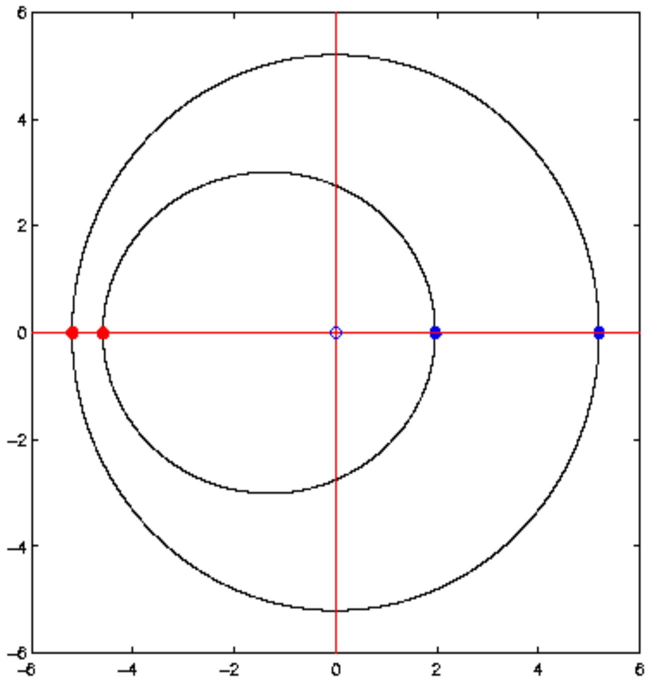
Planetary System Dynamics

Part II

Resonances: Periodic orbits

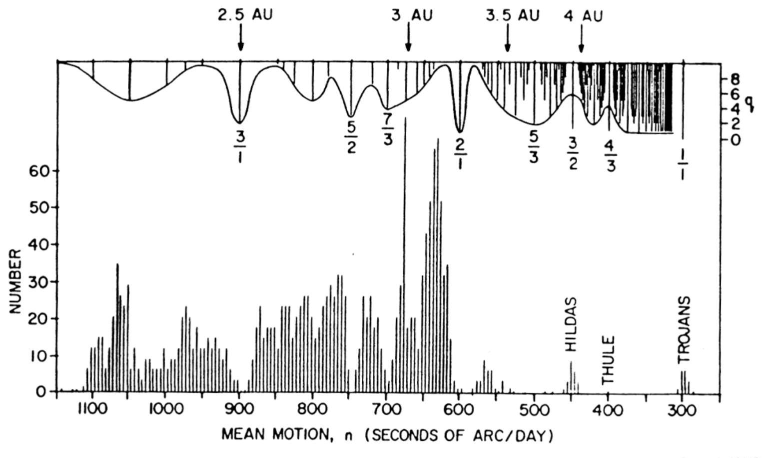


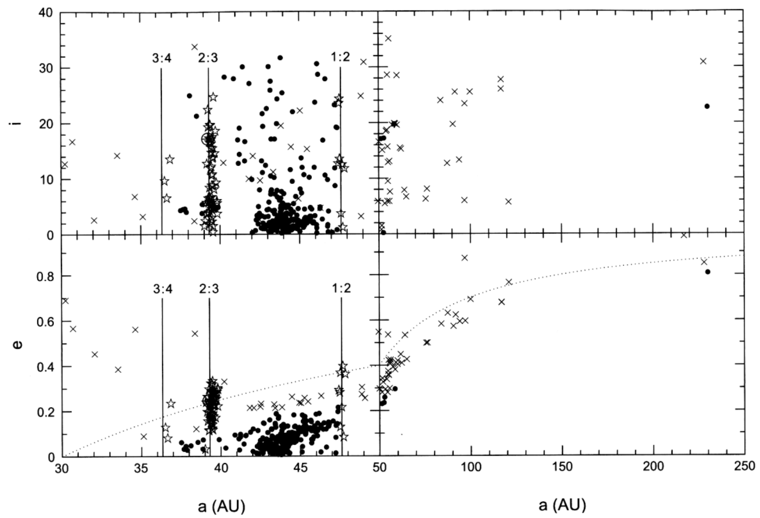
Example: asteroid in 2/1 mean motion resonance with Jupiter

