Dynamics of planet formation

Centrifugal equilibrium

- *Conservation of angular momentum in a contracting cloud ⇒ increase of angular velocity and rotational energy*
- Rotational energy U_{rot} increases faster than gravitational energy U decreases

- *Contraction perpendicular to the spin axis stops when the centrifugal force equals the force of gravity at the equator of the cloud*
- $R_{lim} = R_c$ (centrifugal radius)
- Continued collapse along the spin axis ⇒ flattened disk with radius $R \sim R_c$

Rotation and Contraction

- Potential energy of a spherical, homogeneous cloud: $$U = -\frac{3}{5}\frac{GM^2}{R}$$
- Moment of inertia: $I = \frac{2}{5}MR^2$
- Energy of rotation: $$U_{rot} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} = \frac{5}{4}\frac{L^2}{MR^2}$$
- *Centrifugal equilibrium condition*:

$$R_c\omega^2 = \frac{GM}{R_c^2} \qquad \because R_c = \frac{25L^2}{4GM^3} = \frac{25}{4GM}\times\left(\frac{L}{M}\right)^2$$

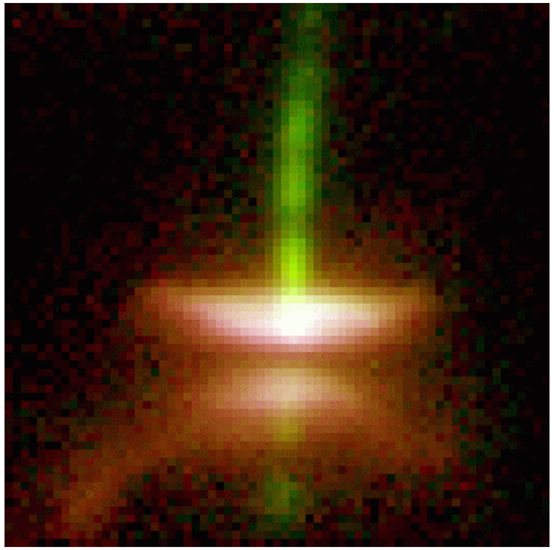
Centrifugal radius

Estimating a rough value of R_c

- Use $L/M = \omega R^2$ from observations of molecular cloud cores
- R=5000 AU and $\omega = 2\times10^{-14}$ s^{-1} leads to R_c~25 AU

- Observations of ***protoplanetary disks*** are consistent with this estimate

Protoplanetary disks



HH 30
"Herbig-Haro object"

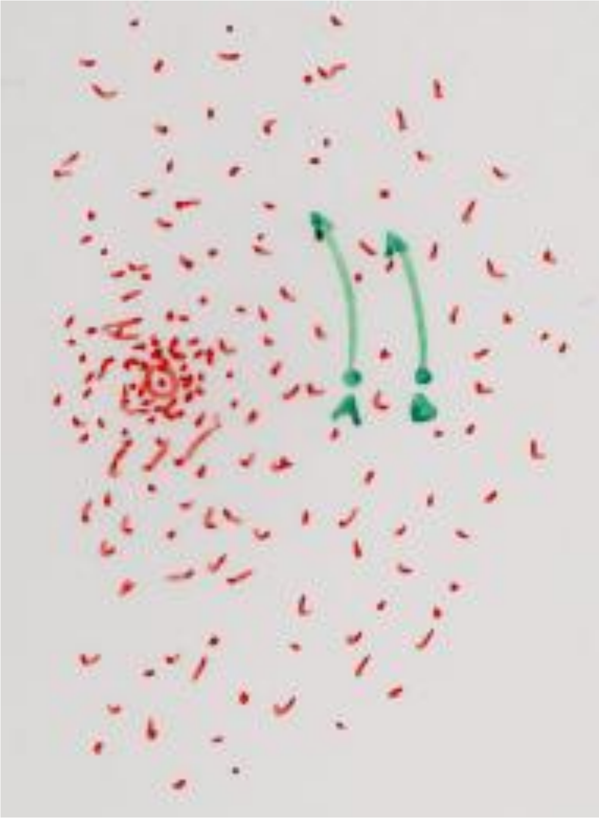
"proplyds"
(Orion nebula)

