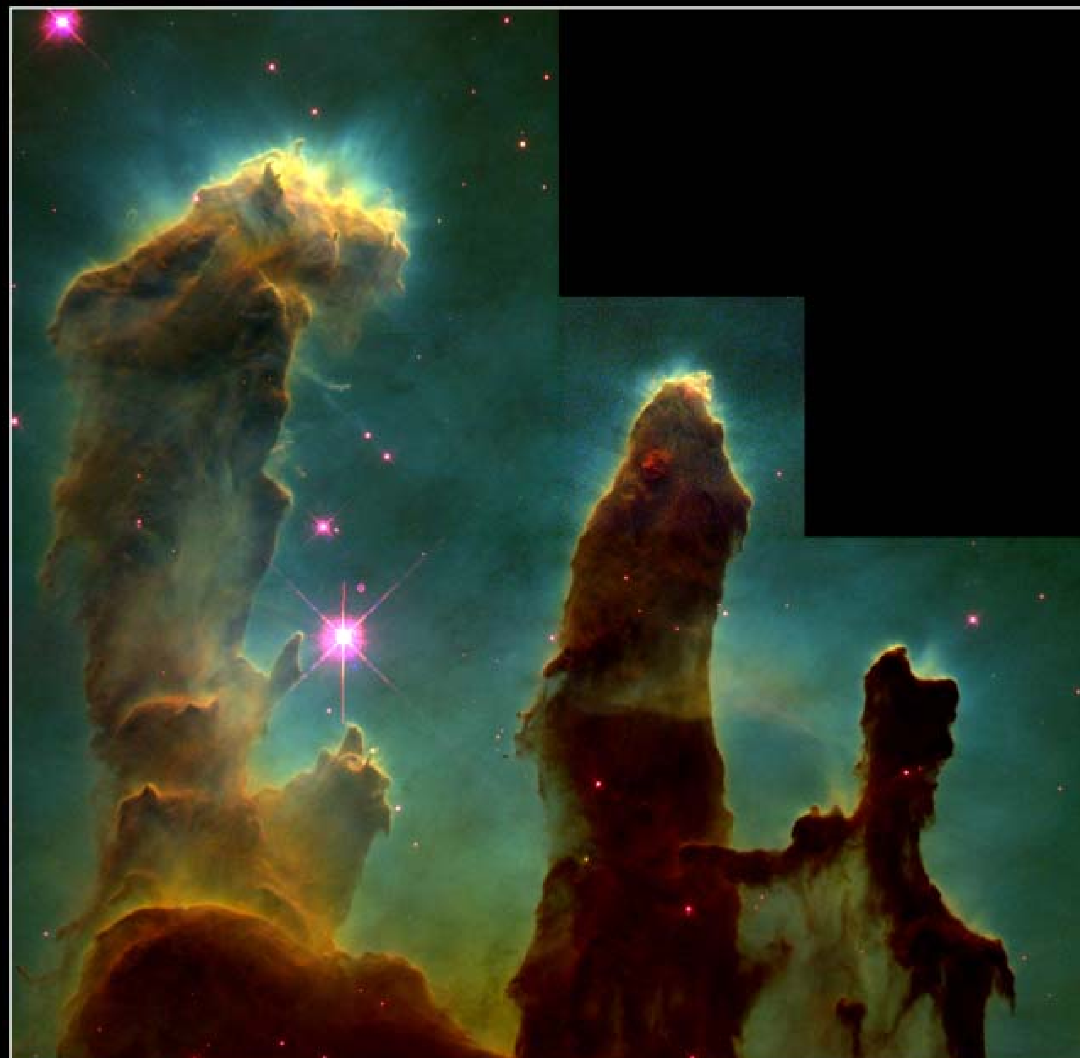
Table 19.1 Typical States of Gas in the Interstellar Medium

State of Gas	*Primary Constituent*	*Approximate Temperature*	*Approximate Density (atoms per cm^3)*	*Description*
Hot bubbles	Ionized hydrogen	1,000,000 K	0.01	Pockets of gas heated by supernova shock waves
Warm atomic gas	Atomic hydrogen	10,000 K	1	Fills much of galactic disk
Cool atomic clouds	Atomic hydrogen	100 K	100	Intermediate stage of star–gas–star cycle
Molecular clouds	Molecular hydrogen	30 K	300	Regions of star formation
Molecular cloud cores	Molecular hydrogen	60 K	10,000	Star-forming clouds