Analogue: ***the planar pendulum***

- Periodic orbits (equilibria) exist for perihelic and aphelic conjunctions
- The perihelic conjunction case is stable and the aphelic one is unstable (chaotic)

Examples of Mean Motion Resonance





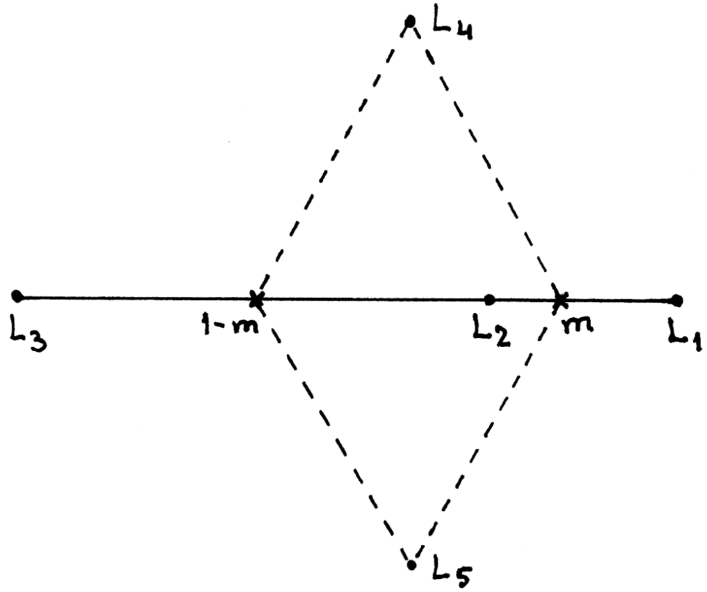
- Asteroid Main Belt: *Kirkwood gaps*
- Outside the MB: *Hildas and Trojans*
- Transneptunian Population: Large group of "*Plutinos*"

Stability of the Solar System

- The truncated part of the series is a possibly non-integrable perturbation
- ***Resonances*** involve both periodic motion and chaos (*Poincaré*): Do the "tori" of quasi-periodic motion in phase space survive in the presence of perturbations?
- *KAM (Kolmogorov-Arnol'd-Moser) theorem*: Quasi-periodic motions dominate the phase space if the perturbations are small enough
- *Nekhoroshev theorem*: Even if chaos exists, the deviations from regular motion are bounded during a finite time

Circular restricted 3-body problem

- A massless body moves in the combined gravitational field of the Sun and one planet, and the orbit of the planet is circular



- ***Lagrange points***: *Equilibrium points for the massless body in the rotating frame following the planet*

