Inverse problems

Doppler Imaging
Lecture 6

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Doppler Imaging of CP stars

CP-stars: stars of spectral type B-F with variable spectral line strength. The first one α^2CVn has been discovered nearly 100 years ago. High resolution observations revealed the changes not only in strength but also in the shape of lines. The successful model of so called oblique rotator has been introduced in the end of 50's: the star is assumed to rotate as a solid body while chemical elements are non-uniformly distributed over the surface.

Forward problem. Assumptions:

- Each observed spectrum is taken during the time much shorter than the rotation period.
- The surface structures do not change throughout the observing run.
- The rotational broadening is larger than the instrumental profile (profiles are resolved).
- The star rotates as a solid body (no differential rotation).
- Chemical inhomogeneities do not affect the continuum.
- The wavelength scale of observations corresponds to the stellar reference frame.

Forward problem. Formulation:

- The residual intensity measured by the telescope/spectrometer is the flux coming from the star at a given rotational phase convolved with the instrumental profile and normalized to the continuum:

$$R^{\mathrm{Obs}}(\lambda,\phi) = \frac{1}{F^{\mathrm{Cont}}(\lambda)} \int_{-\infty}^{+\infty} K(\lambda-\lambda')F(\lambda',\phi)d\lambda'$$

- The flux coming from the star for a given wavelength and rotational phase is:

$$F(\lambda,\phi) = \oint_{\substack{\text{Visible}\\ \text{hemisphere}}} \mu I(\lambda + \Delta\lambda^{\mathrm{M}}_{\mathrm{Dop}})d\sigma$$

The relation between the local abundances and the specific intensity is given by the equation of radiative transfer:

$$\mu \frac{dI_\lambda}{dx} = \kappa_\lambda(M) \cdot (S_\lambda - I_\lambda)$$

$$\kappa_\lambda = \sum_{\mathrm{sp}} N_{\mathrm{sp}} \cdot \kappa_\lambda^{\mathrm{sp}}$$

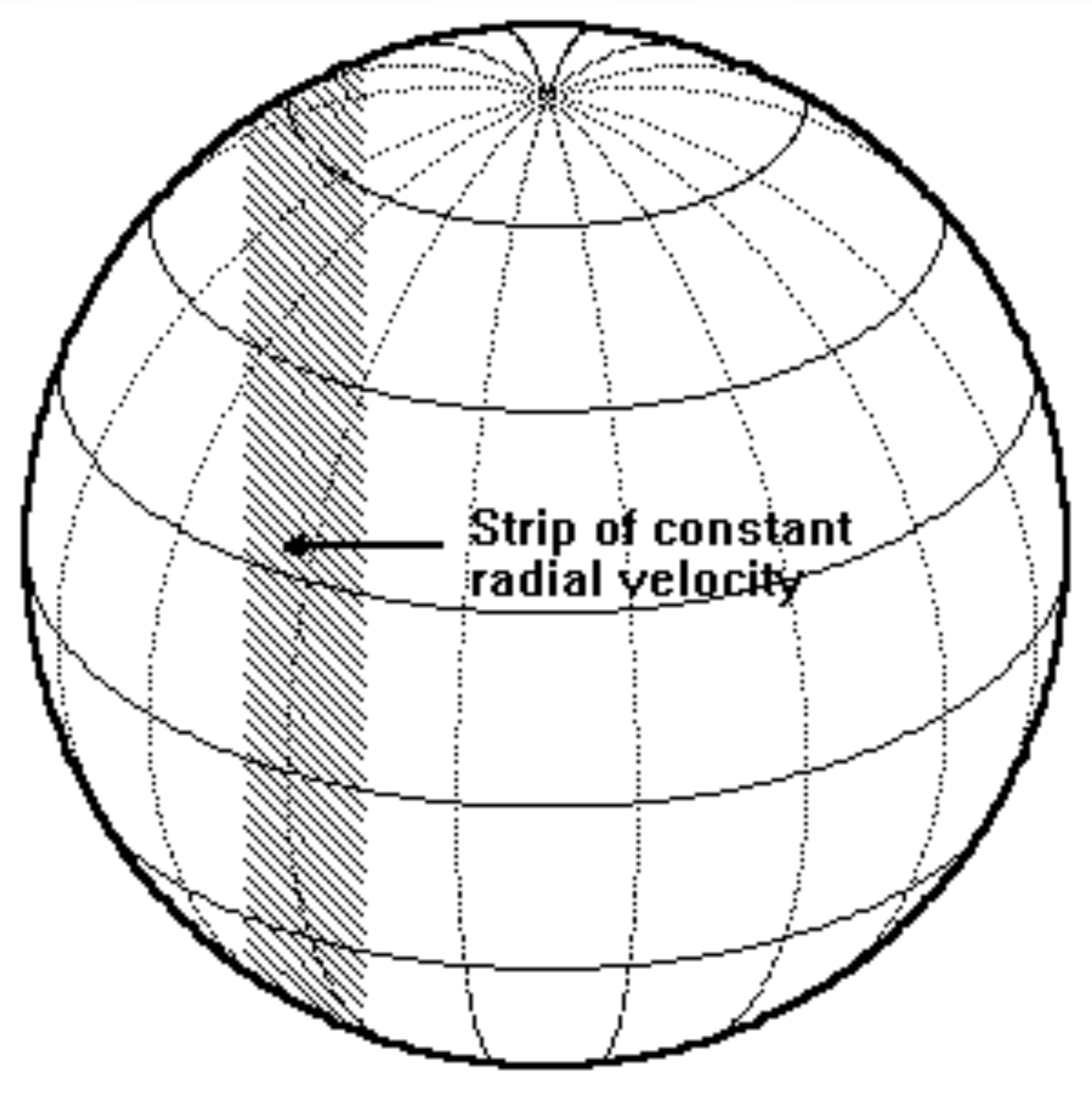
where N_{sp} is the number density and $\kappa_\lambda^{\mathrm{sp}}$ is the absorption coefficient per particle for each species.

The algorithm

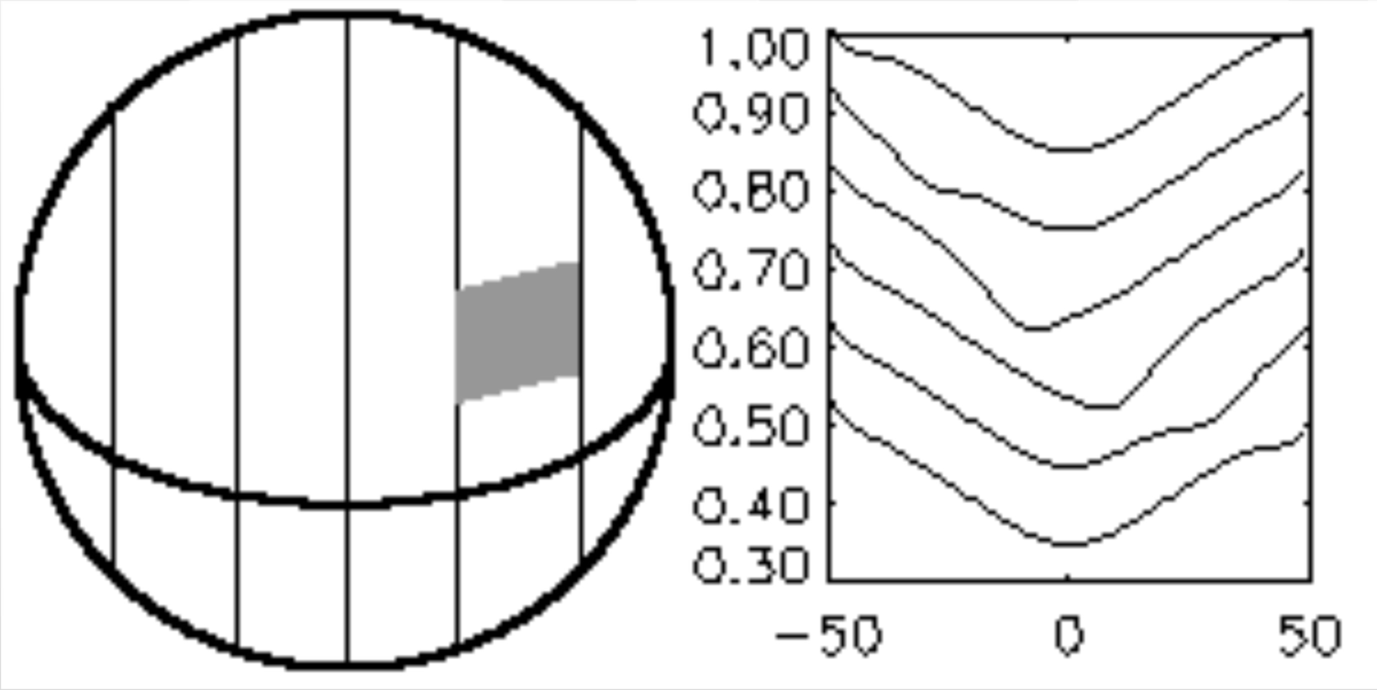
- For a given distribution of abundances compute specific intensities for each surface element at each phase covering the wavelength range: $\left\{\lambda_{Obs}\right\} \pm \Delta\lambda_{Dop}$
- Integrate intensities over the disk to obtain fluxes taking into account Doppler shifts for all phases.
- Convolve the fluxes with the PSF of the spectrometer and normalize to continuum.

Restrictions and efficiency

- The longitudinal resolution is usually determined by the PSF versus $v\sin i$. E.g. for $v\sin i$ = 50 km/s and the PSF=5 km/s we may hope to resolve 40 elements along the stellar equator.
- In order to have a reliable reconstruction we should observe all the elements in many different combinations and Doppler shifts (analogous to the offsets in the "triangle" example). In practice, sensible maps can be constructed already with 10 phases.

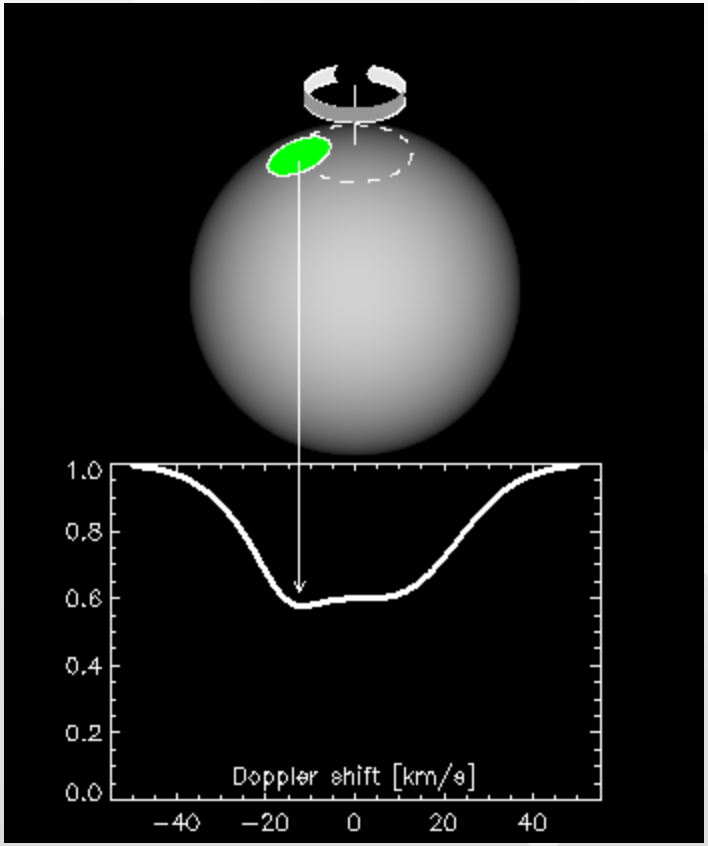

Strip of constant
radial velocity

- The size of the surface element (in longitude) should not be larger than the width of a strip with "constant" (unresolved) radial velocity.
- The S/N ratio must be sufficient to identify the presence of a spot of a given contrast:



- An area near South Pole is not visible at any phase.
- Surface elements above the equator are visible more than half of the rotation.
- The elements near North Pole have small Doppler shifts.
- The best reconstruction must be expected in the belt of intermediate latitudes shifted North of equator by the inclination angle.
- For pole-on and equator-on stars the latitude information is lost.

