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Exercise: A Toy Model for Dust-driven Winds

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Cool luminous giants stars, in particular pulsating AGB stars, show evidence of heavy mass loss through stellar winds with typical velocities of 10 – 30 km/s. The most likely driving mechanism for these outflows is radiation pressure on dust grains which form in the extended dynamical atmospheres of these stars. A realistic quantitative description of these phenomena requires detailed radiation-hydrodynamical modeling, including non-equilibrium dust condensation. The basic dynamical properties, however, can be demonstrated with the following simple toy model (see lecture notes for details):

We consider the dynamical evolution of a single fluid element from an instant when it has just been accelerated outwards by a passing atmospheric shock wave to the point where it is accelerated beyond the escape velocity, or starts falling back towards the stellar surface. Assuming that gravity and radiation pressure are the only relevant forces, the equation of motion (in a co-moving frame of reference) can be written as

$$\frac{du}{dt} = -g_0 \left(\frac{r_0}{r}\right)^2 (1-\Gamma) \qquad \text{with} \qquad \Gamma = \frac{\kappa_H L_*}{4\pi c G M_*} \tag{1}$$

where r is the distance from the stellar center, $u = dr/dt$ the radial velocity, $g_0 = GM_*/r_0^2$ denotes the gravitational acceleration at r_0, κ_H the flux mean opacity, and M_* and L_* are the stellar mass and luminosity, respectively (c = speed of light, G = gravitation constant). Atmospheric shocks due to stellar pulsation or large-scale convective motions set the initial velocity $u_0 = u(r_0)$ of the fluid element in the photosphere, located at $r_0 = R_*$, with the stellar radius defined by $R_* = \sqrt{L_*/(4\pi\sigma T_*^4)}$.

For given stellar parameters, the quantity Γ (ratio of radiative to gravitational acceleration) is determined by the flux-mean opacity κ_H which is a function of distance from the star. In our simple toy model we can distinguish two regimes: (a) the zone of hot dust-free gas close to the photosphere where κ_H is relatively low and, consequently, $\Gamma \ll 1$ (reducing the equation of motion to the free fall case), and (b) the cooler region beyond the dust condensation distance R_c (defined as the distance from the stellar center where the grain temperature is equal to the condensation temperature of the dust material) where κ_H may be dominated by the grain opacities and Γ may exceed the critical value of 1, leading to an outward acceleration.

Assuming a Planckian radiation field, geometrically diluted with distance from the stellar surface, and a power law for the grain absorption coefficient $\kappa_{\rm abs}(\lambda)$ in the relevant wavelength range (around the flux maximum of the stellar photosphere) the condensation distance R_c can be expressed as

$$\frac{R_c}{R_*} = \frac{1}{2}\left(\frac{T_c}{T_*}\right)^{-\frac{4+p}{2}} \qquad \text{where} \qquad \kappa_{\rm abs} \propto \lambda^{-p} \tag{2}$$

and R_* and T_* are the radius and effective temperature of the star. The condensation temperature of the dust T_c and the power law index p depend on the grain material (see Tab. 1 for an example).

In order to compute the total flux mean opacity κ_H which determines the radiation pressure and, consequently, Γ, it is necessary to know the combined opacity of all grains in a fluid element (total cross section per fluid mass). Introducing the efficiency $Q_{\rm rp} = C_{\rm rp}/\pi a_{\rm gr}^2$ (radiative cross section $C_{\rm rp}$ divided by geometrical cross section for a grain of radius $a_{\rm gr}$) and assuming that all grains in a given fluid element have the same size, the total dust opacity can be written as

$$\kappa_{\rm rp}(\lambda, a_{\rm gr}) = \frac{3}{4}\ \frac{A_{\rm mon}}{\rho_{\rm grain}}\ \frac{Q_{\rm rp}(\lambda, a_{\rm gr})}{a_{\rm gr}}\ \frac{\varepsilon_c}{1 + 4\,\varepsilon_{\rm He}}\ f_c\,. \tag{3}$$

where $A_{\rm mon}$ and $\rho_{\rm grain}$ are the atomic weight of the monomer (basic building block of the dust material) and the density of the grain material, respectively. The abundance of the key element of the condensate is denoted by ε_c, the degree of condensation of this key element by f_c, and $\varepsilon_{\rm He}$ represents the abundance of He (see lecture notes for details).

