UPPSALA UNIVERSITET

Lecture 5:
Doppler Imaging of Stars

NORDITA Master Class 2006

Motivation

- Major improvements in the models (e.g. description of physical processes, numerical methods etc.)
- Major improvements of the observations (precision, angular, spectral and time resolution, spectral range from X-ray to IR and radio etc.)
- In order to compare models with observations we learned how to simulate the observations
- The range of spatial and time scales in calculation and observations do not quite match. E.g. in the Sun the two physically important scales: dissipation of turbulent energy and photon mean free path at the surface are under 100 km. 100 km is the best spatial resolution achieved by observations.
- Inverse problem approach attempts to incorporate observations directly into the modelling

Forward problem

- Simulation of observations based on a model is called ***forward problem***
- Radiative transfer in semi-infinite non-absorbing medium: $I(\tau)=\int_0^\tau S(\tau)e^{-\tau}d\tau$
- Convolution: $g(y)=\int f(x)K(y-x)dx$
- More general case can be written in operator form: $g(x)=F(x,f)$
- Special case is for linear operators
  $$F(\alpha\cdot p+\beta\cdot q)=\alpha\cdot F(p)+\beta\cdot F(q)$$

Inverse Problem

- In all previous examples the properties of the model f are connected to the observations via an operator F describing physical relation between the model and the observables g
- In some cases one can construct an inverse operator F^{-1} such that the unknown function f can be found directly from observations g
- Convolution is one example:

$$g = k * f \Rightarrow \mathrm{g} = \mathrm{k} \cdot \mathrm{f} \Rightarrow \mathrm{f} = \mathrm{g} / \mathrm{k}$$

Fourier domain

Ill-posed problems

- Jacques Hadamard introduced the concept of an ill-posed problem in 1932. Examples of an ill-posed problems.
- Ill-posed problem essentially means that F^{-1} does not exist and there are multiple f that fulfill the operator equation