Line profile variation



The inverse problem

I. Assume an initial guess for the abundance map.

II. Compute the local line profiles.

III. Integrate over the disk for each phase taking into account Doppler shifts.

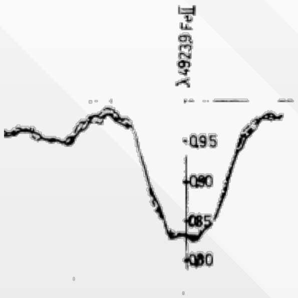
IV. Minimize the functional by adjusting local abundance

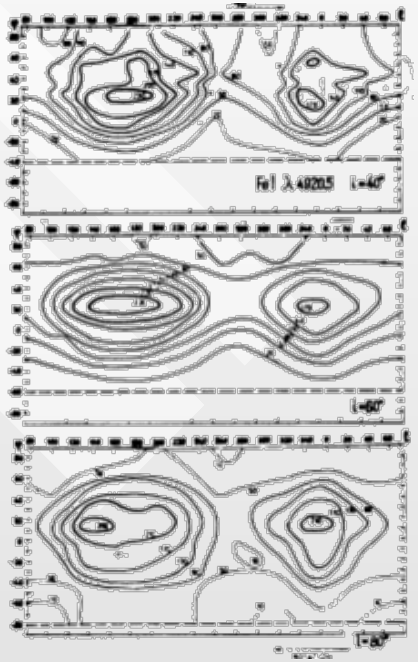
$$\Omega(Z) = \sum_{\lambda,\phi} \omega_{\lambda\phi} \left[R^{\mathrm{Obs}}(\lambda,\phi) - R^{\mathrm{Cal}}(\lambda,\phi) \right]^2 + \Lambda \cdot \mathbf{R}(Z)$$

V. Loop from II until convergence is reached.

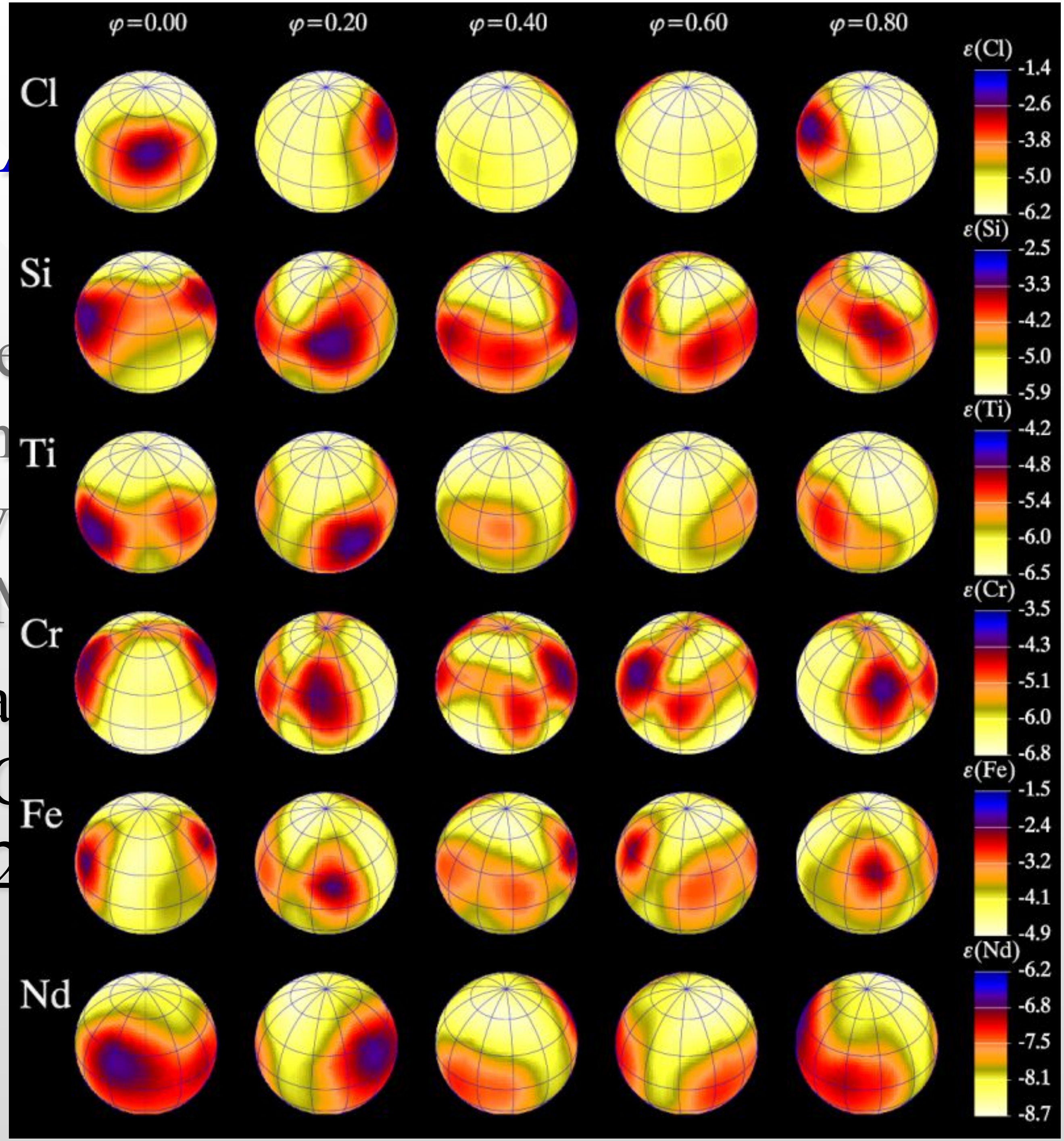
Applications: CP Stars

- One of the first maps (Fe, Cr on εUMa, Khokhlova et al,1981) 30º×30º, Milne-Eddington
- To compare with modern maps (α²CVn, Kochukhov et al, 2002) 3º×3º, true RT





- One of the
(Fe, Cr on
Khokhlov
30º×30º, N
- To compa
maps (α^2C
et al, 2002



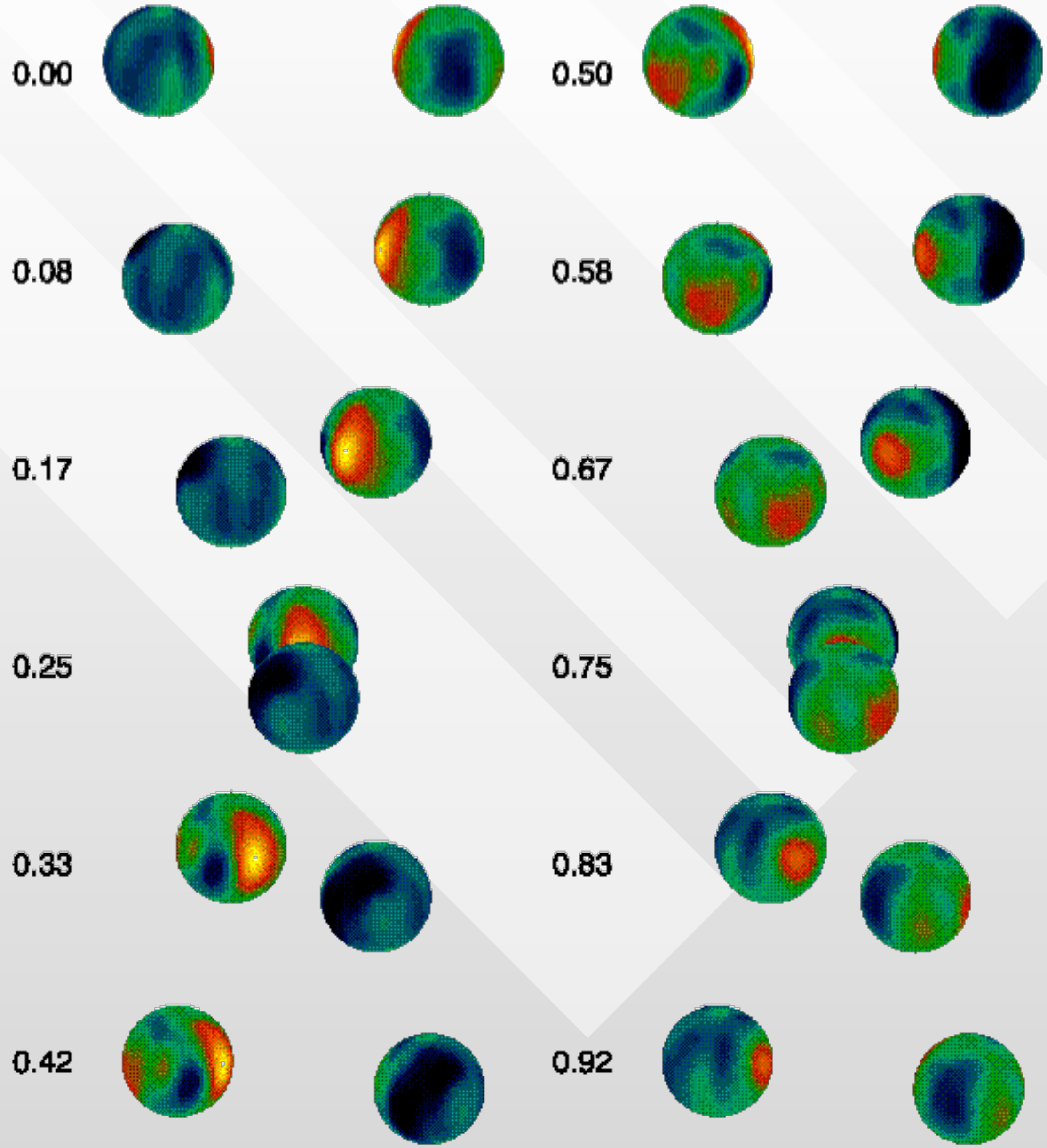
Extension of DI to other objects

- Multiple chemical elements. More than one element requires on the fly radiative transfer and multiple regularising functionals.
- DI of active regions on late-type stars. The temperature variation influences both line intensity and continuum, therefore different spectral lines can behave differently with temperature.