General radii ~ 100 AU

HST pictures

Accretion disk

- Differentially rotating disk - *angular velocity decreases outward*: **SHEAR**
- The shear has a physical effect, if elements at different radial distances "stick" together
- For a gaseous disk the interaction can be described as a **VISCOSITY** (*suppressing relative motion*)



Shearing instability, turbulent flow

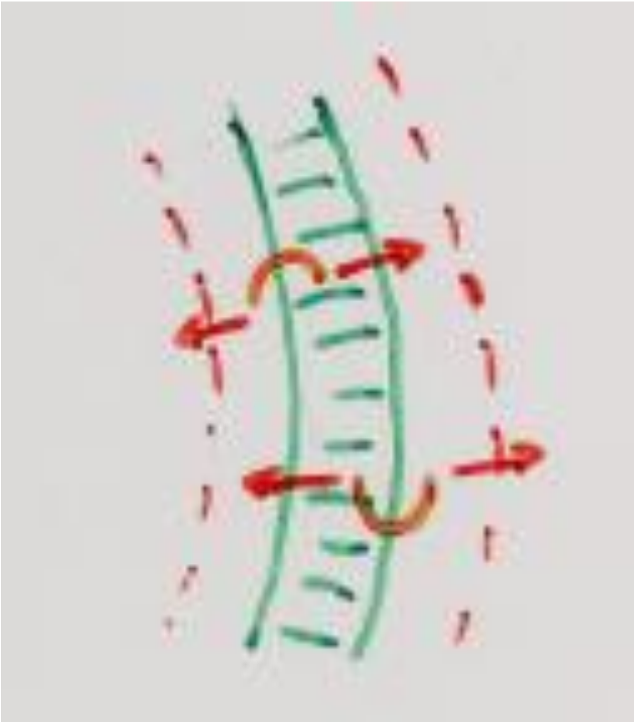
- ***Autobahn example:***

- Low traffic, peaceful drivers – *laminar flow*

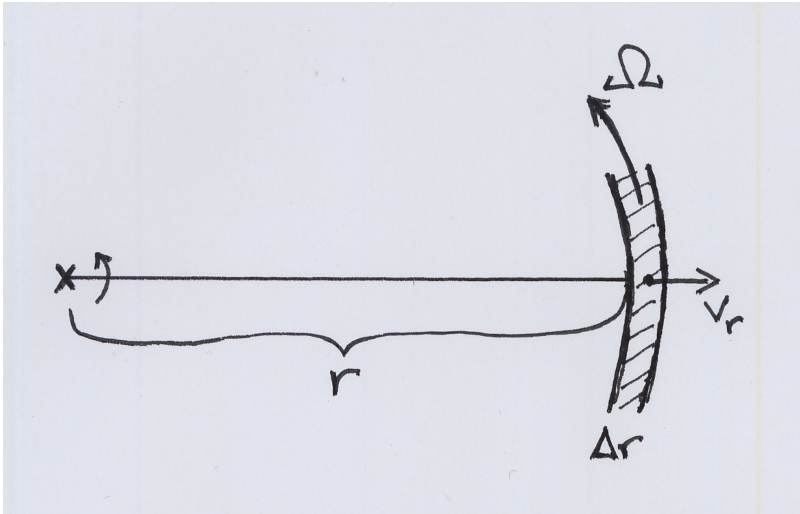
- High traffic, nervous drivers changing lanes – *turbulent flow*

Consequences of shear and viscosity

- The energy of relative motion is dissipated ⇒ **HEATING**
- ***Angular momentum is transported outward***
- *Global tendency for the disk to break up radially*
- Material deprived of angular momentum collects at the center of the disk: **ACCRETION**



Accretion disk evolution



- Consider a *circular annulus* according to the picture
- The surface density is Σ
- Ω decreases with *r*
- In the absence of infall:

$$2\pi r \Delta r \frac{\partial \Sigma}{\partial t} = \frac{\partial(\Delta m)}{\partial t} = -\Delta\left(2\pi r \Sigma v_r\right)$$