This way of writing the dust opacity has the advantage of distinguishing between material properties (constants and optical properties) and the chemical composition of the atmosphere/wind, and allows to take into account that $Q_{\rm rp}(\lambda, a_{\rm gr})/a_{\rm gr}$ (and therefore $\kappa_{\rm rp}$) becomes independent of grain size for particles much smaller than the wavelength λ. For a given grain material and stellar element abundances, the only unknown quantity in this expression of the opacity is the degree of condensation f_c (i.e. the fraction of available material actually condensed into grains). As dust formation in AGB stars usually proceeds far from equilibrium, f_c, in general, needs to be calculated with detailed non-equilibrium methods. Assuming a reasonable value for f_c, however, this equation allows to estimate typical values of the dust opacity, and, consequently, Γ.

For the purpose of our simple toy model, the flux mean opacity in the dusty wind (at $r \geq R_c$) can be approximated by $\kappa_H = \kappa_{\rm rp}(\lambda_{\rm max})$ where the wavelength $\lambda_{\rm max}$ corresponds to the flux maximum of the star. In the dust-free hot gas we assume $\kappa_H = 0$ for simplicity. Consequently, Γ is approximated by a step function with $\Gamma = 0$ for $r < R_c$ and $\Gamma =$ constant (> 0) for $r \geq R_c$ where the value of Γ is computed from the expression in Eq. (1) with κ_H given by the dust opacity.

Assignment

1. **Write a MATLAB code which solves the equation of motion for the toy model of a dust-driven wind** presented above and which plots the resulting trajectories. *Hints*: Note that this problem has similarities with the 'Free Fall' homework which corresponds to the special case of Eq. (1) with $\Gamma = 0$ and $u_0 = 0$). Use the stellar radius R_*, as well as the free fall time and the escape velocity at $r_0 = R_*$ for scaling the distance, time and velocity during numerical integration of the equations (i.e., use dimensionless equations for numerical reasons, as in the free fall example; see *FreeFall.m, track_ff.m*). The step function for Γ can, e.g., be implemented as an if-statement which switches off the Γ-term for $r < R_c$ in the function defining the ODE system (replacing the free-fall version *track_ff.m*).

2. Start with a version using **given values of** Γ **and** R_c and **test the code** by comparing the results to the plots shown in Fig. 1 (see caption for parameter values). Integrate in time to 25 free fall times (about 10.5 years).

3. Implement the formulae given above to **compute** Γ **(**κ_H**) and** R_c **from typical properties of amorphous carbon grains** (see Tab. 1). Compute a trajectory using these new values for Γ and R_c, keeping the same stellar parameters and initial conditions as for the test runs (see figure caption). Compare the result with the test cases and discuss the similarities and differences.

4. Give an **astrophysical interpretation** of the result for the wind model with Γ and R_c based on typical grain properties (see previous item): Plot the velocity in km/s as a function of (a) time in years, (b) distance from the stellar center in units of the stellar radius R_*, (c) same as (b) but distance in astronomical units (to put wind dynamics into a more familiar perspective). Address the following points in your discussion/report:

   - Explain the velocity structure of the wind in terms of forces acting on the fluid element (declining/increasing velocities, leveling out at large distances).
   - Which planetary orbits in the present solar system would be at distances corresponding to the regions inside the star, inside the dust-free dynamical atmosphere, or in the dusty wind?
   - How long would it take until the fluid element reaches the distances of Jupiter, Saturn and Pluto, and what are the values of the flow velocities at these points?
   - Where does the wind exceed the local escape velocity $u_{\rm esc}(r) = \sqrt{2GM/r}$, and is the wind velocity at the distance of Pluto's orbit comparable to typical wind velocities of AGB stars?

5. Test the **influence of stellar pulsation** on wind dynamics by varying the initial velocity u_0. Plot the trajectories and the velocity structures as described above for $u_0/u_{\rm esc} = 0.95$, 0.8 (standard value) and 0.75 (use lower values at your own risk). How does this change the minimum and the final velocity of the flow?

6. Present and discuss the results in the **dedicated exercise session** (week 20).
7. Hand in in a **written report** containing
   - a brief introduction (astrophysical background, toy model)
   - a short description of the applied methods (including tests of the code)
   - a discussion of the results (including plots, comments on the choice of model parameters, comparison of the different cases, interpretations and explanations)
   - a brief summary (conclusions)

   to Susanne Höfner (preferably as a single PDF file, by email) by the end of the course (beginning of June).