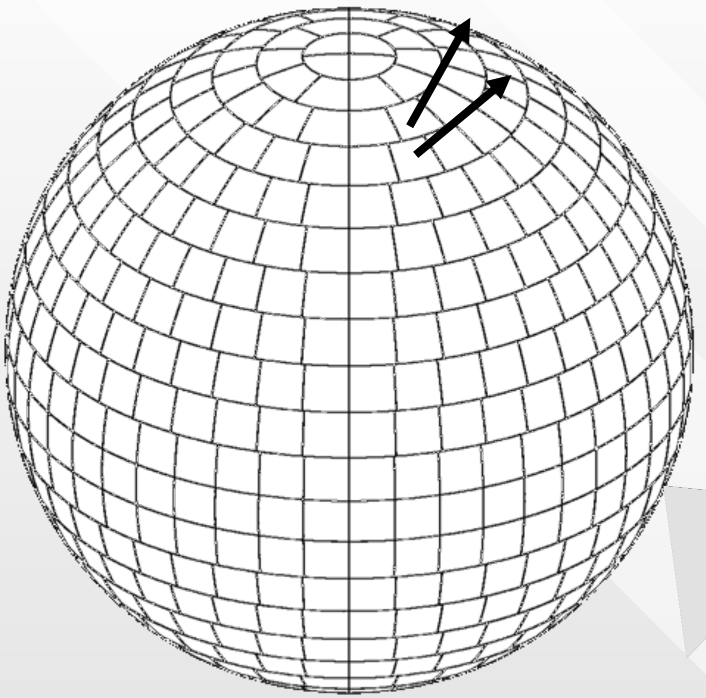
- Eclipsing binaries. The geometry becomes more complicated due to eclipses. Such systems have usually inclinations close to 90 degrees but the eclipse helps to break the symmetry.
- Contact binaries: non-spherical shape, varibale surface gravity.
- Stars with accretion disks: warps, variable accretion, stellar spots due to accretion (bright) and magnetic activity (cool).

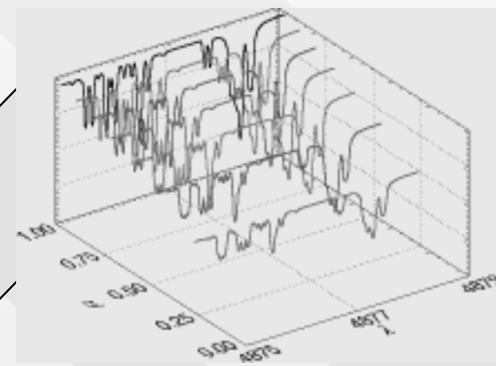
Cool stars: results

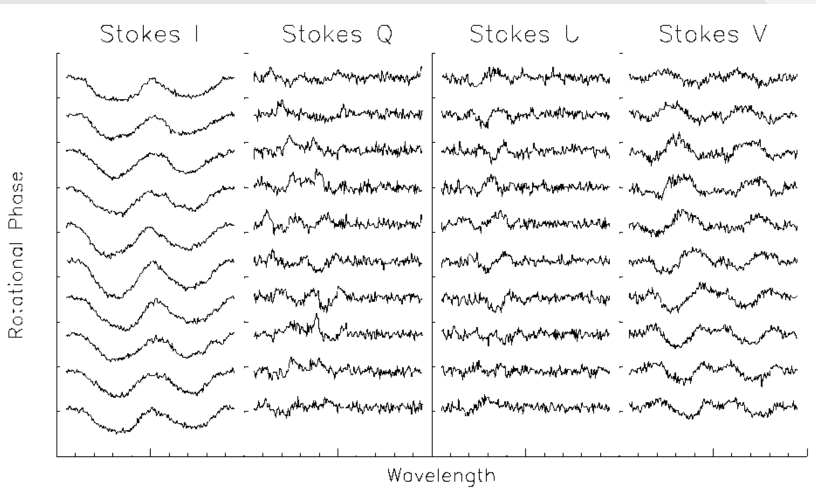
ER Vul



Magnetic Doppler Imaging







Uniqueness of the solution

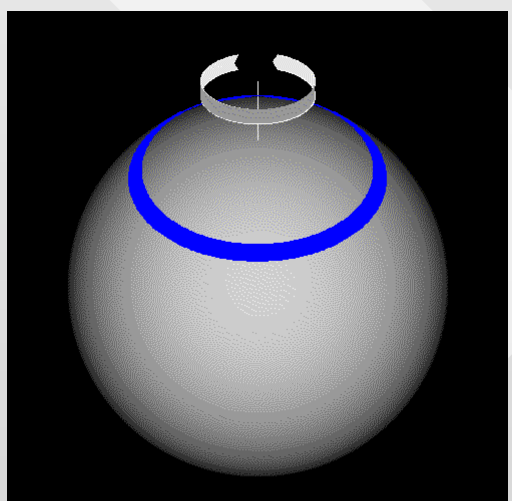
Two questions:

1. Is the relation between local Stokes profiles and local magnetic field unique?
2. Is the inversion of the local Stokes profiles unique?

Inversion of the local profiles

Do we have a unique relation between the disk integrated spectra and the local line profiles?

In other words can we find two surface structures which are undistinguishable for all phases?



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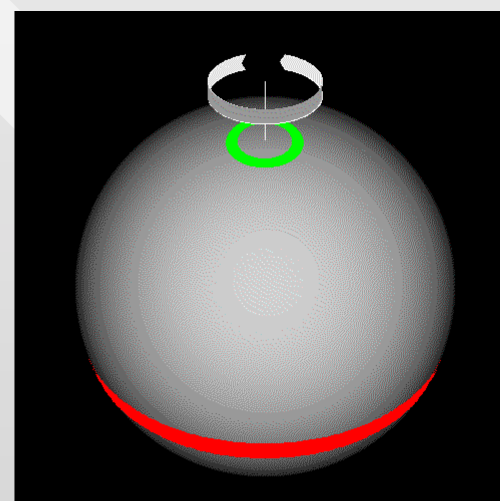


Inversion for 1D periodic functions

Fredholm equation of the 1st kind:

$$g(t) = \int_0^{2\pi} k(t-x) f(x) dx$$

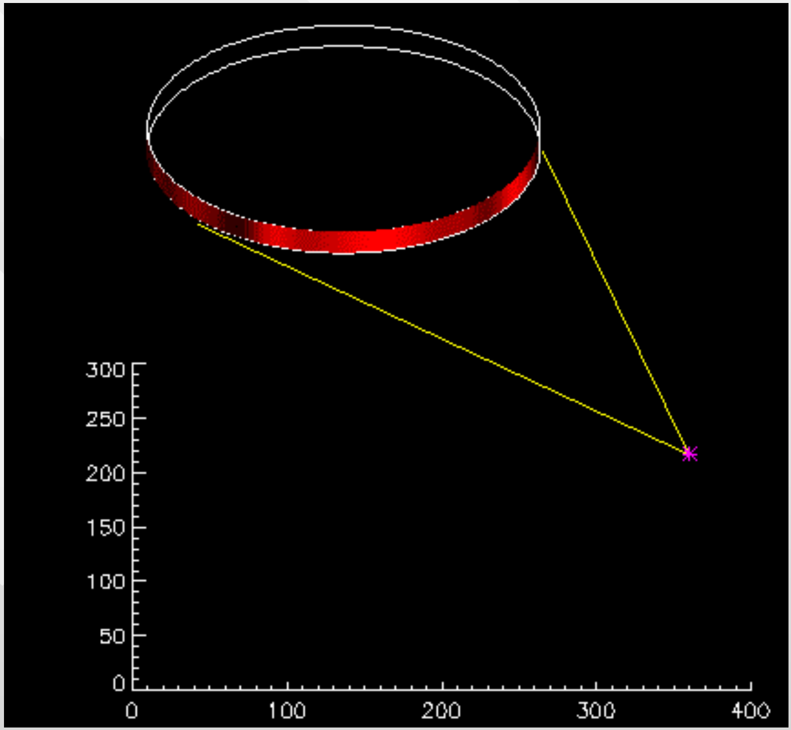
is solved via Fourier transform: $G_n = K_n \cdot F_n$

$$g(t) = \sum_n G_n \cos(2\pi n t)$$

where: $$f(t) = \sum_n F_n \cos(2\pi n t)$$

$$k(t) = \sum_n K_n \cos(2\pi n t)$$

Solution is unique as long as the kernel components exist and are non-zero



Let's look at a convolution for a single latitude:

$$F(\lambda, \phi) = \oint_{\text{Ring}} I(\theta, \rho, \mu) \cdot \underbrace{\mu \cdot \cos\theta \cdot d\rho}_{d\sigma}$$

latitude

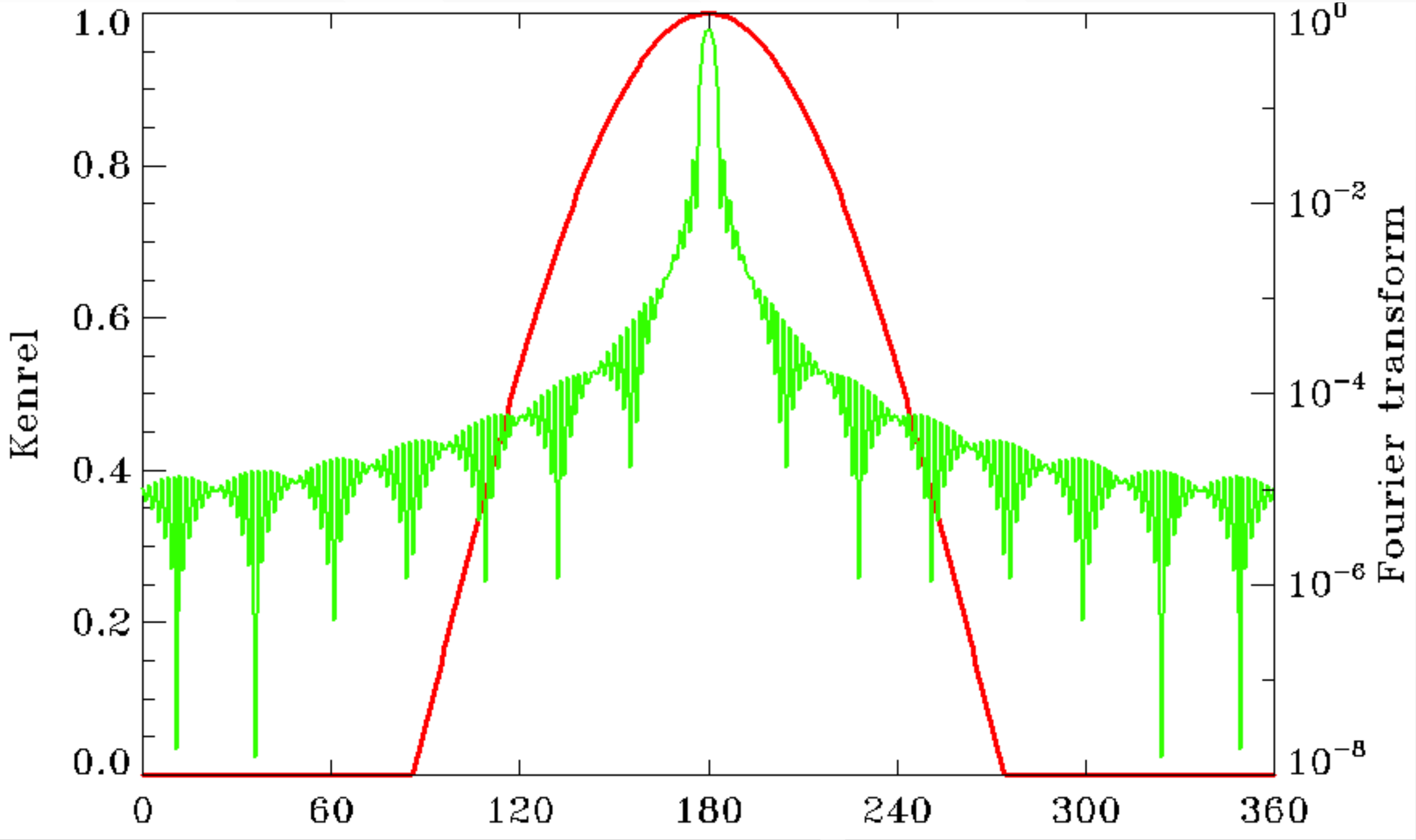
$$\mu = \sin\theta \cdot \cos i + \cos\theta \cdot \sin i \cdot \cos\rho$$