Thus the ***mass continuity equation***:

$$r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}\left(r\Sigma v_r\right) = 0$$

Accretion disk evolution, ctd

Conservation of angular momentum:

$$2\pi r \Delta r \frac{\partial}{\partial t}\left(\Sigma r^2 \Omega\right) = \frac{\partial(\Delta L)}{\partial t} = -\Delta\left(2\pi r^3 \Sigma \Omega v_r\right) + \Delta G$$

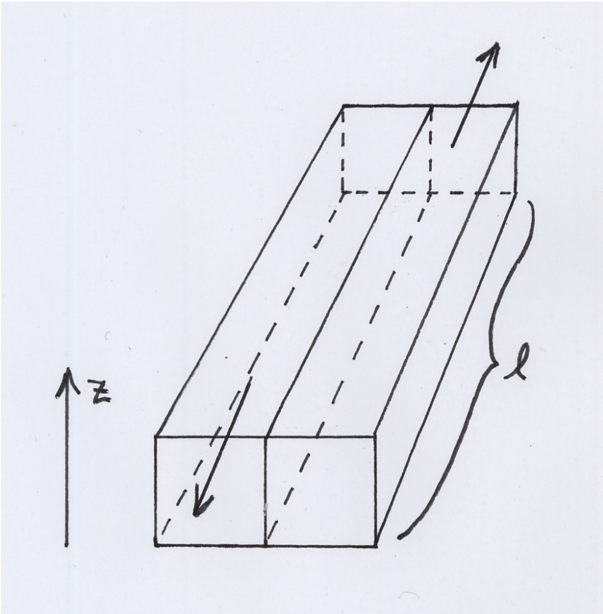
where *G* is the *torque* acting on the inner neighbour

Thus the ***angular momentum continuity equation***:

$$r\frac{\partial}{\partial t}\left(\Sigma r^2 \Omega\right) + \frac{\partial}{\partial r}\left(r^3 \Omega \Sigma v_r\right) = \frac{1}{2\pi}\frac{\partial G}{\partial r}$$

Shear velocity: $$A = r\frac{d\Omega}{dr}$$

Shear and viscosity



- Viscous tension: $p = \rho \nu A$
  where ν is viscosity
- Force acting over length ℓ :

$$F = \ell \int p\, dz = \ell \cdot \nu \Sigma A$$

- Total torque $G = rF$
  with $\ell = 2\pi r$:

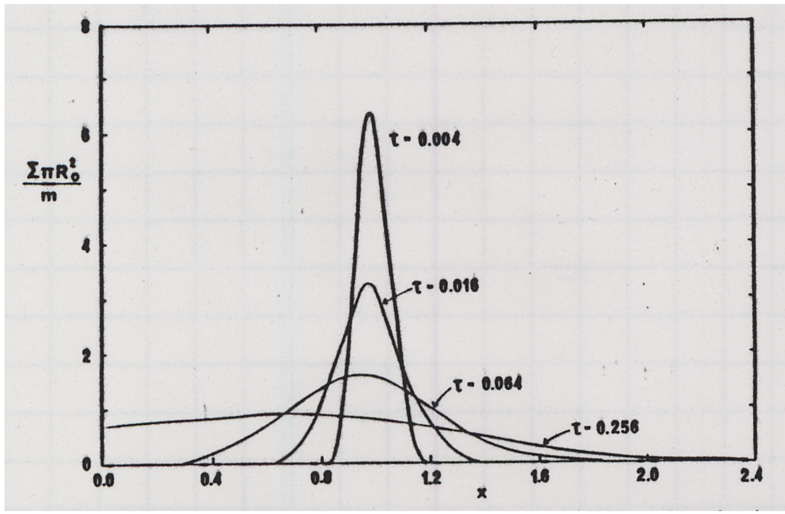
$$G = r \cdot 2\pi r \cdot \nu \Sigma A = 2\pi r^2 \nu \Sigma A = 2\pi r^3 \nu \Sigma \frac{d\Omega}{dr}$$