Star Formation in Dense Clouds



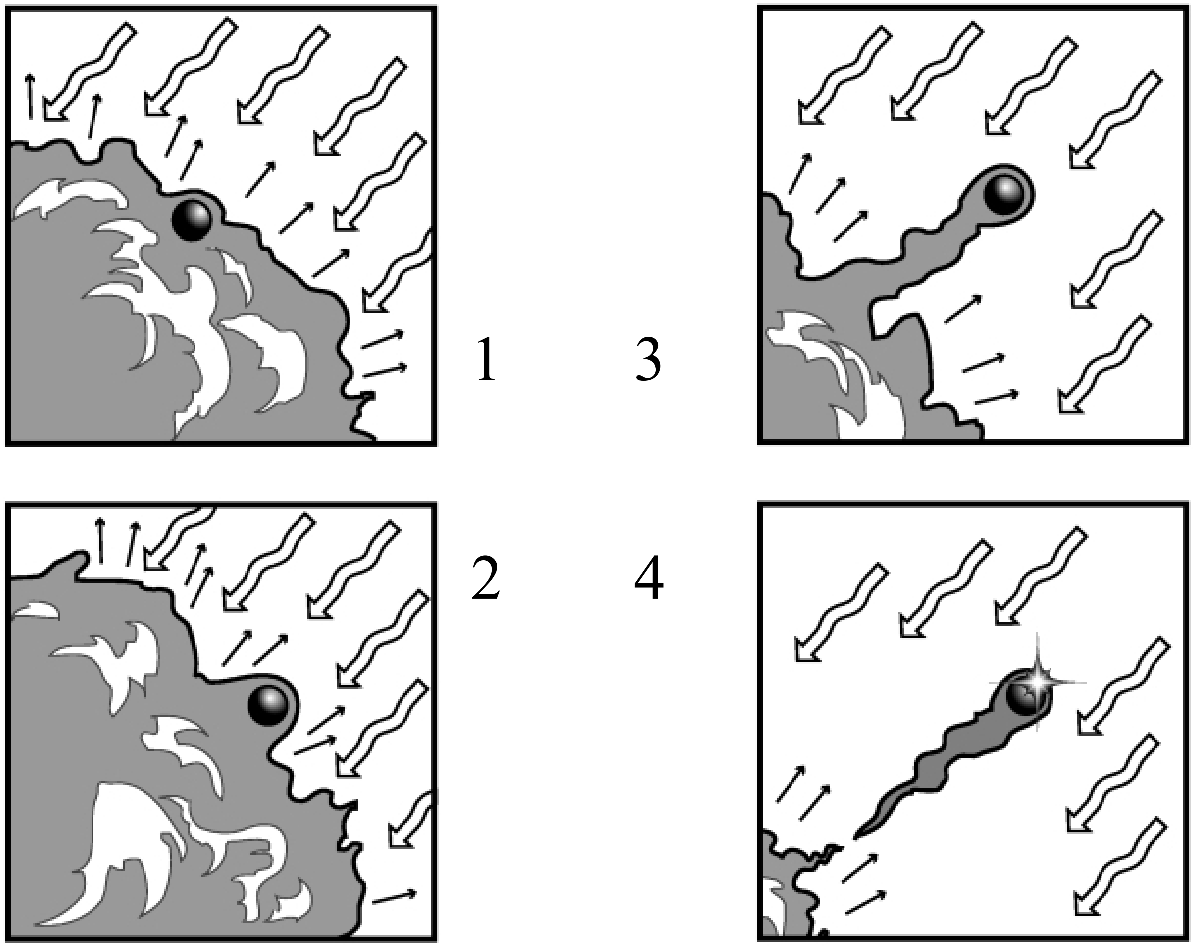
Star Formation in Dense Clouds



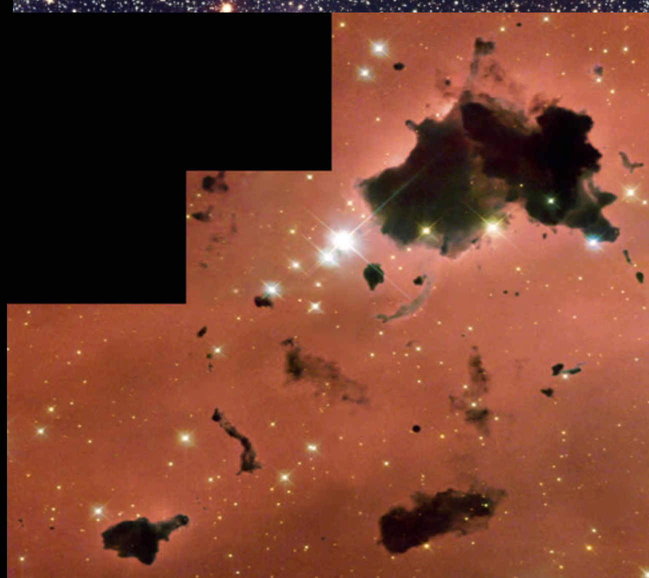
Gaseous Pillars · M16 HST · WFPC2

PRC95-44a · ST ScI OPO · November 2, 1995
J. Hester and P. Scowen (AZ State Univ.), NASA

Star Formation in Dense Clouds



Dense Clouds in the ISM



Is a cloud stable or
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Jeans criterion:

given density, temperature

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critical mass M_J
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How fast will
the cloud collapse ?