longitude

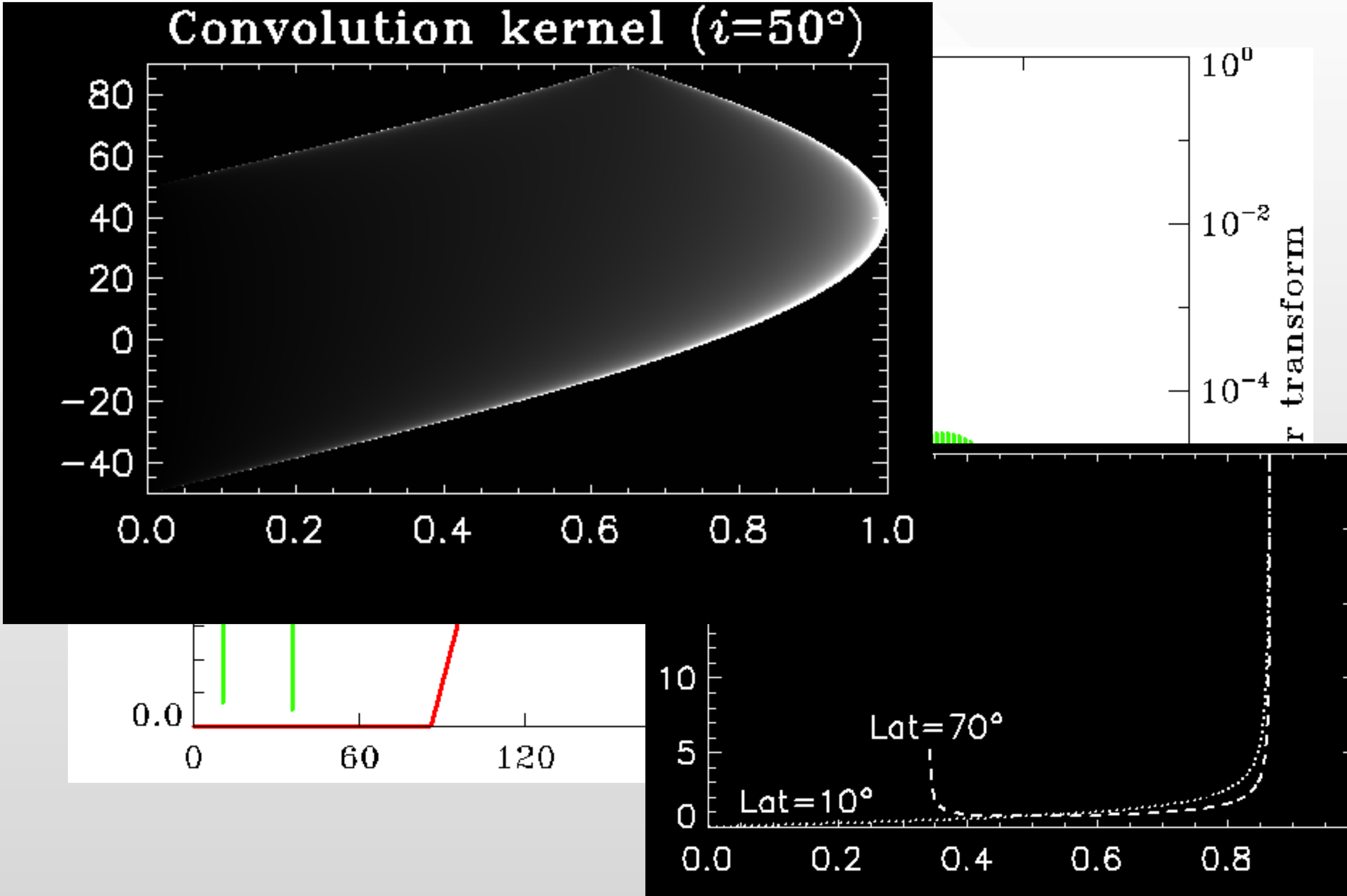
The kernel is simply the projected surface element. How does it depend on μ? What is the area distribution function for μ?

$$\frac{d\sigma}{d\mu} = \frac{2\mu\cos\theta}{\sqrt{(\cos\theta\sin i)^2 - (\mu - \sin\theta\cos i)^2}}$$

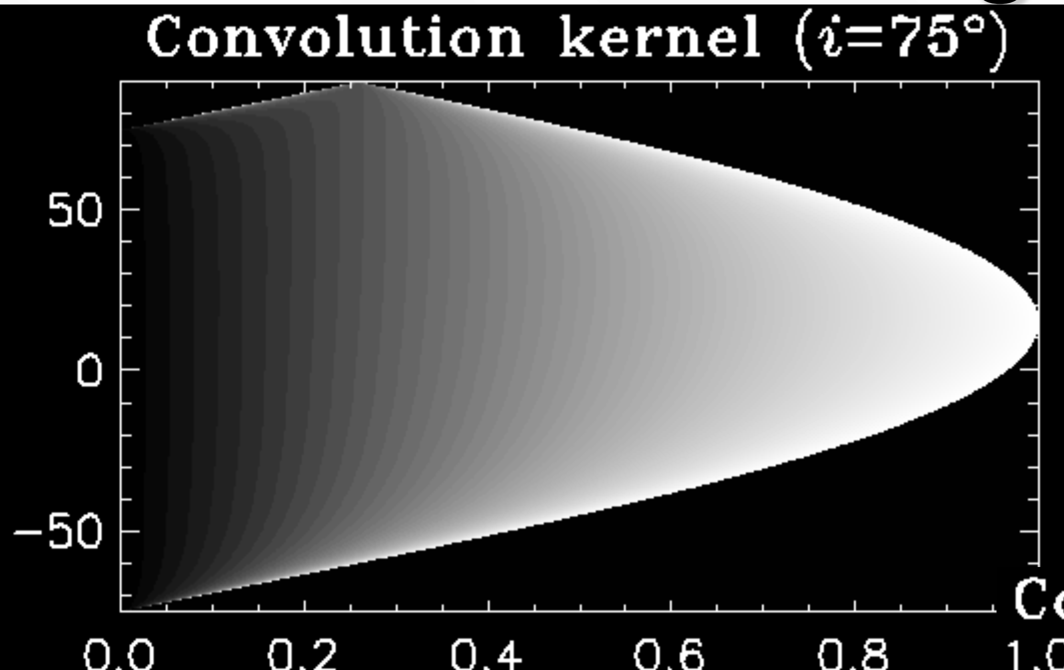
Kernel distribution function



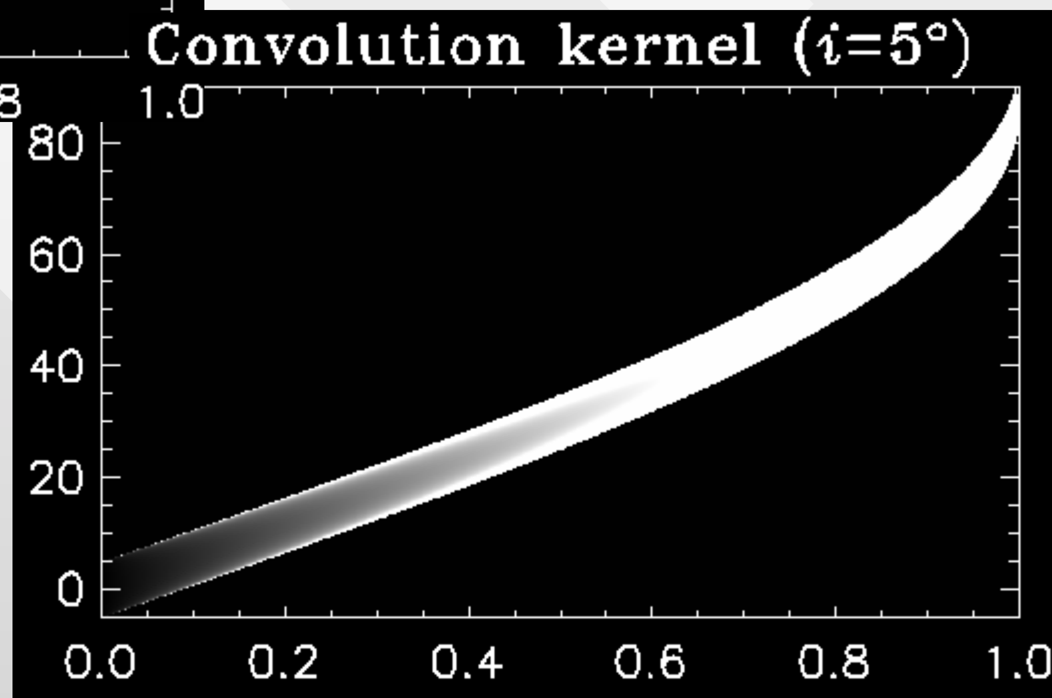
Kernel distribution function



Kernel degeneracy



<u>Type I:</u> linear dependence of kernels for different latitudes



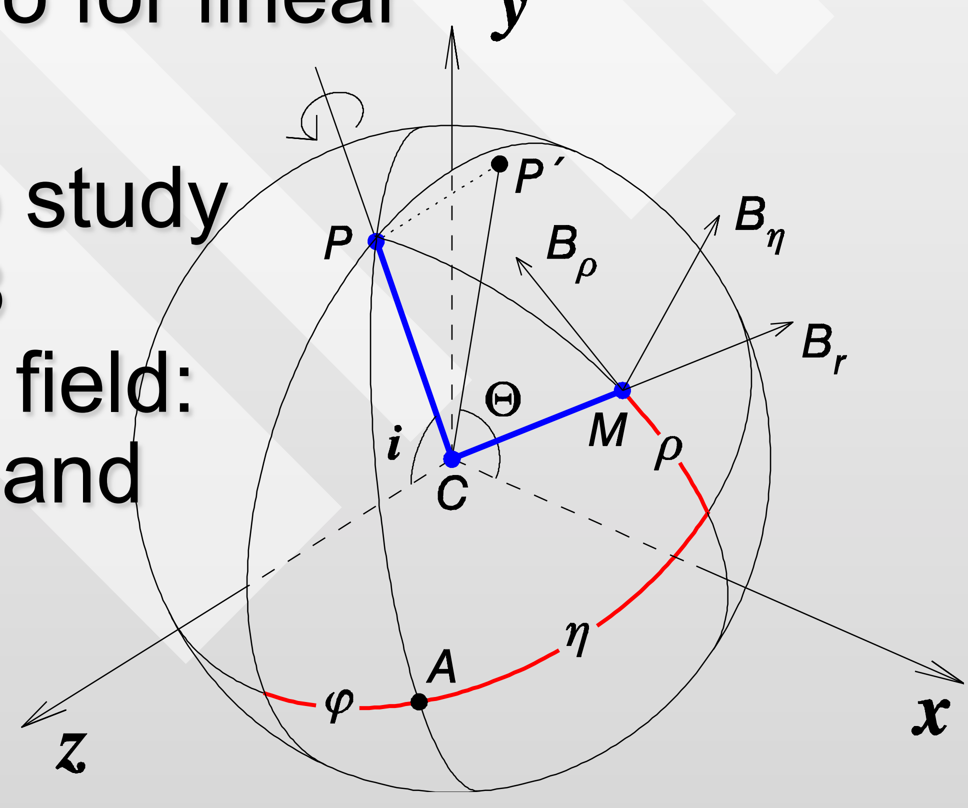
<u>Type II:</u> constant or – shape kernel (zero Fourier components)