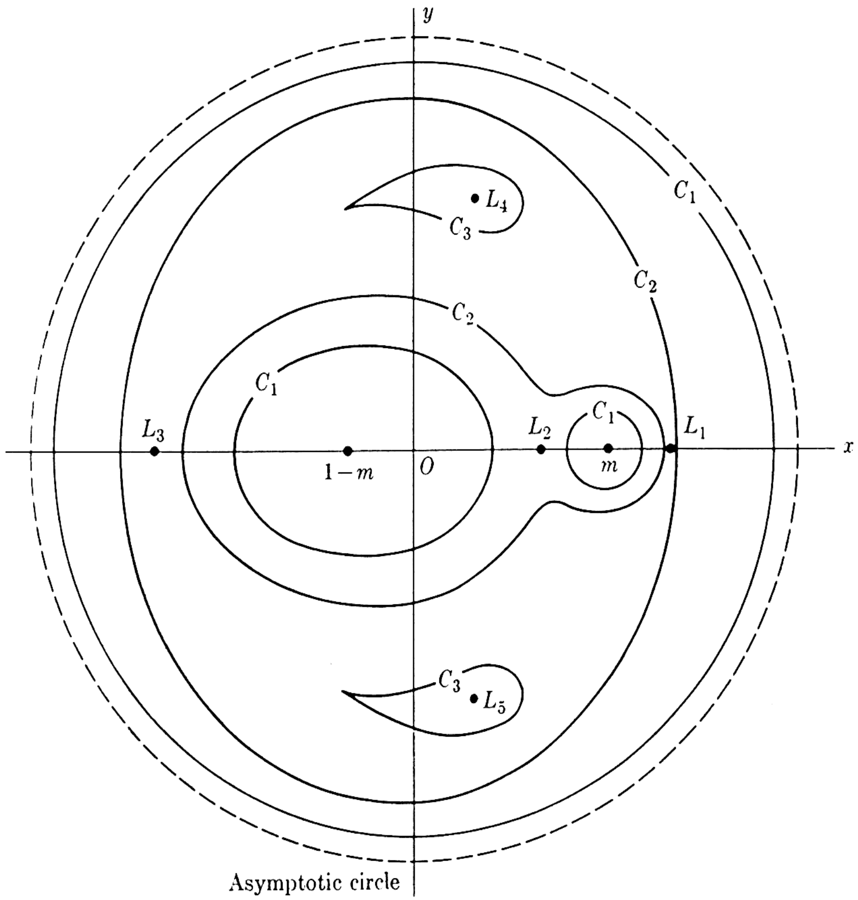
Zero-velocity surfaces

- The force field in the rotating system is conservative ⇒ existence of an ***energy integral***

$$C = \left(x^2 + y^2\right) + 2\left\{\frac{1-m}{\rho_1} + \frac{m}{\rho_2}\right\} - v^2$$

- $v = 0$: ***zero-velocity surfaces***

These cannot be crossed!



Tisserand parameter

- Largest closed zero-velocity surface around the planet: ***Hill sphere***

$$\rho_H = \left(\frac{m}{3}\right)^{1/3}$$

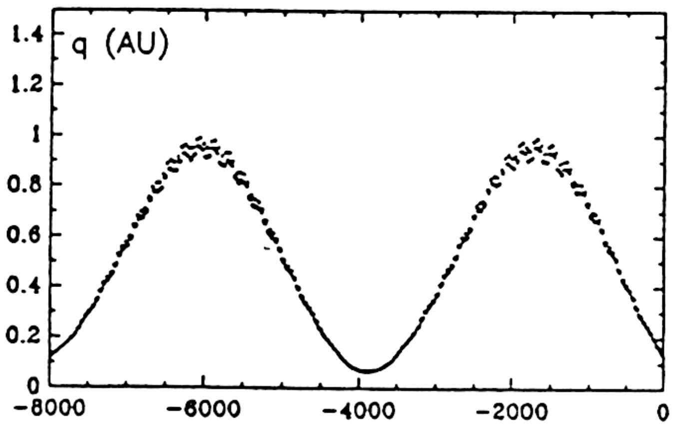
- Orbital energy in the rotating system: ***Jacobi integral***
- Approximation far from the Sun or the planet: ***Tisserand parameter***

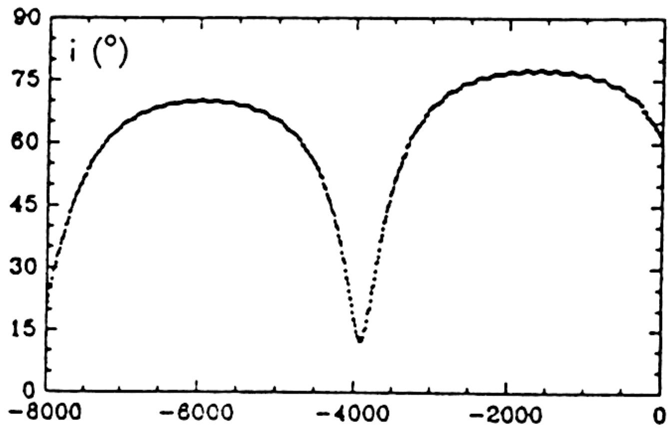
$$T = \frac{a_J}{a} + 2\sqrt{\frac{a}{a_J}(1-e^2)}\cos i$$