Global evolution

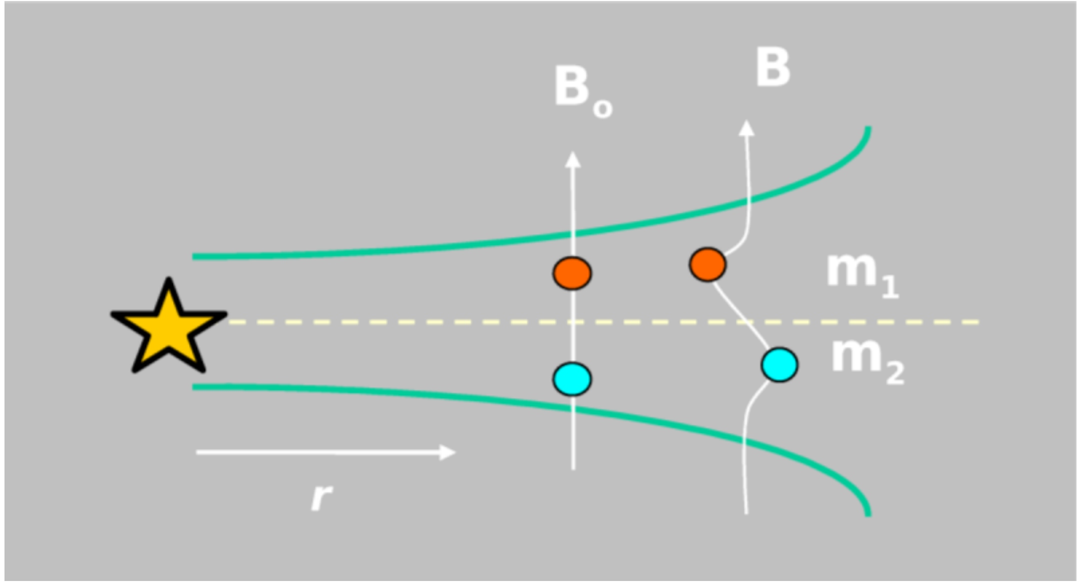
- Combining the equations, eliminating v_r, we get a *diffusion equation* for Σ:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{1}{d/dr(r^2\Omega)} \cdot \frac{\partial}{\partial r}\left[r^3 \nu \Sigma \left(-\frac{d\Omega}{dr}\right)\right]\right\}$$

Example: *a narrow, circular ring gets dispersed inward and outward over time. Most of the mass moves inward, and a small fraction of the mass carries the angular momentum outwards*



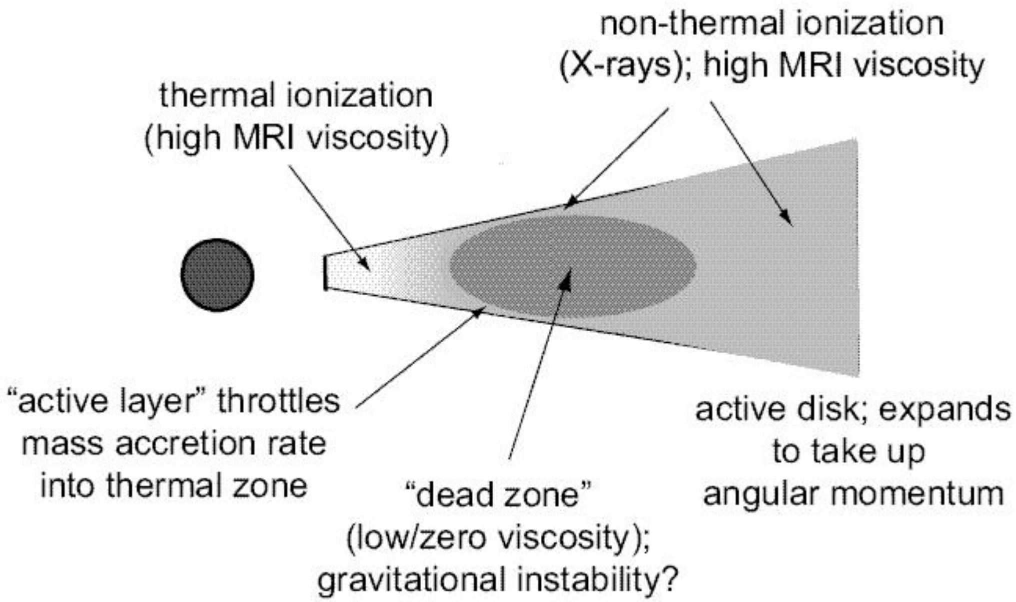
Magneto-rotational instability



Initiates turbulence and transports angular momentum

- *Suppose the gas disk is partially ionized and penetrated by a frozen-in magnetic field*
- If a field line connects two gas parcels at somewhat different radial distance, their differential rotation will stretch the field line, and magnetic tension acts to keep them together ⇒ instability against radial break-up