Kernel properties

1. Kernels are not degenerate for intermediate inclinations
2. Kernel shape(s) does not produce zero Fourier components $\Rightarrow$ unique deconvolution is possible
3. Integration over latitudes may result in degeneracy only if a spectral feature totally disappears for $\mu<\mu_{limit}$
4. So far we ignored Doppler shifts

Magnetic fields

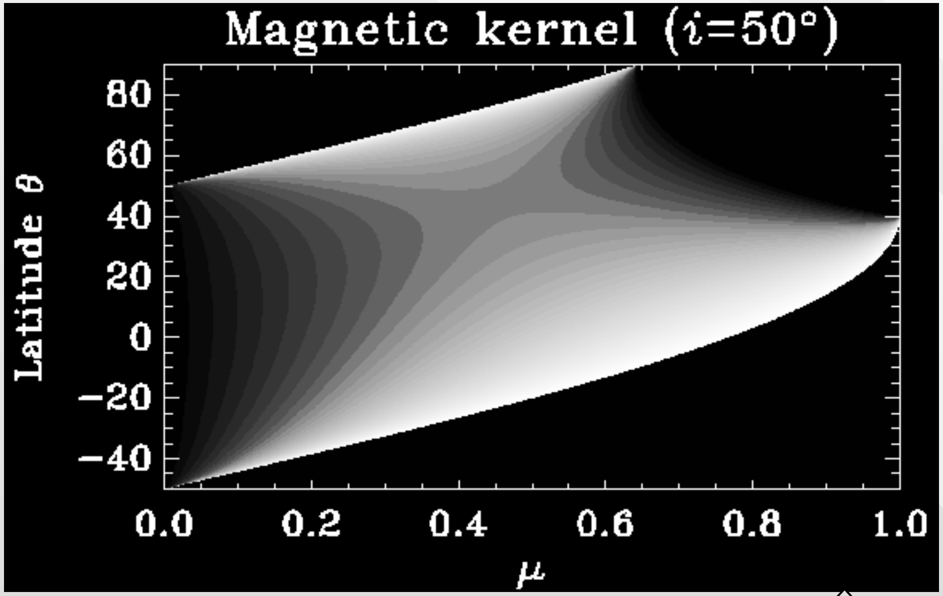
- Magnetic fields split kernel in three separate types: one for circular polarization and two for linear
- The easiest way to study them is to look at 3 components of the field: radial, longitudinal and meridional

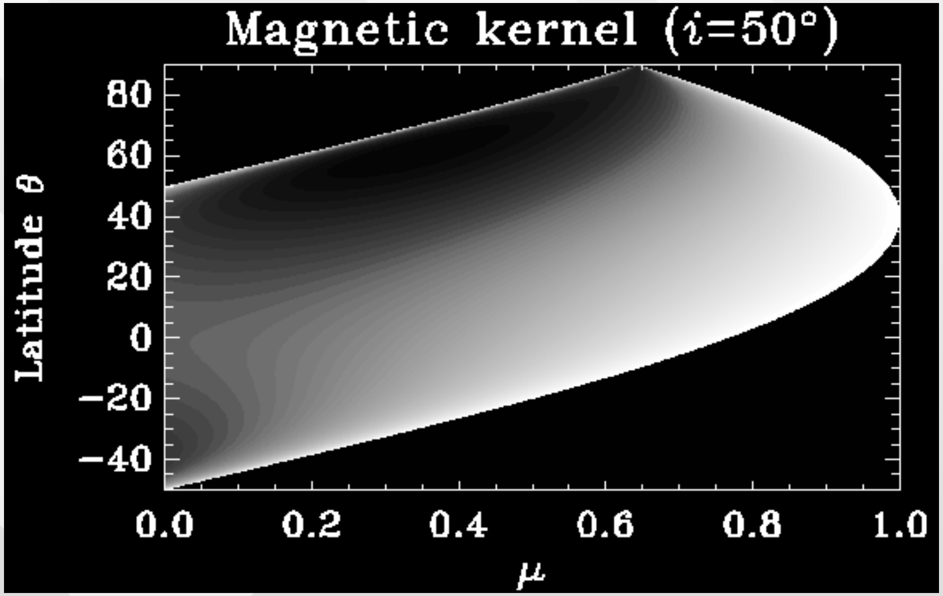


Magnetic kernels for circular polarization

$$\frac{d\sigma}{d\mu} = \frac{\mu^2 \cdot \cos\theta}{\sqrt{\cos\theta \sin i - \left(\mu - \sin\theta \cos i\right)^2}}$$