-E ***H_z***

Kozai cycles

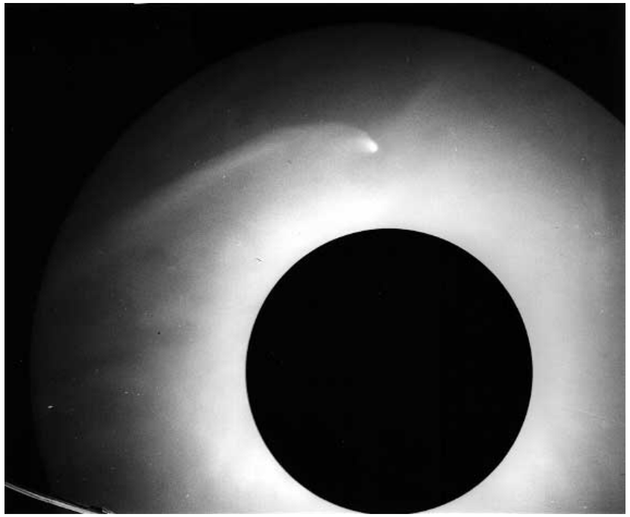
- Without close encounters: both *a* and H_z are constant, but *H* may change
- Coupled (e,ω) and (i,Ω) variations with constant *a*: ***Kozai cycle***

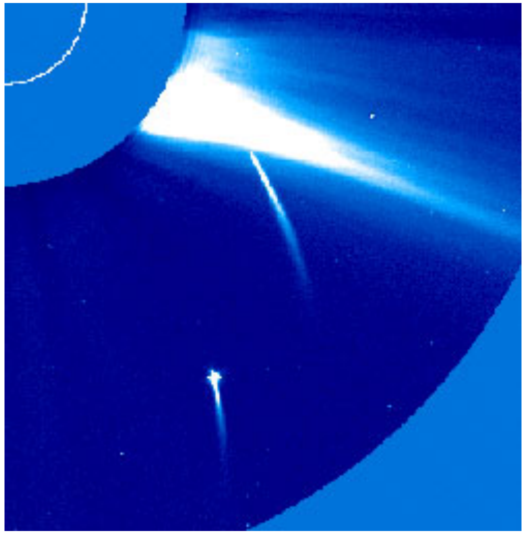




Kozai cycle of comet 96P/Machholz 1

Sungrazing Comets



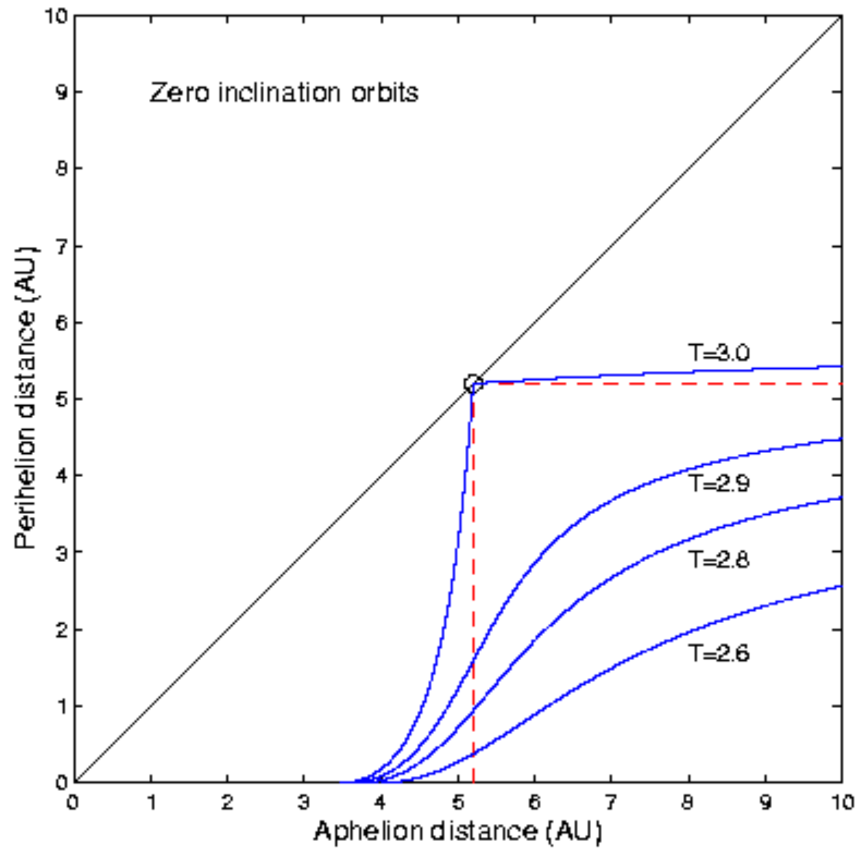


- Comet Ikeya-Seki (1965 S1): groundbased coronograph image
- SOHO image of two comets plunging into the Sun in 1998