Typical Time Scale of the Collapse

Spherical cloud collapsing under its own gravity:

motion of mass shells determined by gravity (free fall), assuming that gas pressure and other forces can be neglected

$\Rightarrow$ free fall time

$$t_{ff} = \left(\frac{3\,\pi}{32\,G\,\rho_0} \right)^{1/2}$$

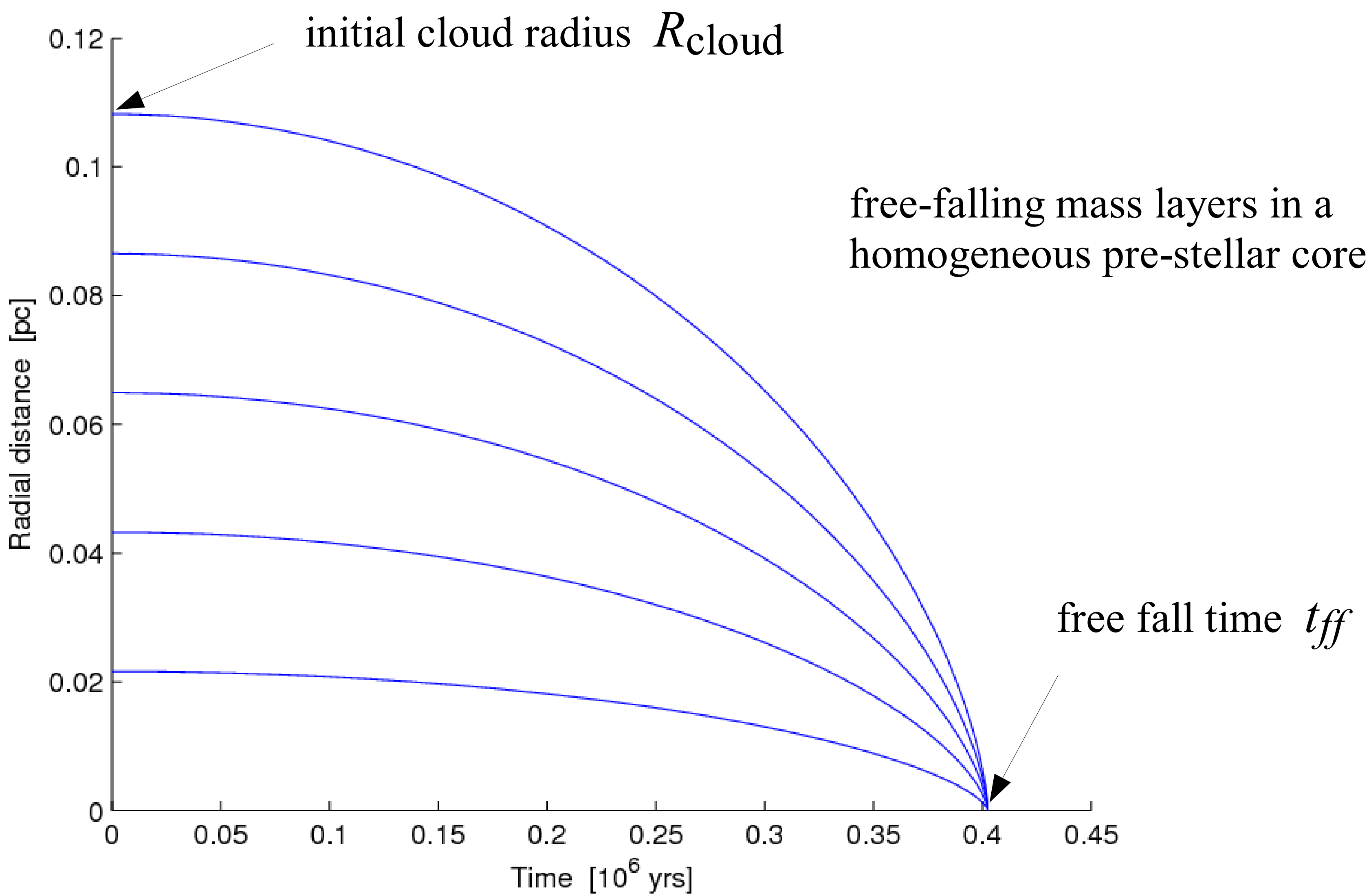
ρ_0 ... initial (mean) density of the cloud