Radial field

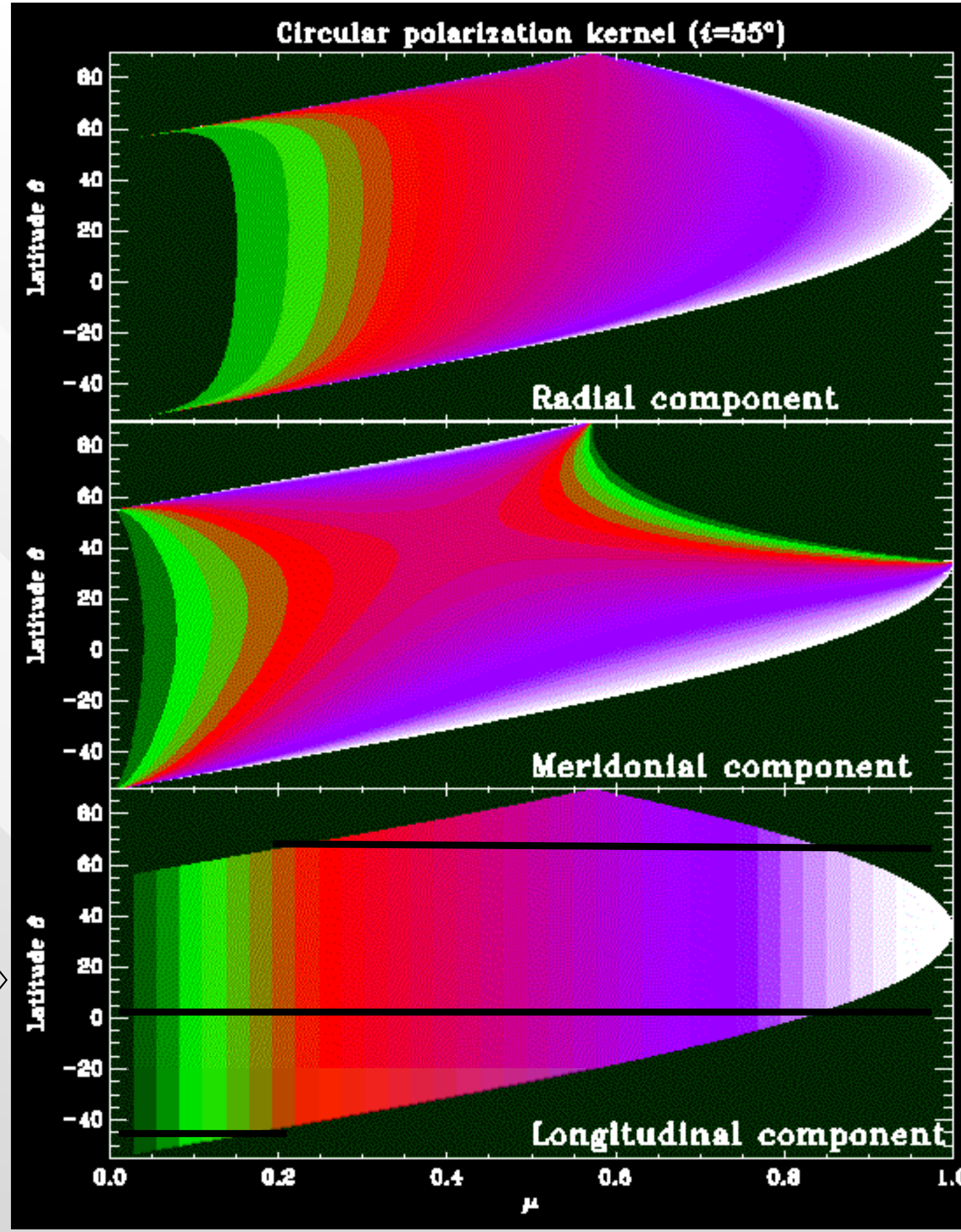




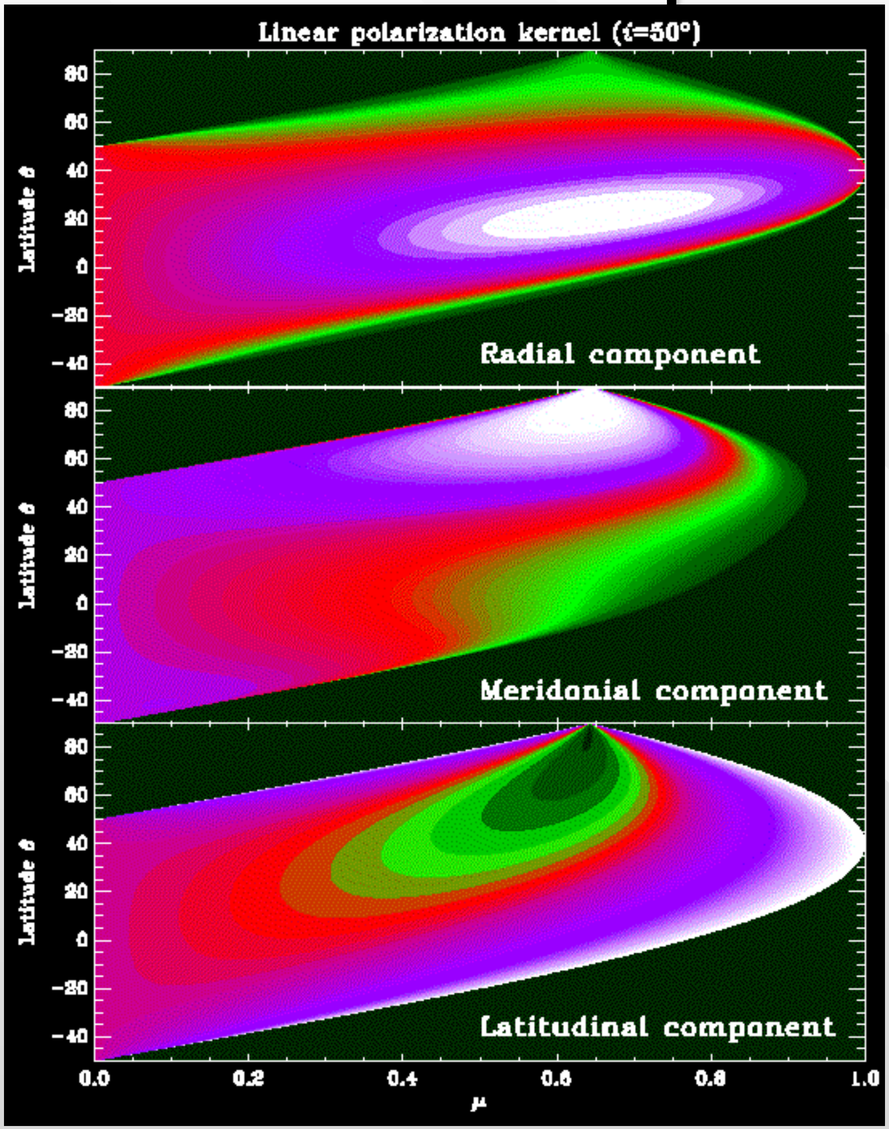
Meridional field

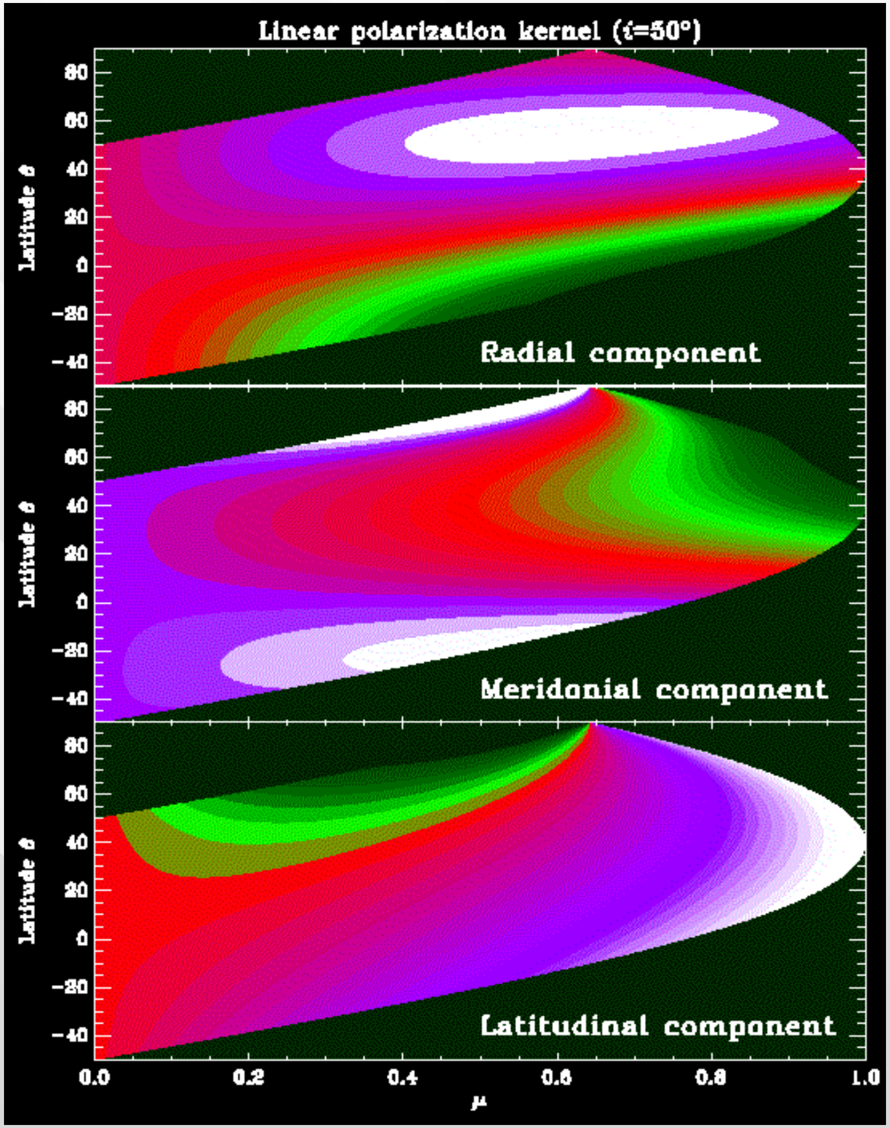
$$\frac{d\sigma}{d\mu} = \frac{\cos i - \sin\theta \cdot \mu}{\sqrt{\cos\theta \sin i - \left(\mu - \sin\theta \cos i\right)^2}}$$