These are results of Kozai cycles

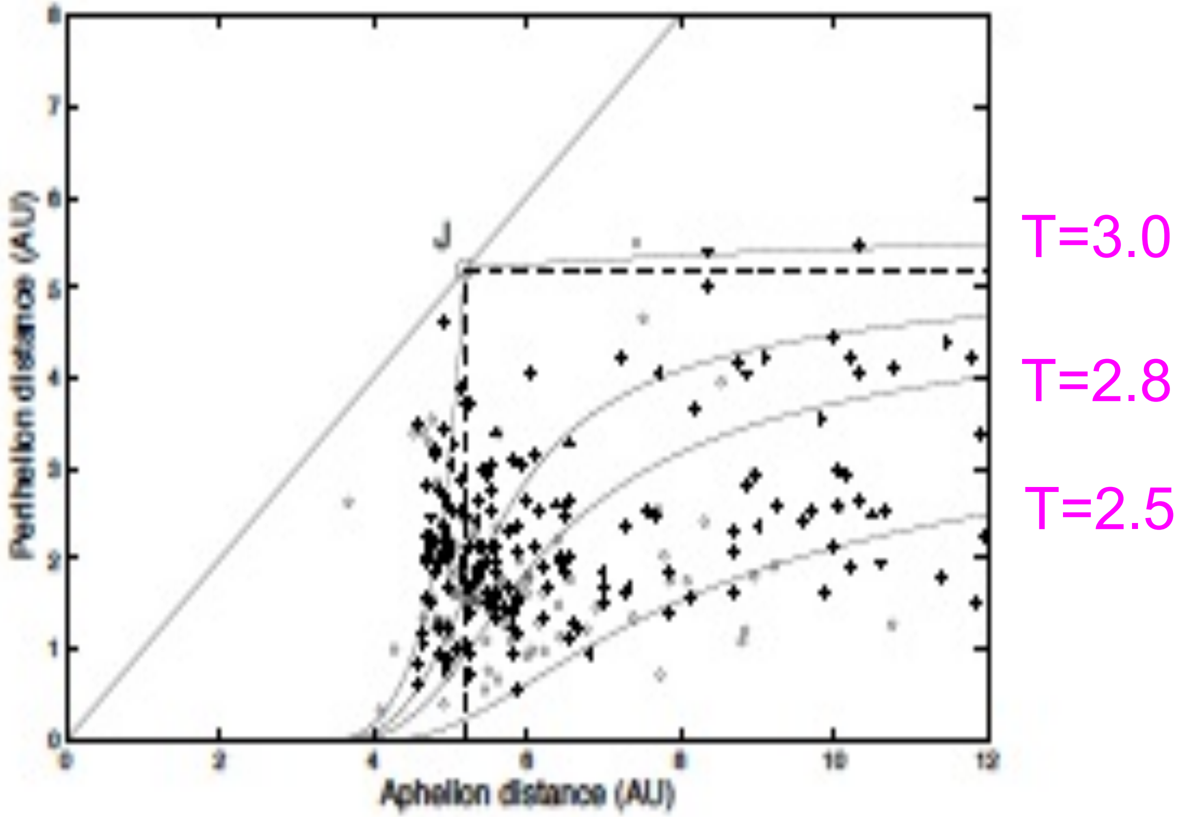
Orbital evolution due to close encounters

- *Many comets have low-inclination orbits and experience close encounters with Jupiter*
- The ***Tisserand criterion*** was used to identify the same comet before and after a large change of the orbit
- It can be used to plot **evolutionary curves** in the *(a,e)* or *(Q,q)* planes