$\rho_0 = M_{\mathrm{cloud}} / (4\pi R_{\mathrm{cloud}}^3 / 3)$

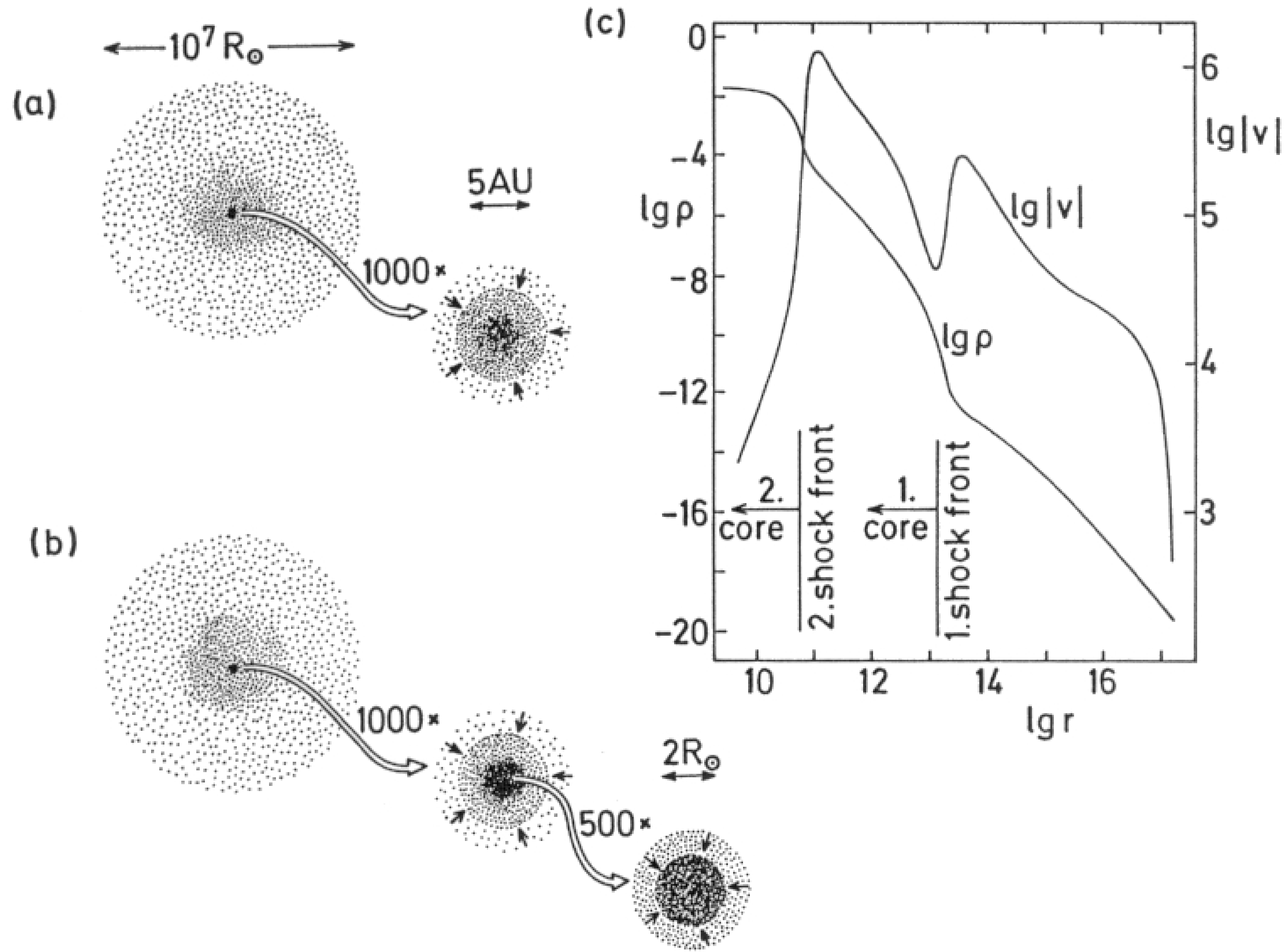
where R_{cloud} is the initial radius of the cloud