Circular polarization alone is not enough

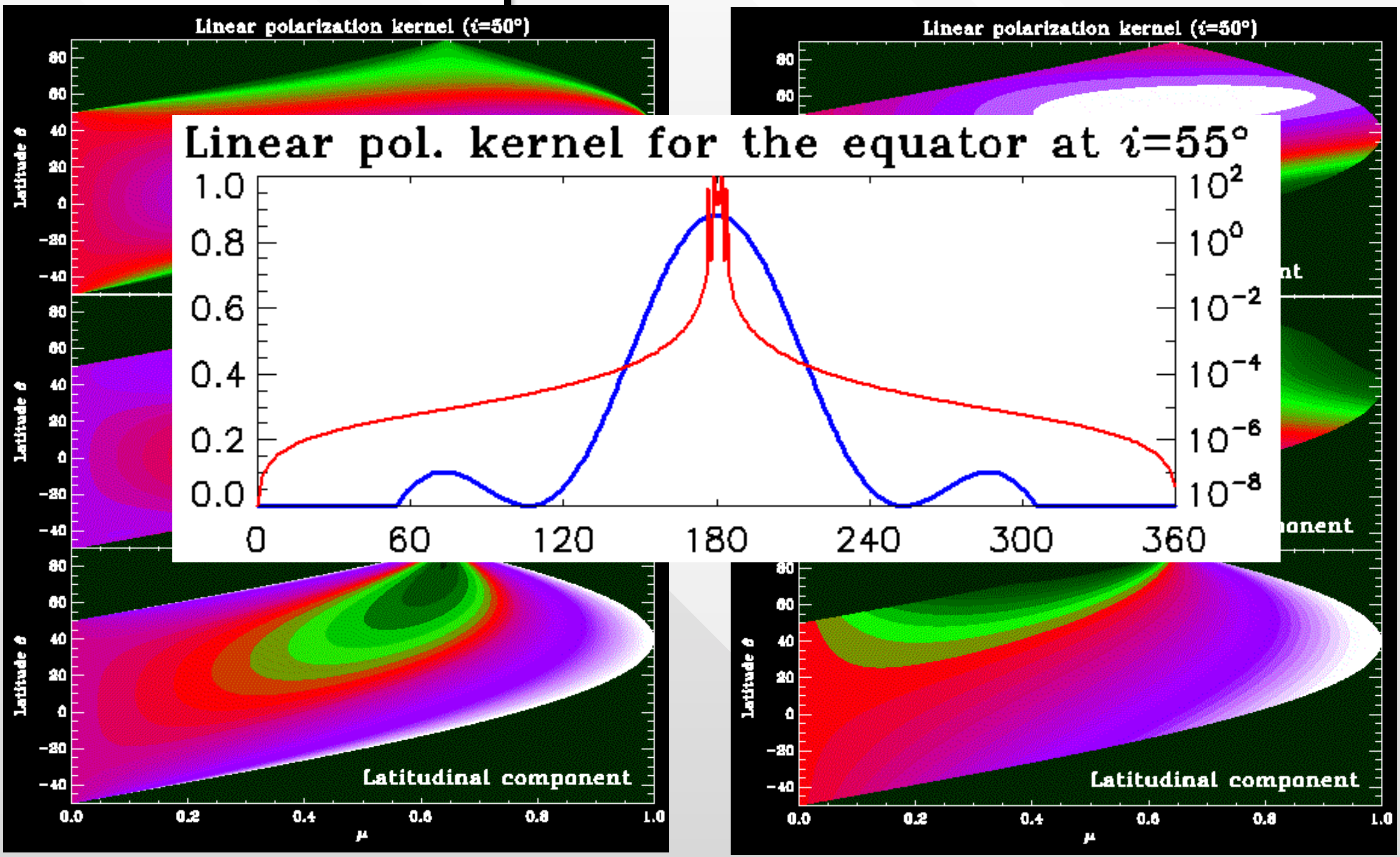


Magnetic kernels for linear polarization

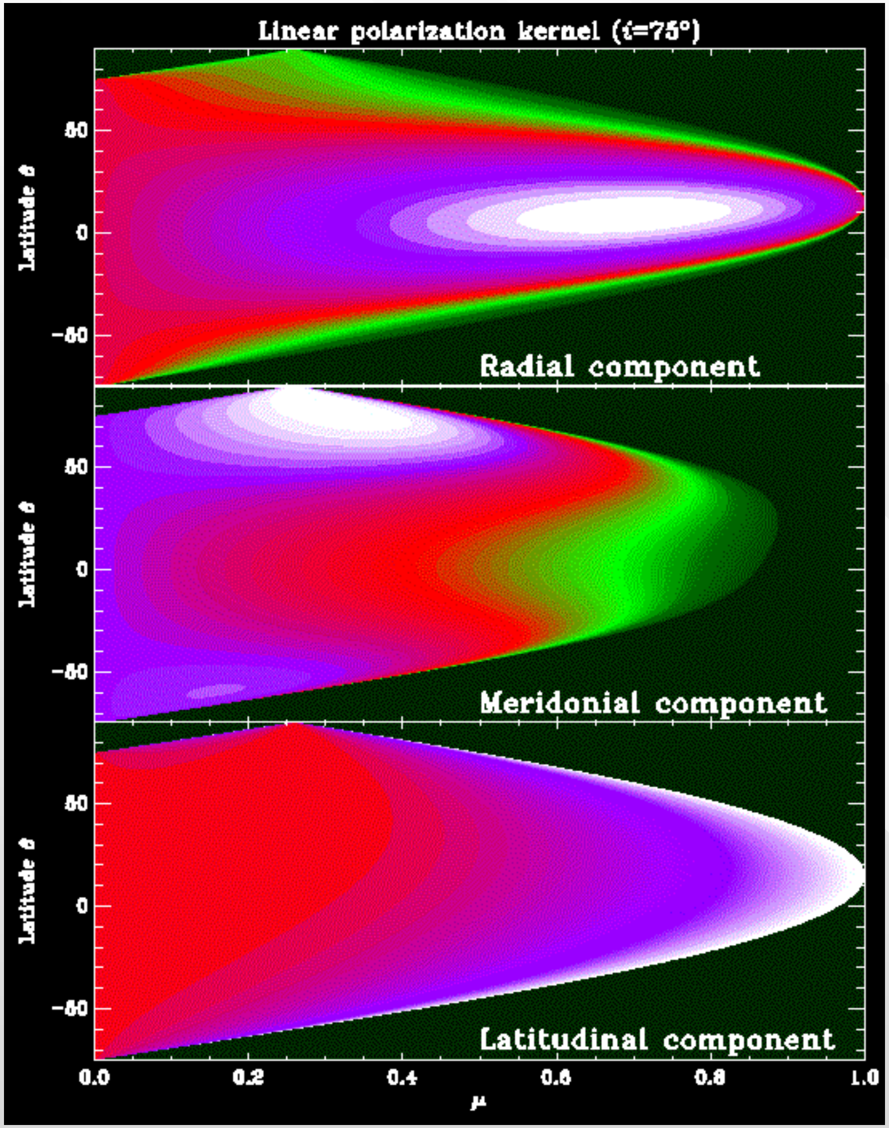


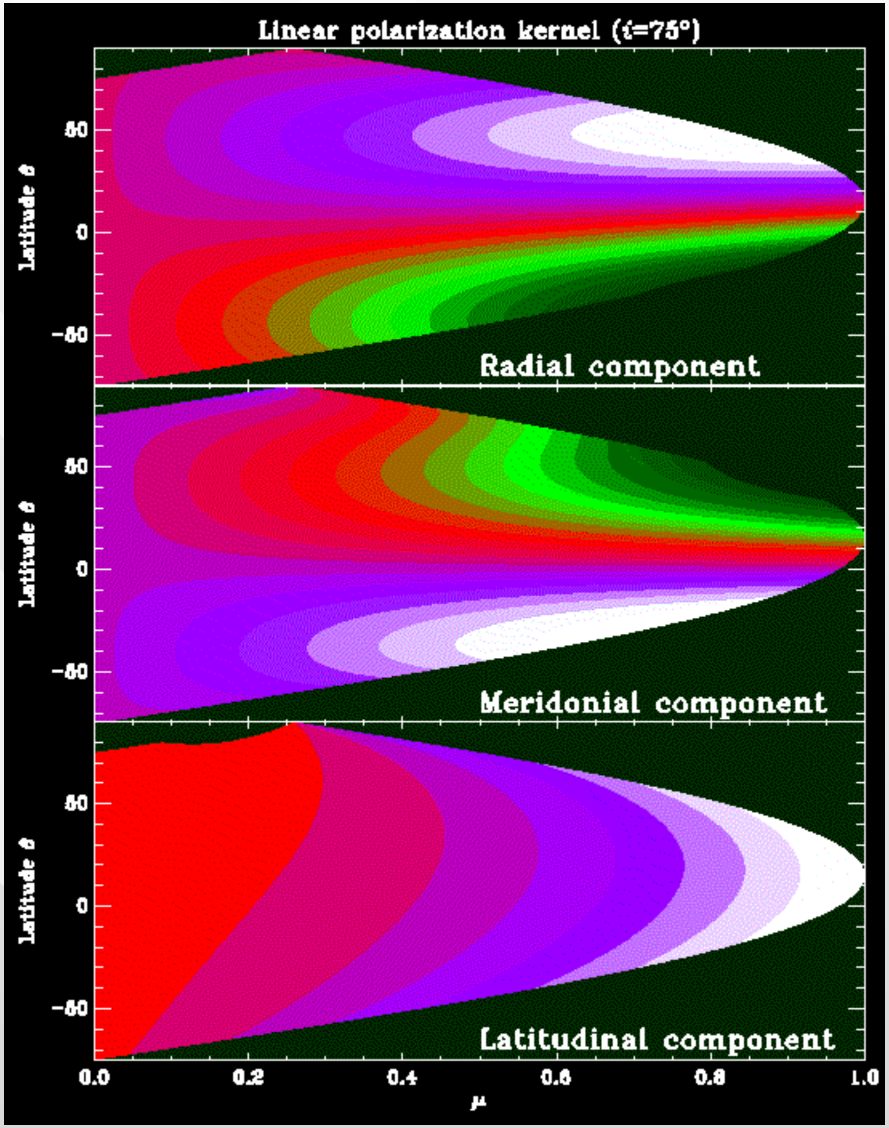


Magnetic kernels for linear polarization

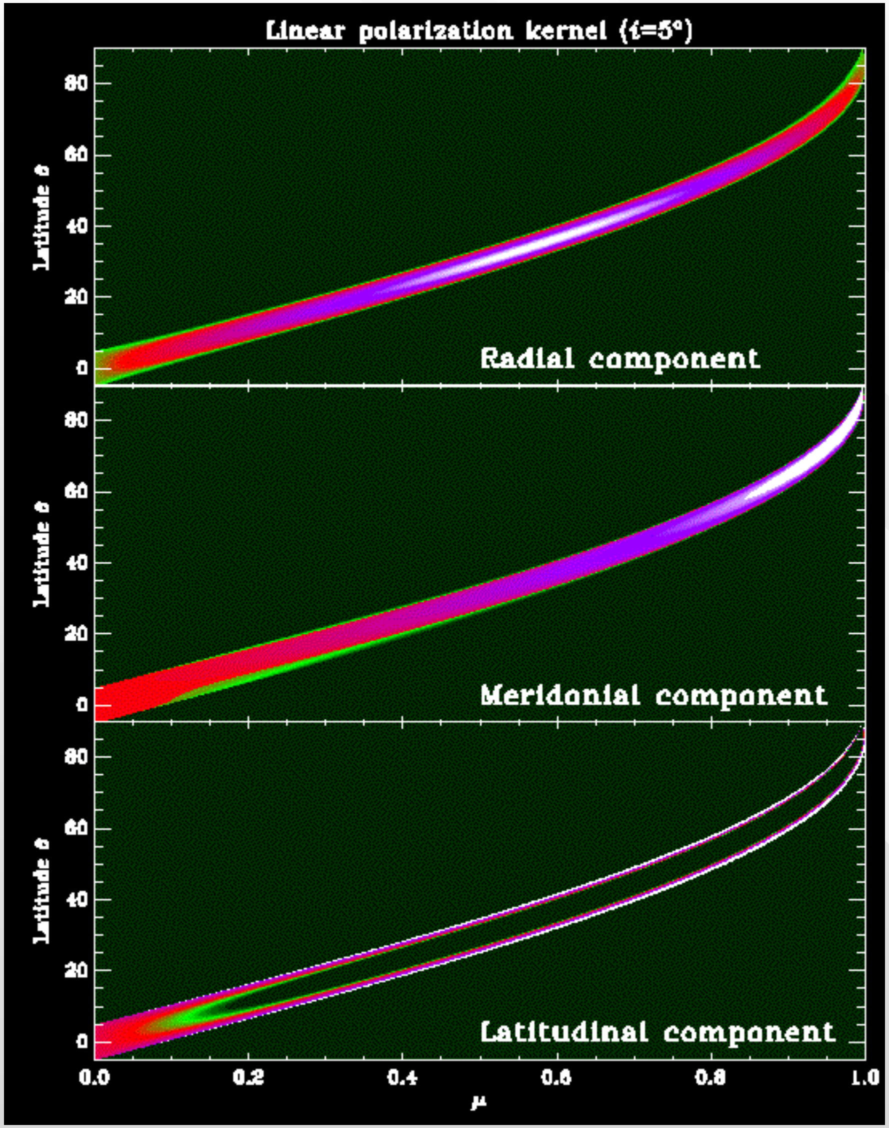


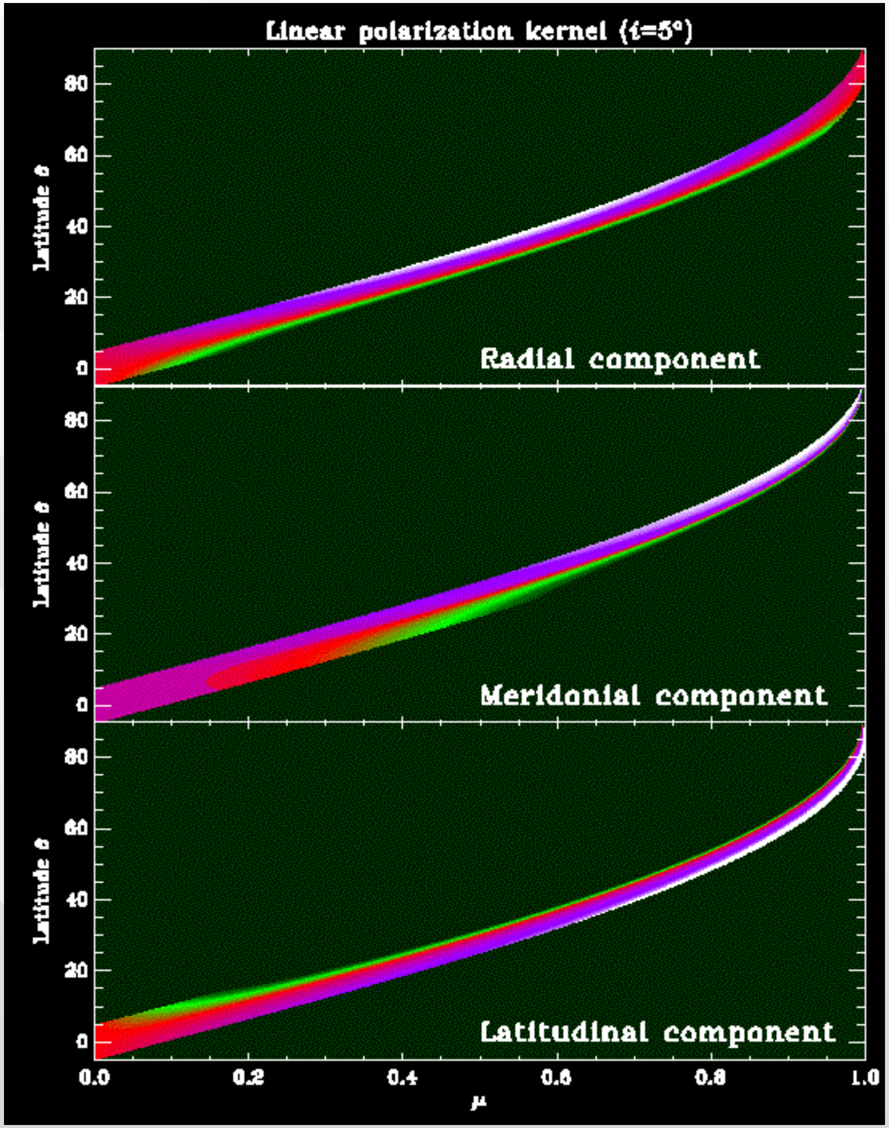
What about difficult inclinations?