Episodic accretion



- A “***dead zone***” may develop inside the disk, where neither thermal motion nor X-rays are able to ionize the gas enough for MRI
- The dead zone would accumulate material from the outside and grow in mass
- There may be *bursts of accretion*, when such clumps fall onto the protostar

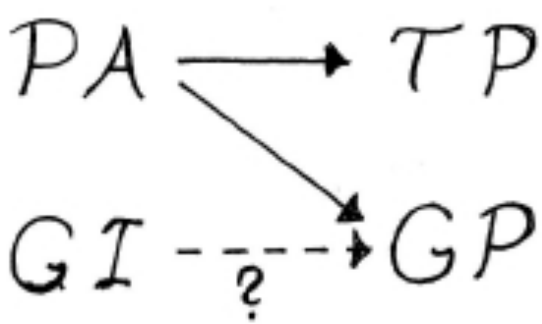
Planetary formation mechanisms

- **Planetesimal Accretion** (PA)
- **Gas Disk Instability** (GI)

Terrestrial planets were certainly formed by PA

For *giant planets*, PA is favoured

But the issue is still open!



What is a planetesimal?

- A ***planetesimal*** is a planetary building block large enough to cause some significant *gravitational attraction* on neighbouring objects (km-size or larger)

- A ***planetary embryo*** is formed from planetesimals by accumulating them into a larger object by local accretion

The eventual planets are much fewer than the embryos

Planetesimal formation

- **Proposed mechanisms**:

  - ***gravitational instability*** of a *thin disk of solid bodies*
  - ***grain accretion*** by collisions with sticking, triggered by *gas-dust interactions*

Gravitational instability

Compare the derivation of the Jeans mass!

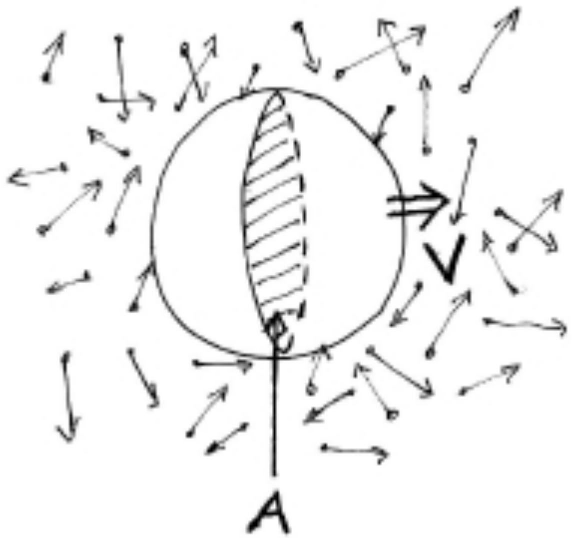
- Consider a *disk in differential, Keplerian rotation* around a central object (Sun)
- *Perturb the mass distribution locally; find the response of the disk as a spiral wave*
- The dispersion relation (frequency vs wave number) indicates ***instability*** (imaginary frequency) *for large wavelengths, if the velocity dispersion is small enough*
- Hence an instability so that local density enhancements can grow ⇒ ***planetesimals***

Grain growth

- **Mechanisms for grain collisions**:

- *thermal* (Brownian) motions
- *turbulent* gas motions (grains carried along by eddies)
- gravitational *settling* into the midplane
- *inward drift* due to solar gravity and gas drag