and M_{cloud} its total mass

Typical Time Scale of the Collapse



Protostellar Collapse



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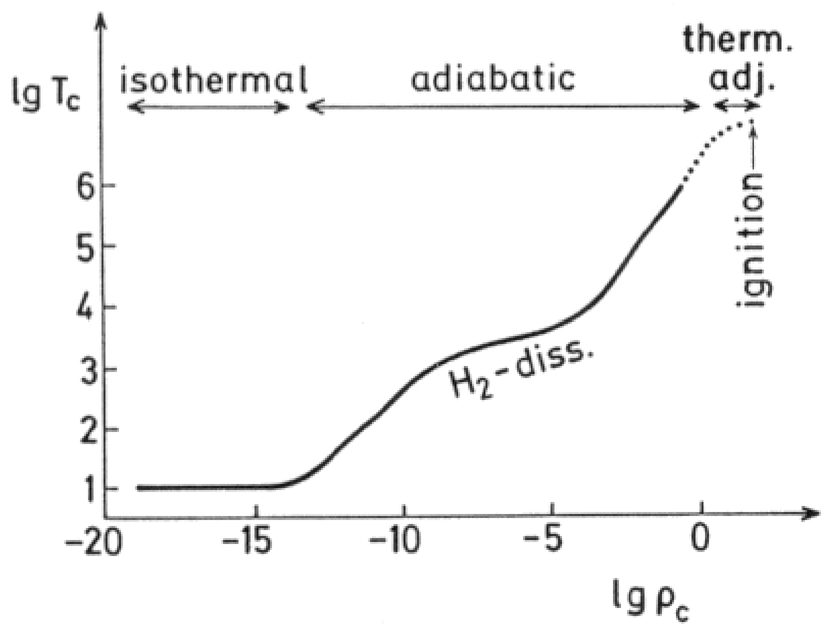
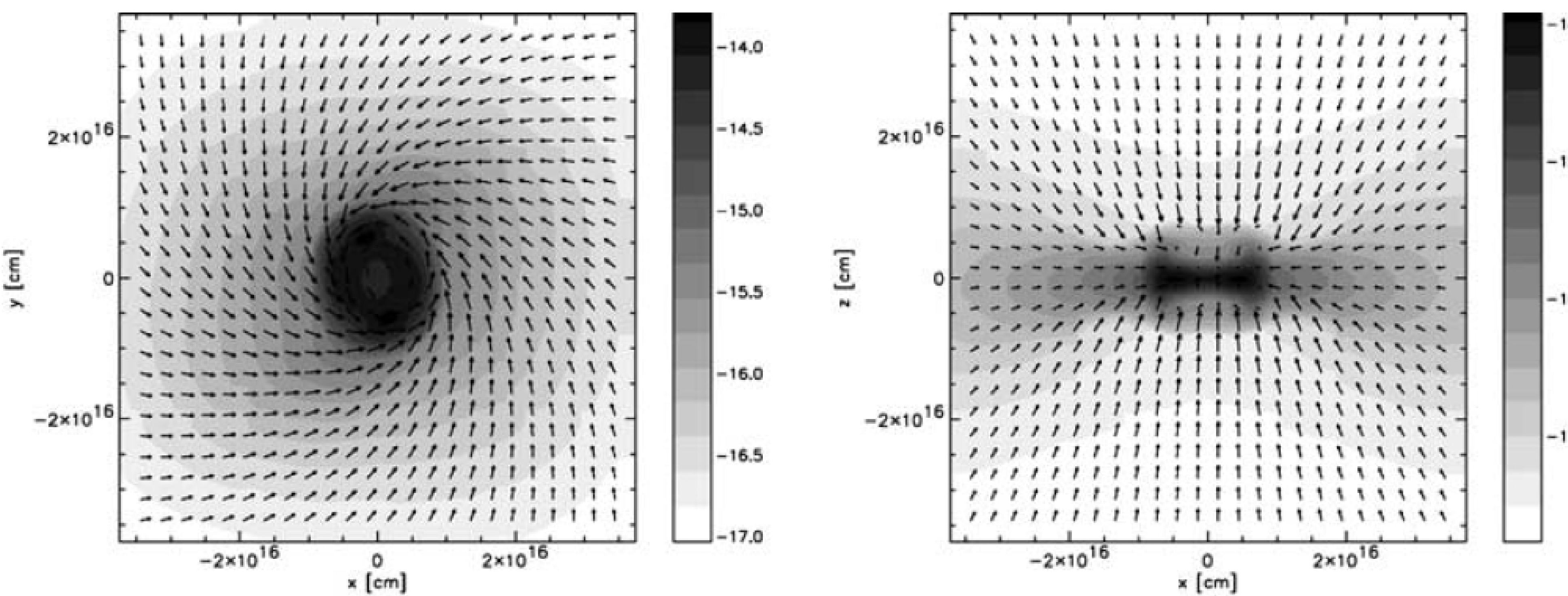


... but reality is more complicated than that:

