Timescales

Kinematics: Study of motions within systems (e.g., rotation)

Dynamics: Description of motions in terms of forces
(e.g., with respect to mass distribution)

How do systems evolve?

What are the timescales?

Timescales and Processes

- Dynamical timescale (crossing & free-fall times)
- Kelvin-Helmholtz timescale
- Quasi-equilibrium states

  Phase mixing and violent relaxation
- Relaxation
- Equipartition
- Ejection and evaporation
- Gravothermal instability

Dynamical Timescale

How can we estimate the dynamical timescale for a system?

- Free fall in system's gravitational field
- Orbit around the center
- Particle (or sound wave) crossing system

$$t_{dyn} = \frac{r_{char}}{v_{char}} \sim \left(\frac{R^3}{GM}\right)^{1/2}$$

Timescale for "mechanical" motions

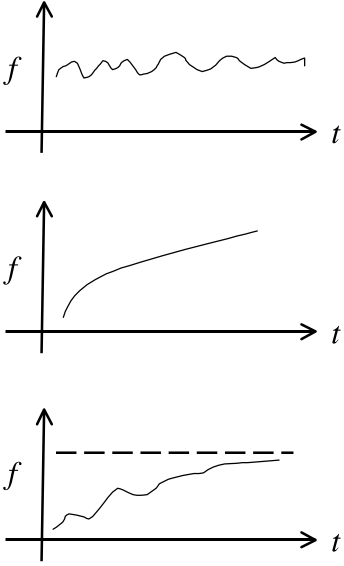
Kelvin-Helmholtz Timescale

How long would it take for a system of luminosity L to radiate away its gravitational energy?

$$t_{KH} = \frac{E_{grav}}{L} = \frac{GM^2}{RL}$$

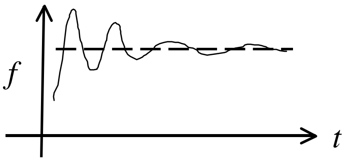
Timescale for release of gravitational energy…
…is also the thermal timescale

Quasi-Stationary Equilibria



Shape, size, mass distribution and nature of stellar motions (given by distribution function) vary "very little" with time.

$$t_{quasi} \;\square\; 30\, t_{cross}$$



"Collisionless Relaxation" Processes

Phase mixing: coarse-grained phase-space density decreases (distribution function when measured on larger scales).

- Analogy: 20% rum and 80% Coke in finite volume; 100% rum or Coke in infinitesimal volume elements.
- Stellar energies do not change but phase-space distribution looks smoother.

Violent relaxation: rapid process (e.g., caused by fast changes in gravitational potential) in which a stellar dynamical system relaxes from an initial state to a quasi-equilibrium state.

- Stellar energies (and position and velocity distribution) changed in a way that is *not* dependent on stellar mass.
- Still controversial. (Simulations: dependence on initial state, and not necessarily spherical.)

Relaxation time

A system that is removed from statistical equilibrium by an external agent will, once left to itself, return to the previous state of statistical equilibrium (e.g., Maxwell distribution for a gas in a closed vessel).

Relaxation time for a stellar system is the time after which:

- changes in velocities from encounters become of same order as velocities themselves; *or*
- stars deflected (e.g., ~ 1 radian) from original directions.

$$t_{relax} \, \square \, \frac{N}{24 \ln N} t_{cross}$$

Stars "forget" their initial velocities (speed & direction).
Effects of many small deviations > effects of few strong ones.

Equipartition

Relaxation □ equipartition of kinetic energy
(stars with more lose KE to stars with less)

$$m_1 \left\langle v_1^2 \right\rangle = m_2 \left\langle v_2^2 \right\rangle$$

- More massive stars lose KE and sink towards center.
- Less massive stars gain KE and move to outer parts.

Equipartition thus contributes to "mass segregation"
(radial distribution of stars by mass)

Ejection and Evaporation

$$v_{escape}^{2} = \square 2V_{0} = 4\langle v^{2} \rangle$$

- **Ejection**: velocity change from a single close encounter gives a star a speed greater than the escape speed.

$$t_{ej} \square 10^{3} \ln(0.4N)\, t_{relax}$$

- **Evaporation**: series of weaker, more distant encounters increases energy of a star, until a single weak encounter gives it a slightly positive energy that allows it to escape.

$$\frac{dN}{dt} \equiv \square \frac{N}{t_{evap}} = \square\square \frac{N}{t_{relax}}$$

$$t_{evap} \square 300\, t_{relax}$$

Gravothermal instability

$$C \equiv \frac{dE}{d\overline{T}} = \square \frac{3}{2} Nk$$

Negative heat capacity: when system loses energy, it grows hotter.

Inner parts lose energy and shrink; temperature (velocity dispersion) increases; energy transferred outward by encounters.

$$t_{core} = 330\, t_{relax}(r = 0)$$

Process may stop due to formation of hard binaries…