Gas drag



- Consider a *grain moving sub-sonically* through a gas with molecular mass m_g and thermal speed v_g. The drag force acting on the grain is:

$$F_D = n_g A V \times C_D m_g v_g$$

Coupling time scale

- Solving the equation of motion $$M\frac{dV}{dt} = F_D$$

  we get: $$V = V_o \exp\left\{-C_D \frac{A}{M} n\mu u_{th} t\right\} = V_o \exp\left\{-\frac{t}{\tau_c}\right\}$$

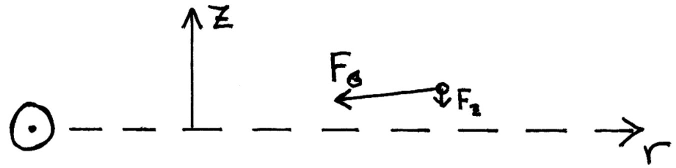
  with $$\tau_c = \frac{M}{AC_D \rho_{gas} u_{th}} \quad (\rho_{gas} = n\mu)$$

- For a spherical grain with radius R_{gr} and density ρ_{gr}, the *gas-grain coupling time scale* is roughly: $$\tau_c \approx \frac{\rho_{gr} R_{gr}}{\rho_{gas} u_{th}}$$ *(very short!)*

Grain settling to the midplane

- Grav. force on a grain of mass *M*:

$$F_G = M\Omega^2 r \qquad \left(r^3\Omega^2 = GM_{sun}\right)$$



- *z* component:

$$F_z = M\Omega^2 z$$

Ω = local frequency of disk rotation

- Gas drag force on a grain with radius R_{gr} moving at vertical velocity *u*:

$$F_D = \left(\pi R_{gr}{}^2 n u\right) \times \left(C_D \mu u_{th}\right)$$

C_D~1 = gas drag coefficient

u_{th} = random velocity of a gas molecule in the direction opposite to *u*

Settling time scale

- With $\rho_{gas}=n\mu$ = density of the gas disk and ρ_{gr} = density of the grain, force balance yields:

$$\rho_{gas} u_{th} u_{eq} = \frac{4}{3} R_{gr} \rho_{gr} \Omega^2 z$$

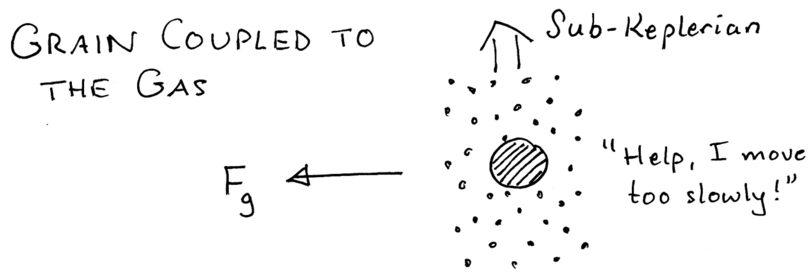
- With $u_{eq} = dz/dt$, we get: $z = z_o \exp(-t/\tau_s)$, introducing the *sedimentation time scale*:

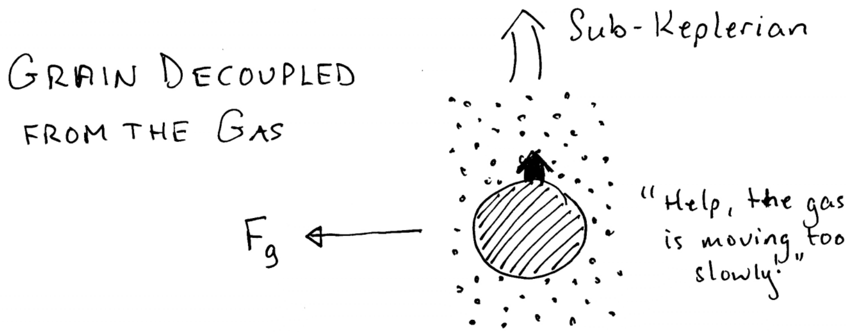
$$\tau_s \approx \frac{z}{u_{eq}} = \frac{3\rho_{gas} u_{th}}{4 R_{gr} \rho_{gr} \Omega^2}$$