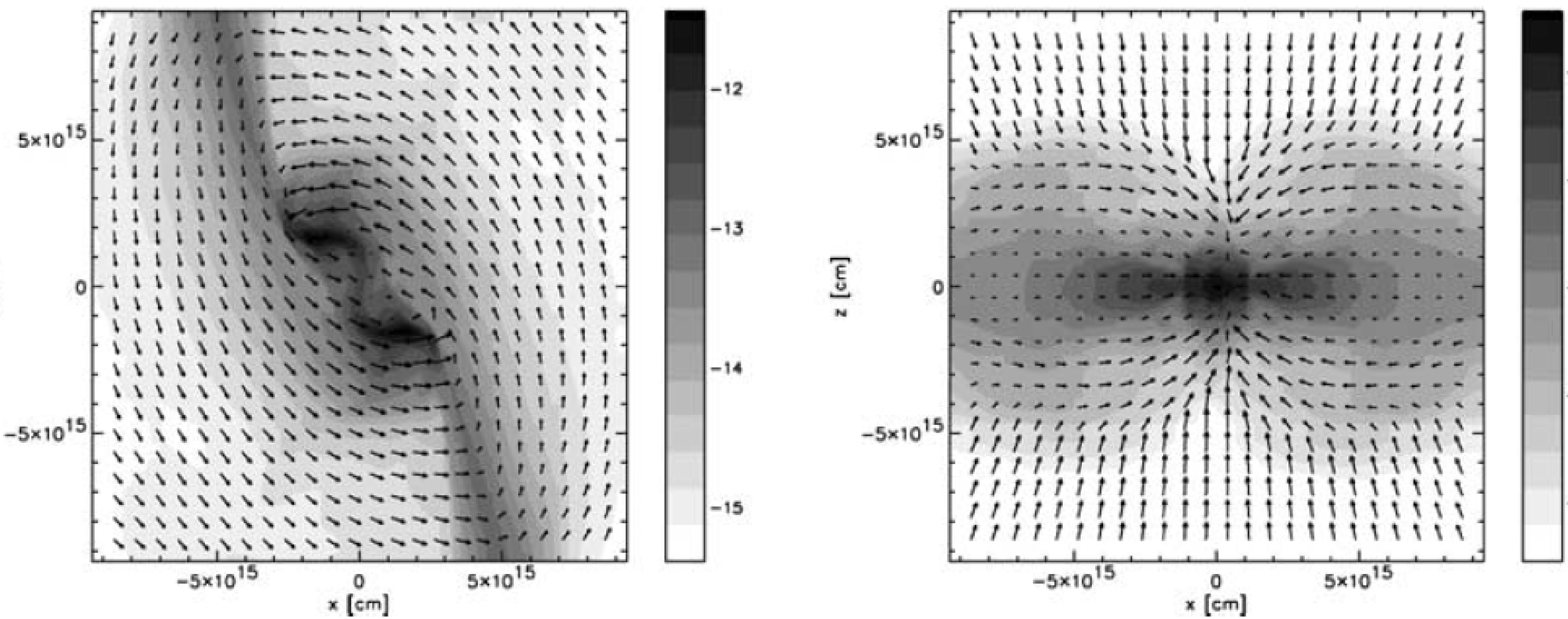
- influence of nearby stars (radiation, winds)
- rotation & turbulence on various scales
- young stellar objects: disks and jets ...

Protostellar Collapse



simulations by Banerjee et al.(2004):
density in the disk plane and perpendicular to it