… and pole-on:


Linear polarization kernel (i=5°)
Latitude θ
Radial component
Meridonial component
Latitudinal component
μ


Linear polarization kernel (i=5°)
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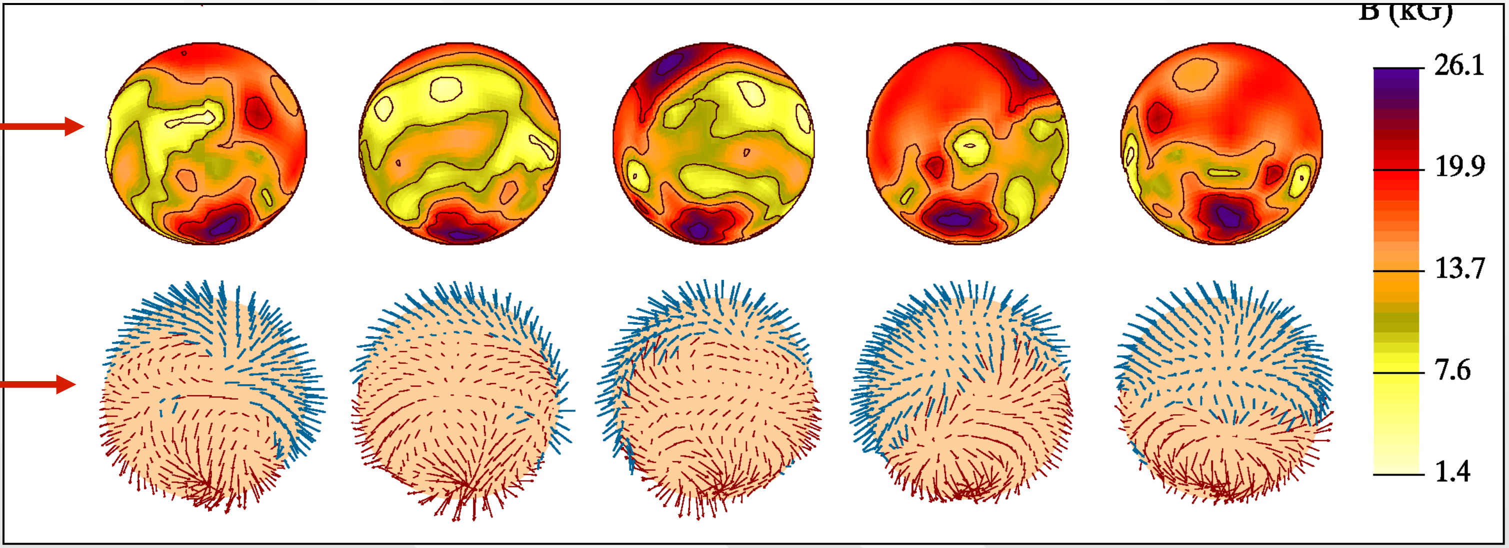
μ

CP stars: MDI results

Magnetic field topology of 53 Cam inferred with MDI

field strength



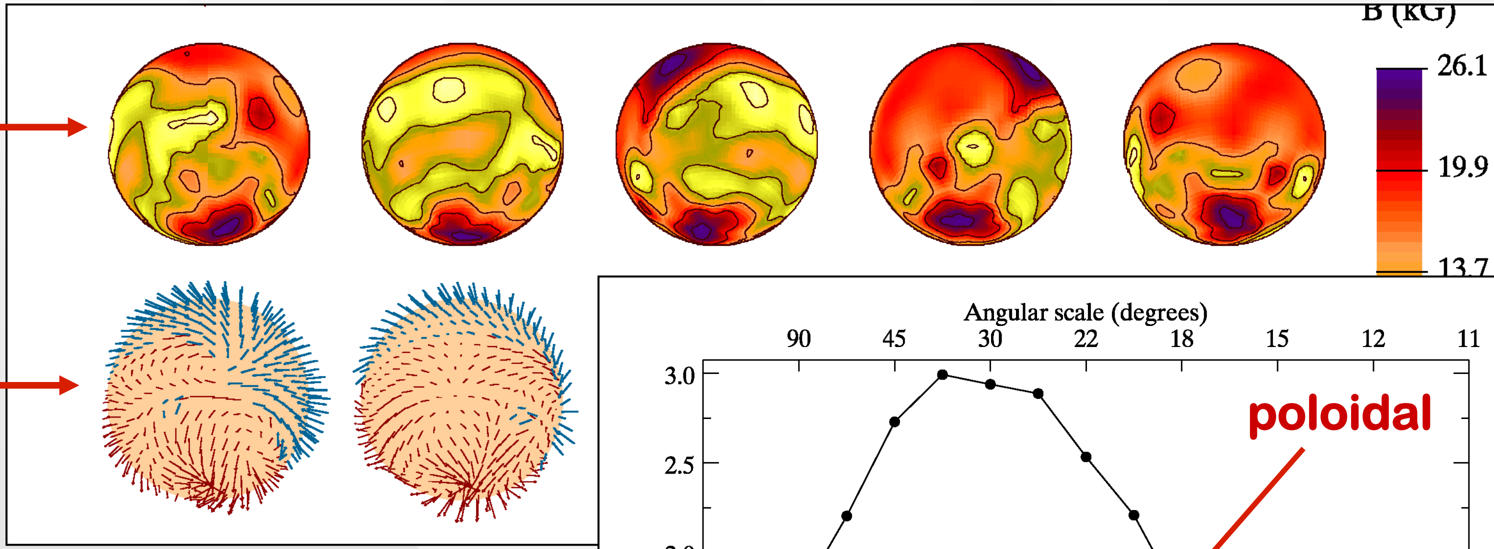
field orientation

inward
outward

Kochukhov et al. (2004)

CP stars: MDI results

Magnetic field topology of 53 Cam inferred with MDI



field strength

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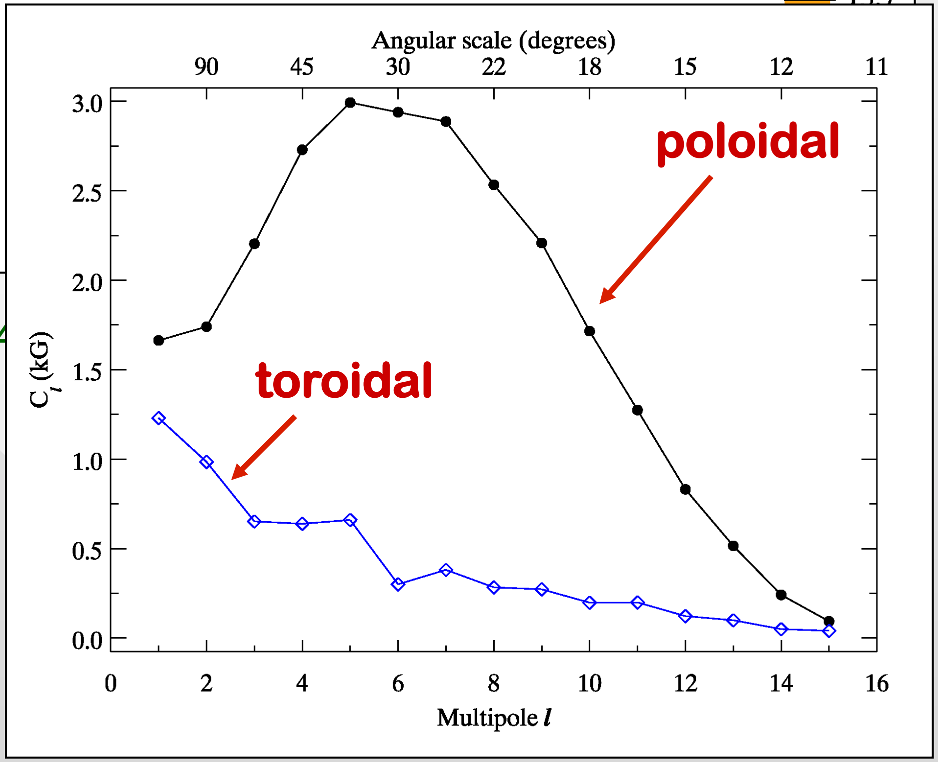
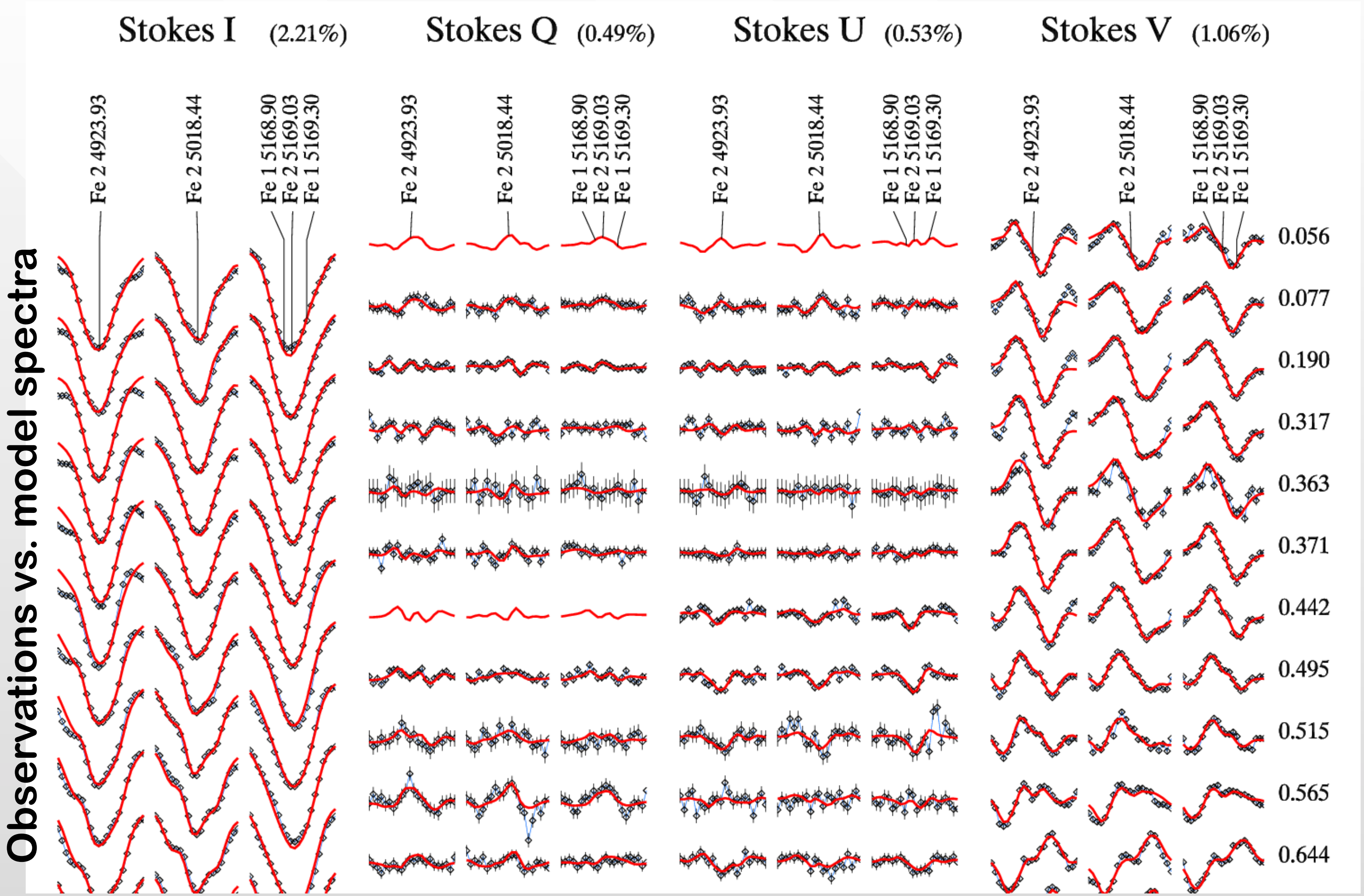


multipolar expansion

Spectra reconstruction for 53 Cam



Other applications of DI

Velocity fields and pulsation mode identification (radial pulsator HR3831)