Capture routes into the Jupiter family

The Jupiter Family

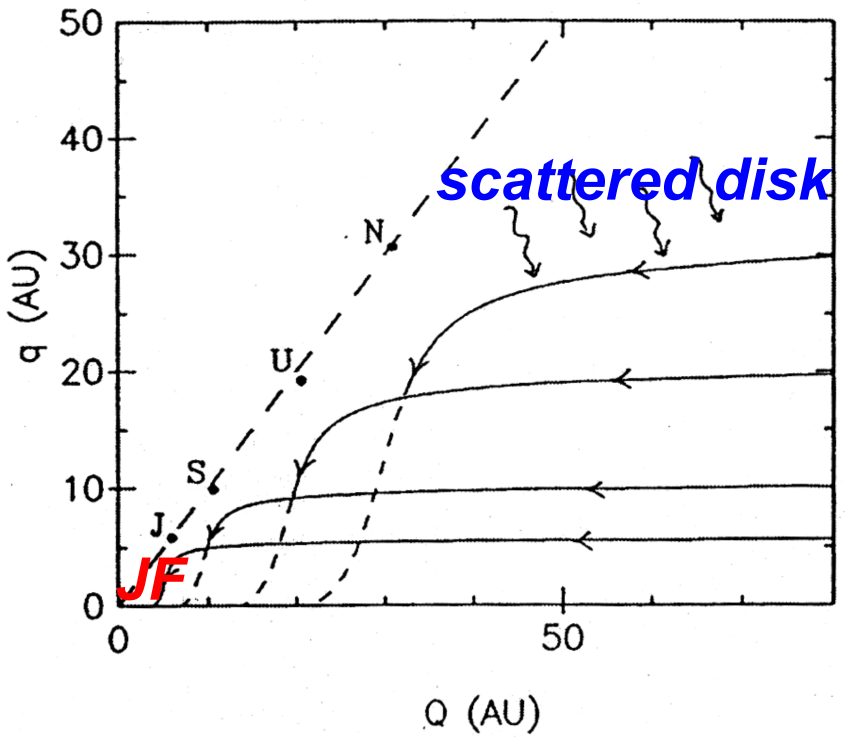


Circles: discovered before 1980

Pluses: discovered after 1980

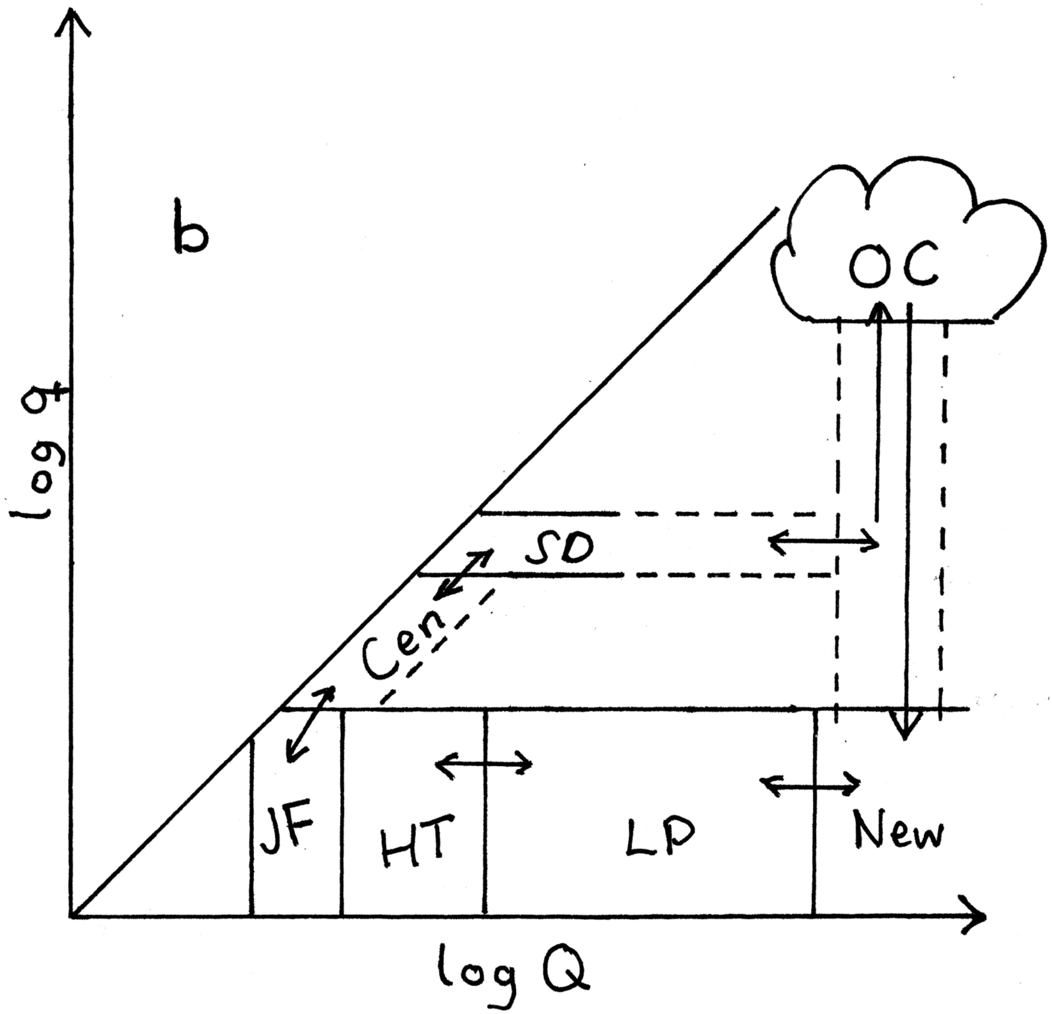
Capture of the Jupiter Family

- Low-inclination routes with $T_p \approx 3$
- "Handing down" process: Neptune-Uranus-Saturn-Jupiter
- Most efficient source: the Scattered Disk



Starts from the TNO population
Goes via Centaur population

Comet Dynamics roadmap



Oort Cloud

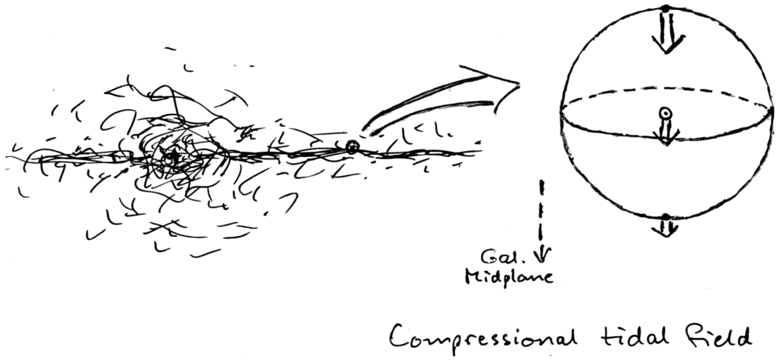
Observed comets

Oort Cloud dynamics

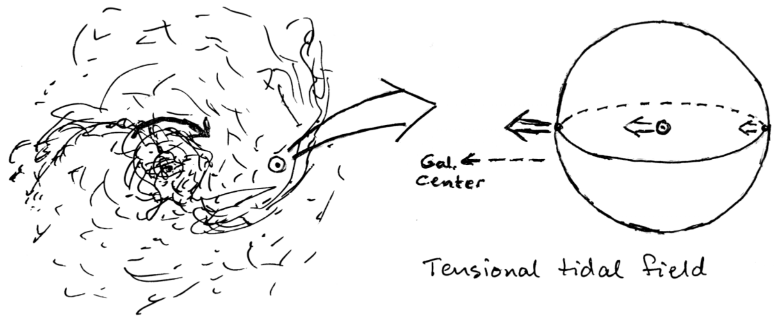
- Comets are orbiting at distances ~ 10^4 - 10^5 AU from the Sun
- Their orbits are continually perturbed by (1) *the Galactic tides*; (2) impulses due to *passing stars*
- These perturbations are important to feed new comets into observable orbits

Galactic tides

- ***The disk tide***
  - Gravitational potential of the Galactic disk
  - Induces *regular oscillations of q and i_G* (small enough *a* for integrability)



- ***The radial tide***
  - In-plane, central force field of the Galaxy
  - *Changes also the semimajor axis and may eject comets*



The Disk tide

- The *q* oscillations of the disk tide are an important mechanism to inject comets into the inner Solar System
- Heisler & Tremaine (1986) developed an analytic theory of the regular disk tide by orbital averaging