Protostellar Collapse



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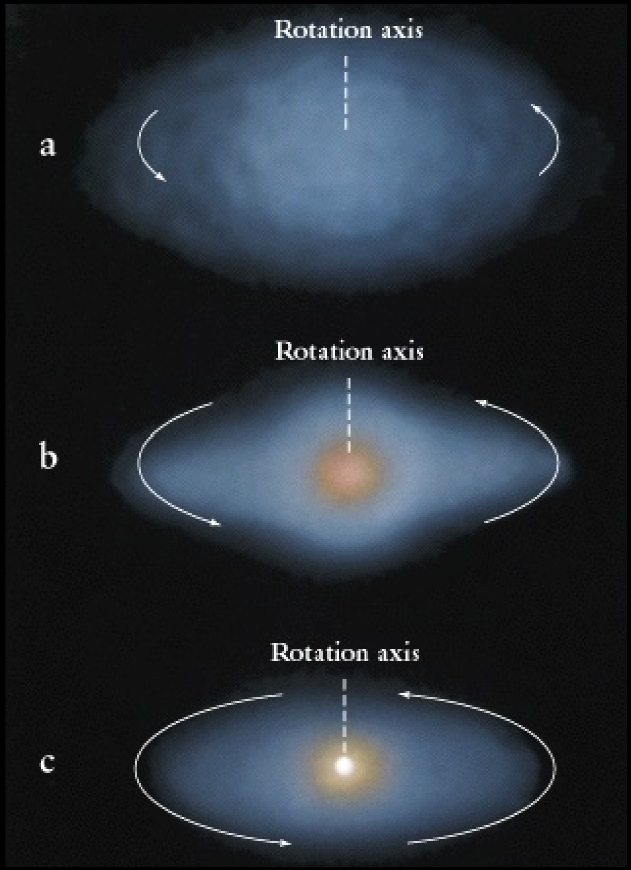
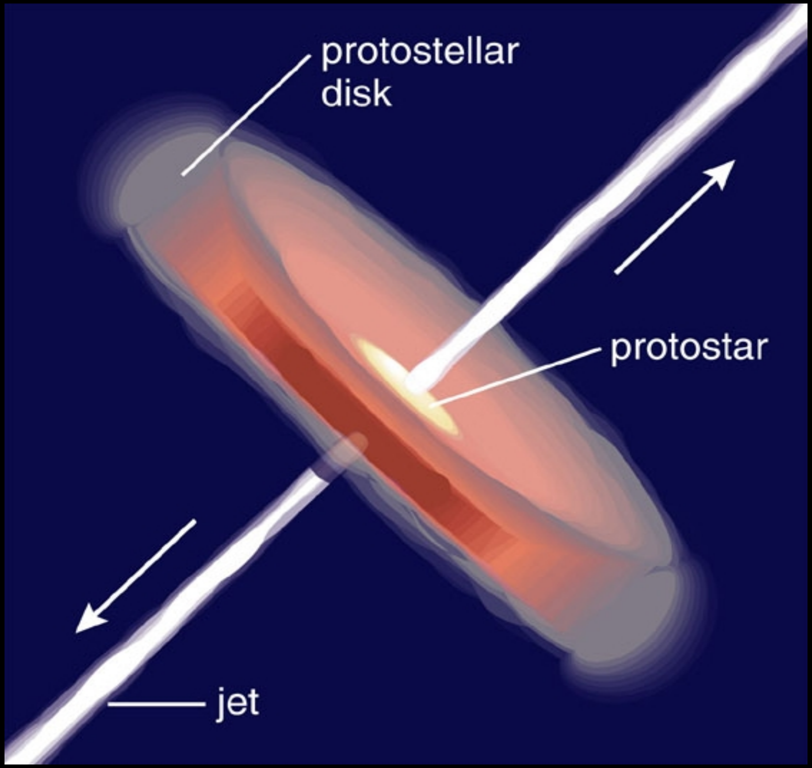
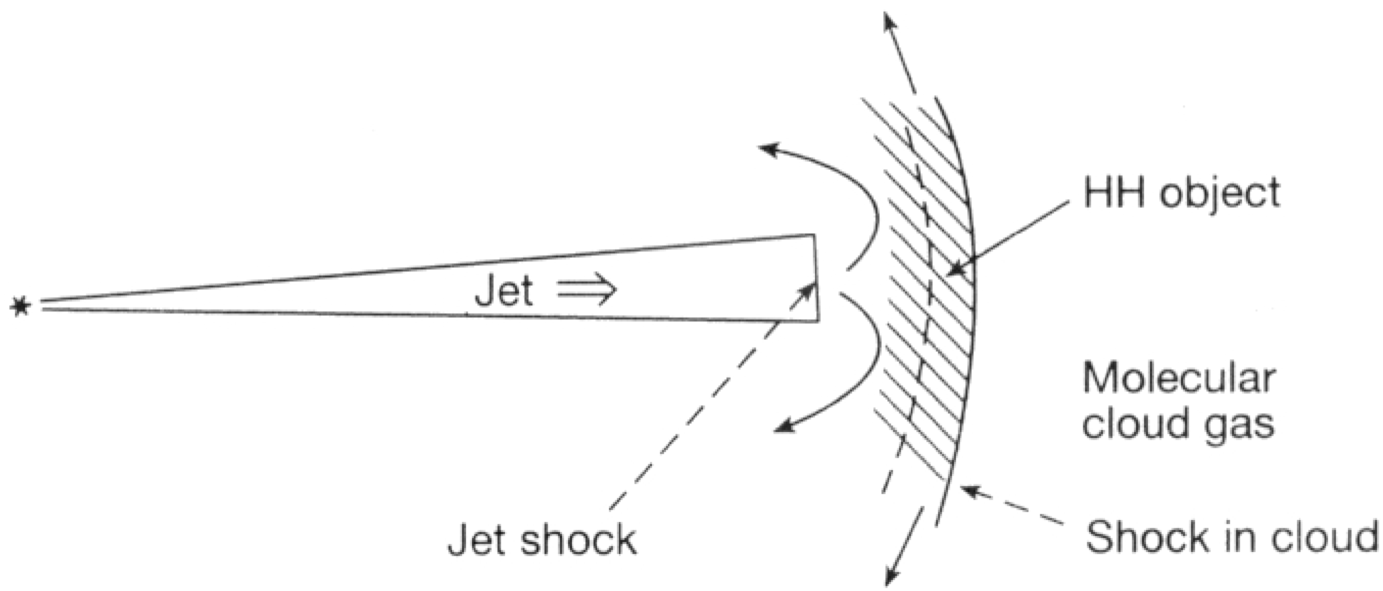
Young Stellar Objects: Disks & Jets