~1000 yr for cm-sized grains at Jupiter's orbit

Radial drift

- *The gas circulates at sub-Keplerian speed being pressure supported, but the grain is not*
- When the grain is small and coupled to the gas, it is dragged toward the Sun by moving too slowly
- When the grain is larger and decoupled from the gas, it moves at Keplerian speed and feels a gas drag. Thus it loses speed and spirals toward the Sun





This radial drift causes the grain to sweep up smaller ones

Drift time scale

- Orbital angular momentum: $L = M\sqrt{GM_{sun}r}$
- Drag torque: $\tau = rF_D = r\rho_{gas}Au^2C_D$
- Decay time scale: $T_d = \dfrac{L}{\tau} \approx R_{gr}\dfrac{\rho_{gr}}{\rho_{gas}}u^{-2}\sqrt{\dfrac{GM_{sun}}{r}}$

- Taking *u* (the difference between Kepler and pressure-supported speeds) as 1% of the circular speed:

$$T_d \approx R_{gr}\frac{\rho_{gr}}{\rho_{gas}}\cdot 10^4\sqrt{\frac{r}{GM_{sun}}}$$

A few hundred years for a 1-meter boulder **_Problem!_**

Planetary accretion

- **Planetesimal to planet evolution**:

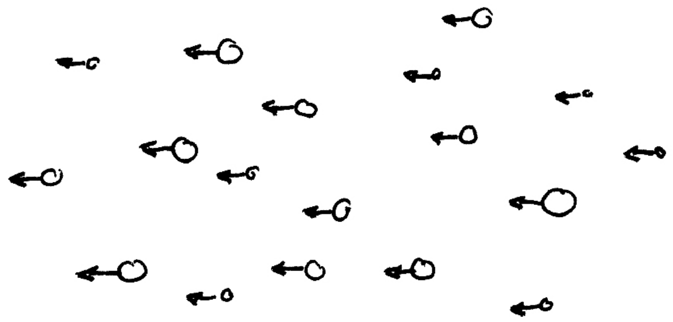
- *Collective growth* of size distribution and relative velocity distribution in an enormous population of initially roughly equal-sized planetesimals
- *Runaway growth*: the largest body grows much more rapidly than the rest of the population $\Rightarrow$ quick formation of planet embryos, moving in concentric orbits separated by a few Hill radii from each other

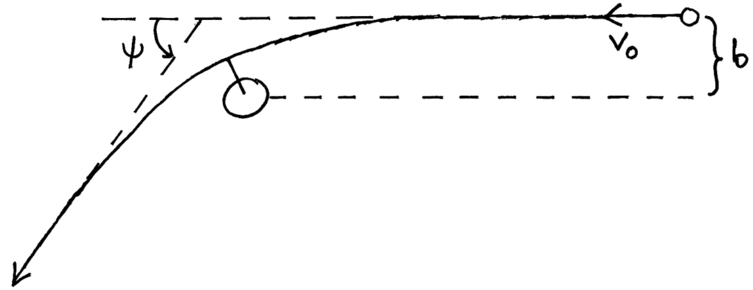
Phenomena caused by the gravity of planetesimals:

Stirring and Focussing

Close encounters

- The number of planetesimals per unit area is so large that "congestion" occurs ⇒ frequent close encounters
- Gravity causes hyperbolic deflections with $\psi = \psi(b, v_o, m)$; this increases the orbital eccentricities





Stirring and focussing

- The typical eccentricity/inclination obtained by hyperbolic deflections ("gravitational stirring") determines the average relative velocity v_o at close encounters
- If m_1 is the largest planetesimal mass (r_1 is the radius), we have:

$$v_o^2 = \frac{Gm_1}{\theta r_1} = \frac{1}{2\theta} \cdot v_{e,1}^2$$ (V.I. Safronov)

  where v_e is the escape speed and θ~2-5
- From angular momentum conservation we get:

$$v_o \cdot r_c = \sqrt{v_o^2 + v_e^2} \cdot r$$

  where r_c>r is the maximum impact parameter for collision ("gravitational focussing"), and we get the cross-section:

$$A_c = \pi r^2 \left(1 + \frac{v_e^2}{v_o^2}\right) \Leftarrow \textit{gravitational focussing}$$