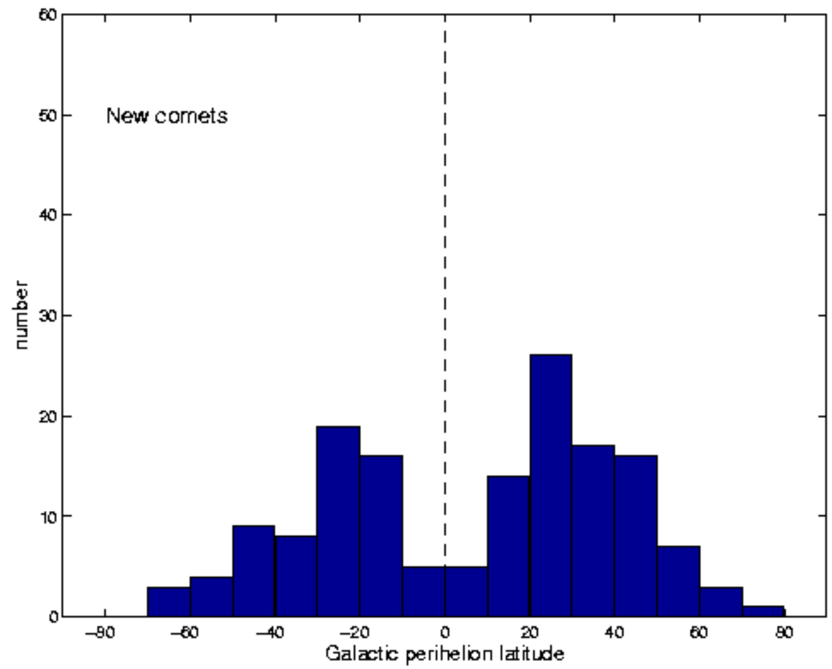
$$|\Delta q| \propto q^{1/2} \cdot a^{7/2} \cdot |\sin 2\beta_G|$$

Tidal change of q per orbital revolution due to the disk tide

Gal. latitude of perihelion

Imprint of the disk tide

- Delsemme (1987) recognized a double-peaked distribution of Galactic latitudes of perihelia of new comets, indicating the *importance of the disk tide for comet injection*

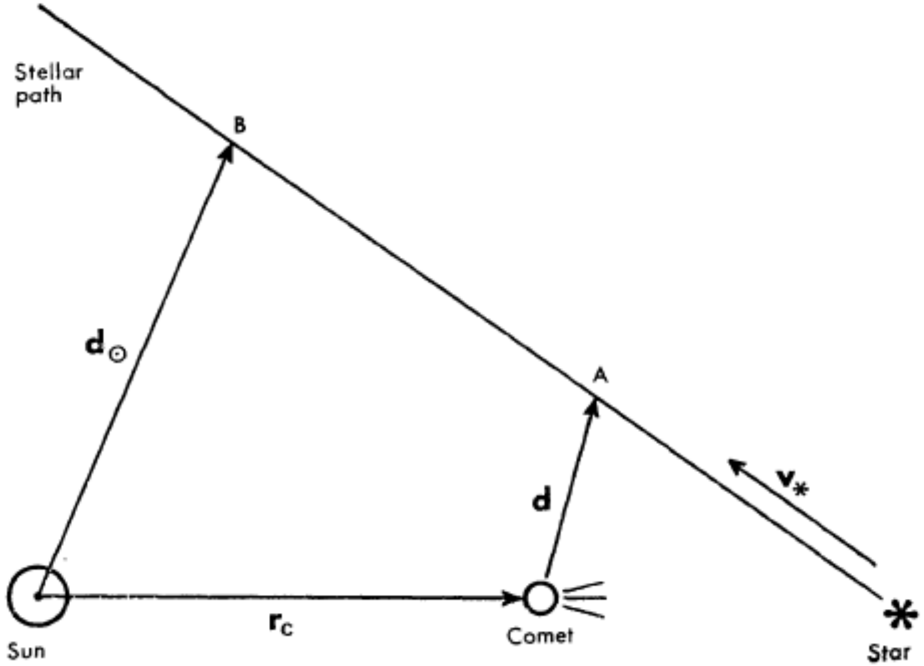


recent orbit sample

Impulse Approximation

- Heliocentric impulse (velocity change $\mathbf{I}=\Delta\mathbf{v}_c$):

$$I = \frac{2GM_*}{v_*}\left\{\frac{\hat{\mathbf{d}}}{d} - \frac{\hat{\mathbf{d}}_\odot}{d_\odot}\right\}$$



Rickman (1976)

Assumptions: the comet and the Sun are at rest; the star passes at constant speed along a straight line