Nonlinear Dynamics and Shock Formation

Burgers' equation can be interpreted as an **equation of motion for a pressureless fluid** (see lecture). In contrast to the momentum equation of gas dynamics, internal forces (pressure gradients) and external forces (e.g. gravity) are neglected, but the basic non-linearity of the advection term is kept, leading to the possibility of shock formation (in contrast to the wave equation). The equation **describes the evolution of a velocity profile** *v(x,t)* and can be written in conservation form as

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{2}v^2\right) = 0$$

The **MATLAB routine *Burgers.m*** solves the equation using a simple explicit upwind (donor cell) numerical scheme. Several different types of **initial conditions for the velocity profile** can be chosen using the parameter ***i_dat* (1 = Gaussian, 2 = slope, 3 = triangle)**, and scaled/shifted using the parameters (preset values shown here)

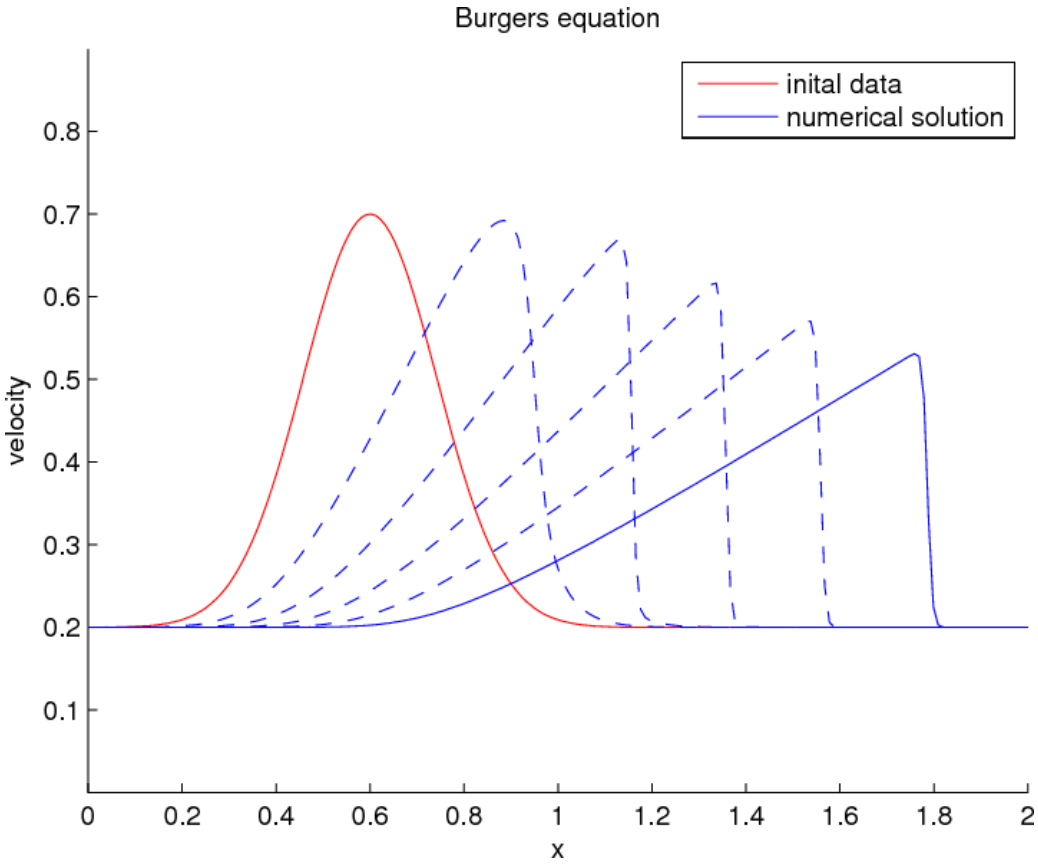
```
v_min = +0.20;  % maxium velocity
v_max = +0.70;  % minimum velocity

f_tip   = 0.30; % location of gaussian and triangle
f_gauss = 0.40; % width parameter for gaussian
f_width = 0.20; % width parameter for triangle and slope
```

while the spatial range and the time interval are set by the parameters *x_min*, *x_max* and *t_stop* (starting time *t_start* = *0*).

Homework (due April 26, 2010):

- First, run the routine ***Burgers.m*** without any modifications and **check that the result looks like the included figure**. Pre-selected type of profile: Gaussian (*i_dat* = *1*), other parameters as given above.
- Make **two runs with modified parameters**:
  (a) *v_min* = *+0.00* and *v_max* = *+0.50,* with *f_tip* = *0.30*
  (b) *v_min* = *-0.20* and *v_max* = *-0.70,* with *f_tip* = *0.70*
  (note: *v_min* > *v_max* makes sense here, they are just parameters, not the actual minimum and maximum of *v*)
- Hand in (email: Susanne.Hoefner@fysast.uu.se, preferably in pdf format) the resulting **plots together with a short text describing and explaining the differences** between the original version (see included figure) and cases (a) and (b).



Initial data (Gaussian, red line) and time evolution of the velocity profile (snapshots at 5 instants of time, blue lines) obtained with the unmodified routine *Burgers.m*.