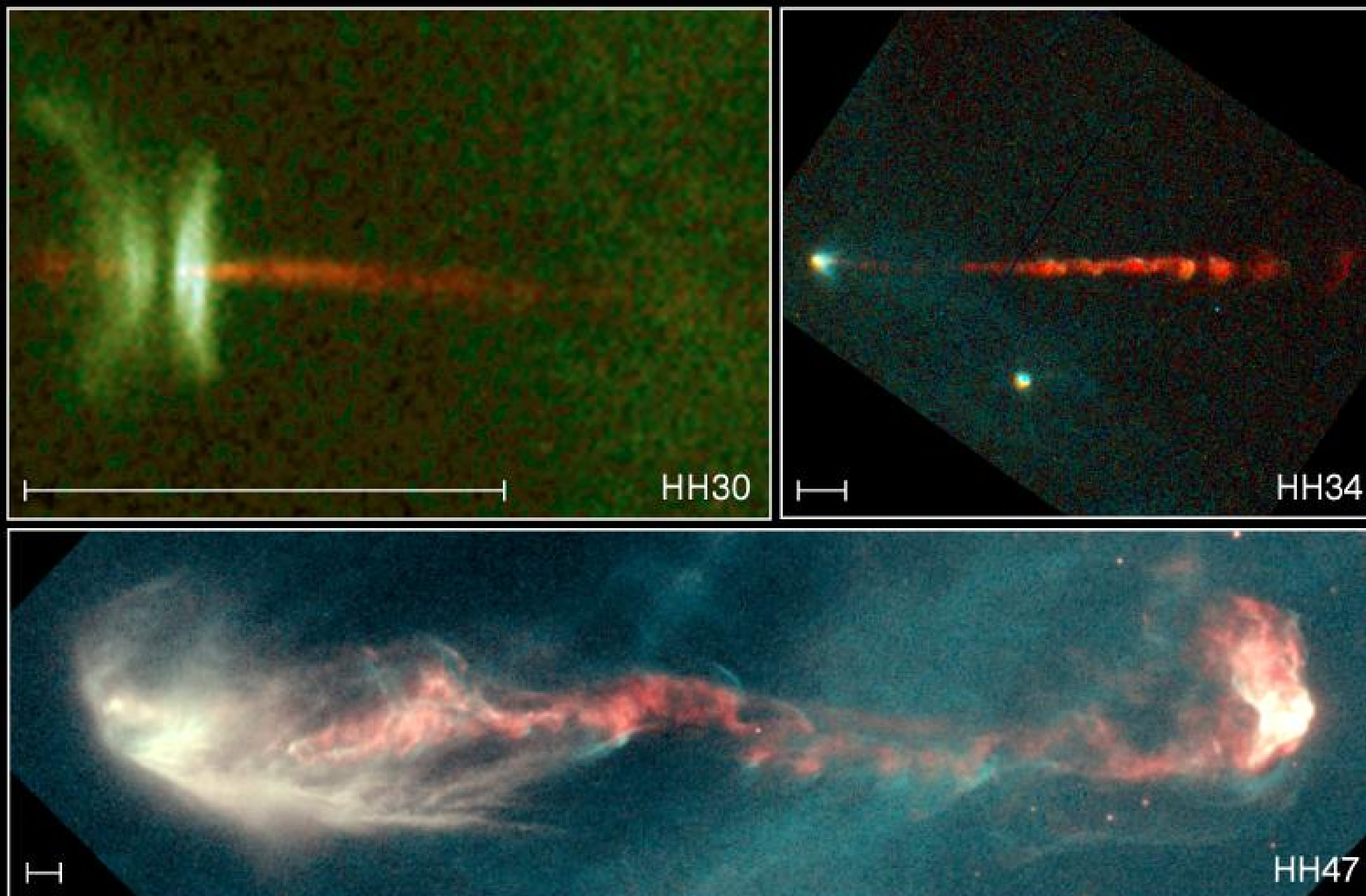


Young Stellar Objects: Disks & Jets



A jet model for the formation of Herbig–Haro objects. The dashed curve is the contact discontinuity which separates shocked cloud gas from shocked jet gas.

Young Stellar Objects: Disks & Jets



Jets from Young Stars HST · WFPC2

PRC95-24a · ST ScI OPO · June 6, 1995

C. Burrows (ST ScI), J. Hester (AZ State U.), J. Morse (ST ScI), NASA