Astrophysical Dynamics: Exercise

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1 Gravitational focussing and hyperbolic deflection

The task is to find the critical impact parameter that leads to a grazing impact as a function of the encounter velocity, when a massless body encounters a planet.

The way this should be done is to use a `Matlab` code, and in order to do that, you must pick up three files from the course web page:

- `grfocus.m`
- `newton.m`
- `initia.dat`

Only the last one is to be edited by you. The program does the following. It integrates the motion of the massless particle according to Newton's law of gravity, as attracted by a planet that is treated as a point mass. The integration starts at 50 million km from the planet and runs for 1000 days, enough to approach the planet and recede to at least a similar distance if only the initial velocity is larger than one km/s. The integrator takes small steps, when the acceleration is large but may take very long steps, if the acceleration is small.

The program gives as output: the four input parameters (see below), and then a parameter called q, which is the smallest distance from the center of the planet – *i.e.*, the point mass used in the calculation – in units of the planetary radius. Therefore, if $q < 1$, an impact onto the planetary surface occurs, but if $q > 1$, the particle passes outside the planetary surface and experiences a hyperbolic deflection. Of course, if $q = 1$, the limiting case of a "grazing impact" occurs.

You are supposed to use three of the Solar System's planets for this investigation: Jupiter, the Earth and Mercury. Start by finding their masses expressed in solar masses and their radii in km. Then prepare for the integrations by choosing a range of initial velocities and impact parameters – *i.e.*, the distance from the planet at which you aim, when launching the particle. Before each run, you will edit the file `initia.dat`, which contains four numbers:

- the impact parameter in km
- the initial velocity in km/s
- the inverse value of the planet/Sun mass ratio
- the radius of the planet in km

The file copied from the web page contains an example setup that will yield a nearly grazing impact with Jupiter. You can do the editing while running `Matlab`. To run the code, type `grfocus` after the prompter. The input parameters will be listed as `dis`, `vel`, `ima`, and `rad`, respectively, and then comes `q`. For each chosen value of `vel`, repeat the calculations with different values of `dis` until you reach $q = 1$ to a good approximation. Then note the critical value of `dis`, and repeat the procedure with another value of `vel`.

For each planet, produce a diagram showing the critical impact parameter versus the encounter velocity. Then compare these results for the three different planets, and note the features that you find most striking.

Then, for each combination of critical impact parameter and velocity, you should compute the hyperbolic deflection angle using the formula in the lecture notes. Take care to use the proper units. You may use AU and AU/day for distance and velocity, respectively, and then use $0.2959 \cdot 10^{-3}$ for $GM_{\odot}$, multiplying by the respective planet/Sun mass ratio.

Using those results, plot the maximum deflection angle (corresponding to grazing impacts) as a function of velocity for each planet, and comment on these results.

2 Giant planet gravity assist calculation

Suppose we want to send a spacecraft to Neptune, but we cannot afford the fuel to launch it directly into an orbit that reaches out to Neptune, so we want to use so-called *gravity assist* manoeuvres, passing close to the other giant planets and using hyperbolic deflections to accelerate the spacecraft.

Therefore, consider the following approximations. All planets are supposed to move on circular orbits in the same plane. Start by finding the co-called Hohmann transfer orbit with perihelion at the Earth's orbit and aphelion at Jupiter's orbit (supposed to be at $a = 5.20$ AU).

- What is the semi-major axis and eccentricity of this orbit?
- How long time will it take to reach Jupiter this way?
- Which extra velocity along the Earth's orbital motion is required in order to enter this orbit?

Then proceed to a calculation of an optimal hyperbolic deflection to reach Saturn, supposed to move at $a = 9.54$ AU, as follows:

- Find the velocity, by which the spacecraft approaches Jupiter, supposing the planet and spacecraft reach the spacecraft's aphelion point at the same time.
- Compute the value of the impact parameter, for which Jupiter will deflect the spacecraft into a new orbit with aphelion at Saturn's distance! This can be done by turning the planet-spacecraft relative velocity vector immediately by the angle that corresponds to a given impact parameter, and then computing the new heliocentric velocity vector and from this the new perihelion and aphelion distances. Find the appropriate impact parameter by trial and error.
- Check if such a deflection is possible without impacting Jupiter!

Repeat the last steps for the next deflections by Saturn and Uranus, assuming Uranus to move at $a = 19.18$ AU and Neptune at $a = 30.06$ AU. It is probably too complicated for you to calculate the times it will take for the different transfers between the giant planets, but if you are able to do it, this will be appreciated.

Finally, check if any shortcuts can be made! For instance, would Jupiter be able to deflect the spacecraft directly to Neptune or Uranus, or would this mean too large deflection angles and thus collision with the planet?