Table 1: Typical properties of amorphous carbon grains (which are probably driving the winds of C-rich AGB stars) and of the surrounding gas.

symbol	value (SI)	comment
T_c	1500 [K]	condensation temperature
p	1	power law index of the absorption coefficient
$Q_{\rm rp}/a_{\rm gr}$	$2 \cdot 10^6$ [1/m]	efficiency/grain radius near the flux maximum
$A_{\rm mon}$	12	atomic weight of the monomer (C)
$\rho_{\rm grain}$	$1.85 \cdot 10^3$ [kg/m^3]	density of the grain material
ε_c	$3.3 \cdot 10^{-4}$	abundance of excess carbon not bound in CO
$\varepsilon_{\rm He}$	0.1	abundance of He (by number)
f_c	0.2	degree of condensation ($0 \leq f_c \leq 1$)

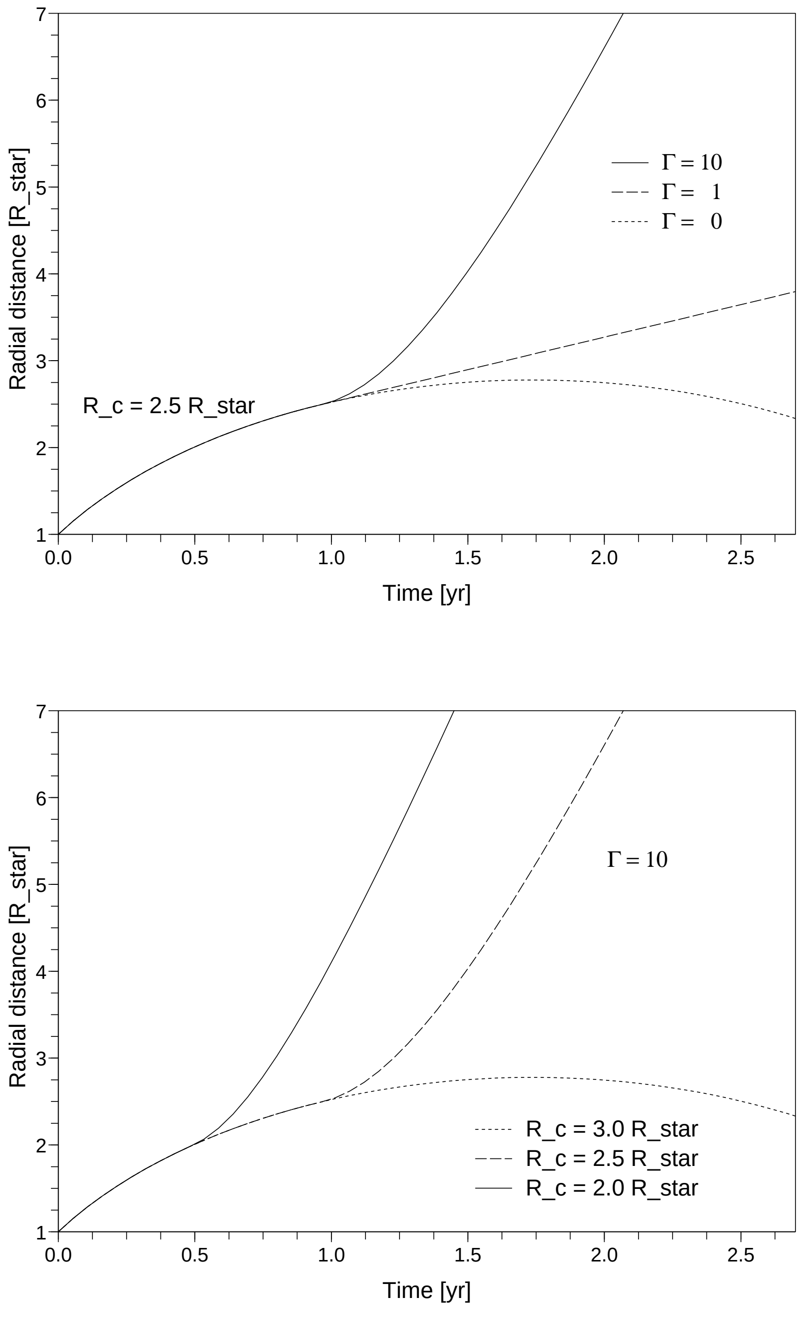

Figure 1: Position of a fluid element (i.e., its distance from the stellar center) as a function of time. The velocity is determined by assuming a distance-dependent Γ in the equation of motion, i.e., $\Gamma = 0$ for $r < R_c$ (below the condensation distance, no dust), and $\Gamma =$ constant at distances beyond R_c. Both, the value of Γ (varied in upper panel, for constant R_c) and the condensation distance R_c (varied in lower panel, for constant Γ) play a crucial rule for the onset of a stellar wind. Stellar parameters $M_* = 1\,M_\odot$, $L_* = 7000\,L_\odot$, $T_* = 2700\,\mathrm{K}$; initial conditions $r_0 = R_*$ and $u_0 = 0.8\,u_{\mathrm{esc}}(r_0)$.