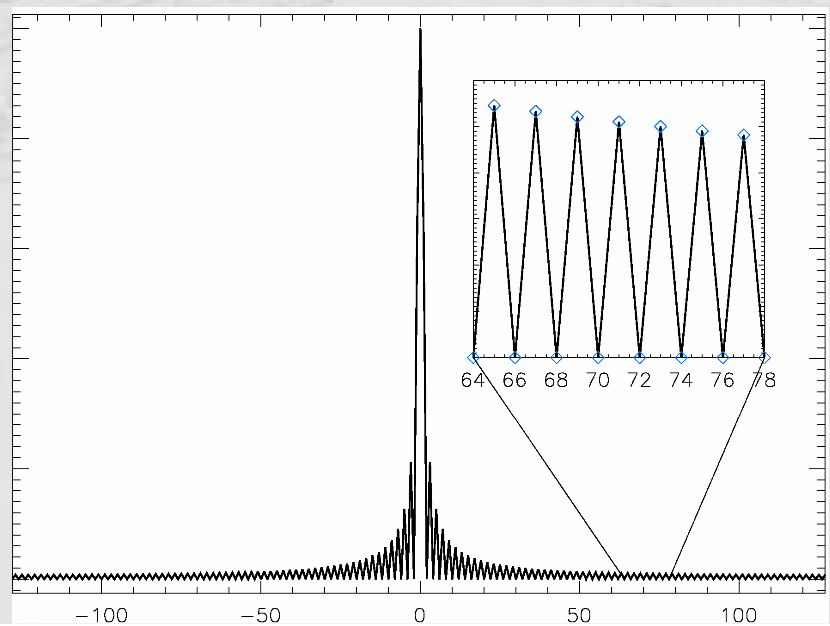
$$\mathrm{K}f = g$$

Solving an inverse problem

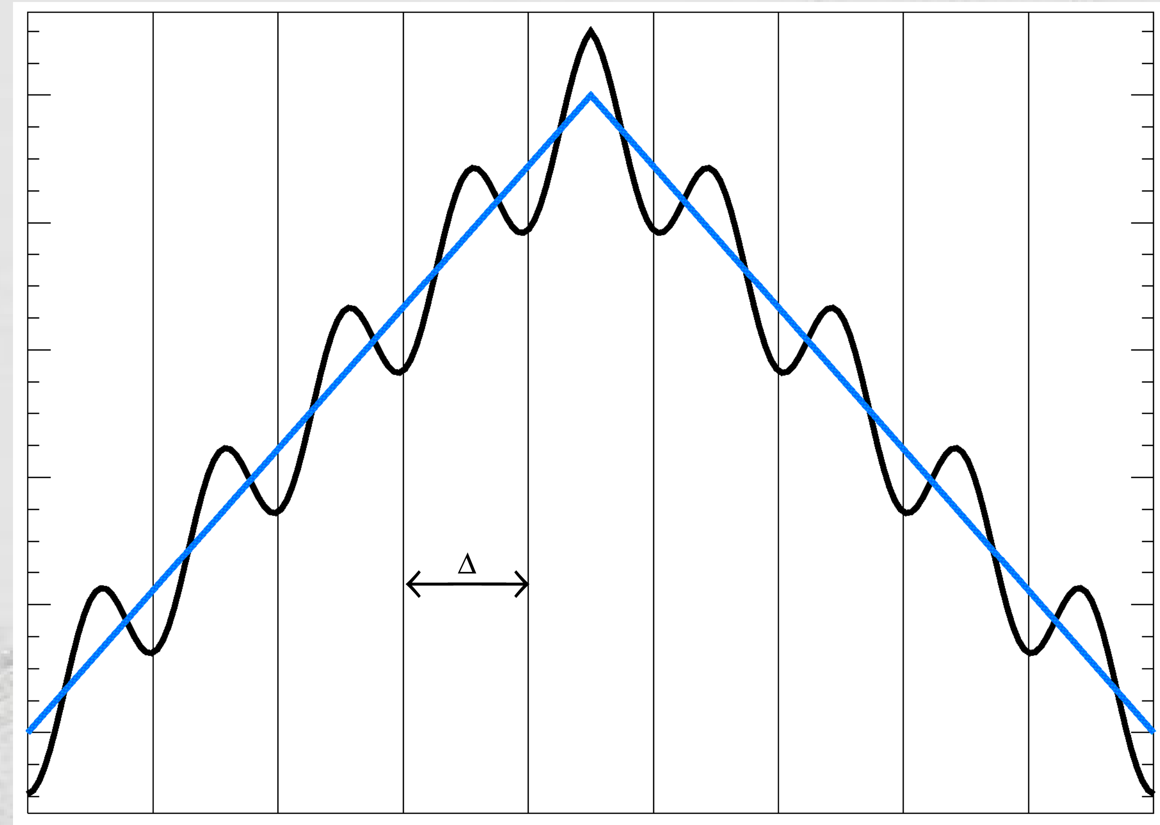
- Fixed pixel sampling is a convolution-type equation but cannot be solved easily (why?):

$$g_i = \int_{-\infty}^{+\infty} \Pi(x - \Delta \cdot i, \Delta) f(x) dx, \quad \Pi(x, \Delta) = \begin{cases} 1, & -\Delta/2 < x < +\Delta/2 \\ 0, & \text{elsewhere} \end{cases}$$

- Mathematically this problem has -shape kernel
- The Fourier transform of such function (amplitude) looks like this (every 2nd Fourier component is zero):



Clearly, with a single set of observations we will not be able to distinguish the following two functions:



although the *g*'s for them are identical.

Solving an ill-posed problem

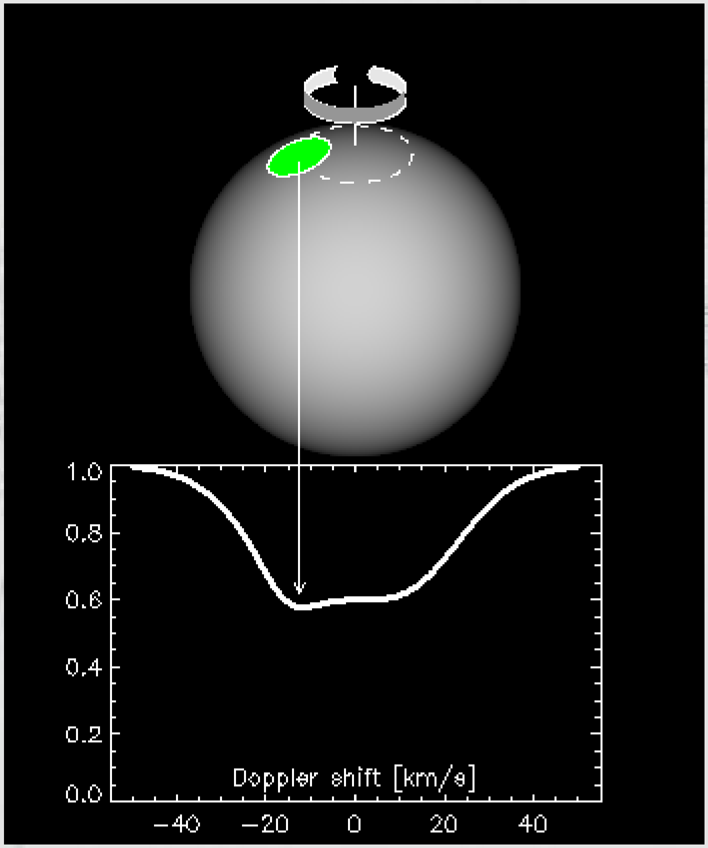
- Instead of constructing F^{-1} we solve an optimization problem:

$$\Phi(f) = \|g - Ff\| + \Lambda \cdot Rf = \min$$

- R is a special (regularization) operator that restricts the space of possible solutions ensuring uniqueness
- Λ is the so-called *Lagrangian multiplier* that establishes the balance between the two parts of Φ
- The goal is to find function f which realizes the minimum of Φ

Doppler Imaging of Stars

- Now we want to apply this to stars.
- 3D structures in stellar atmospheres affect the emerging spectra due to temporal modulation.
- Temporal modulations provided by pulsation (radial direction) or rotation (tangential direction). Here is how it works:



DI formulation

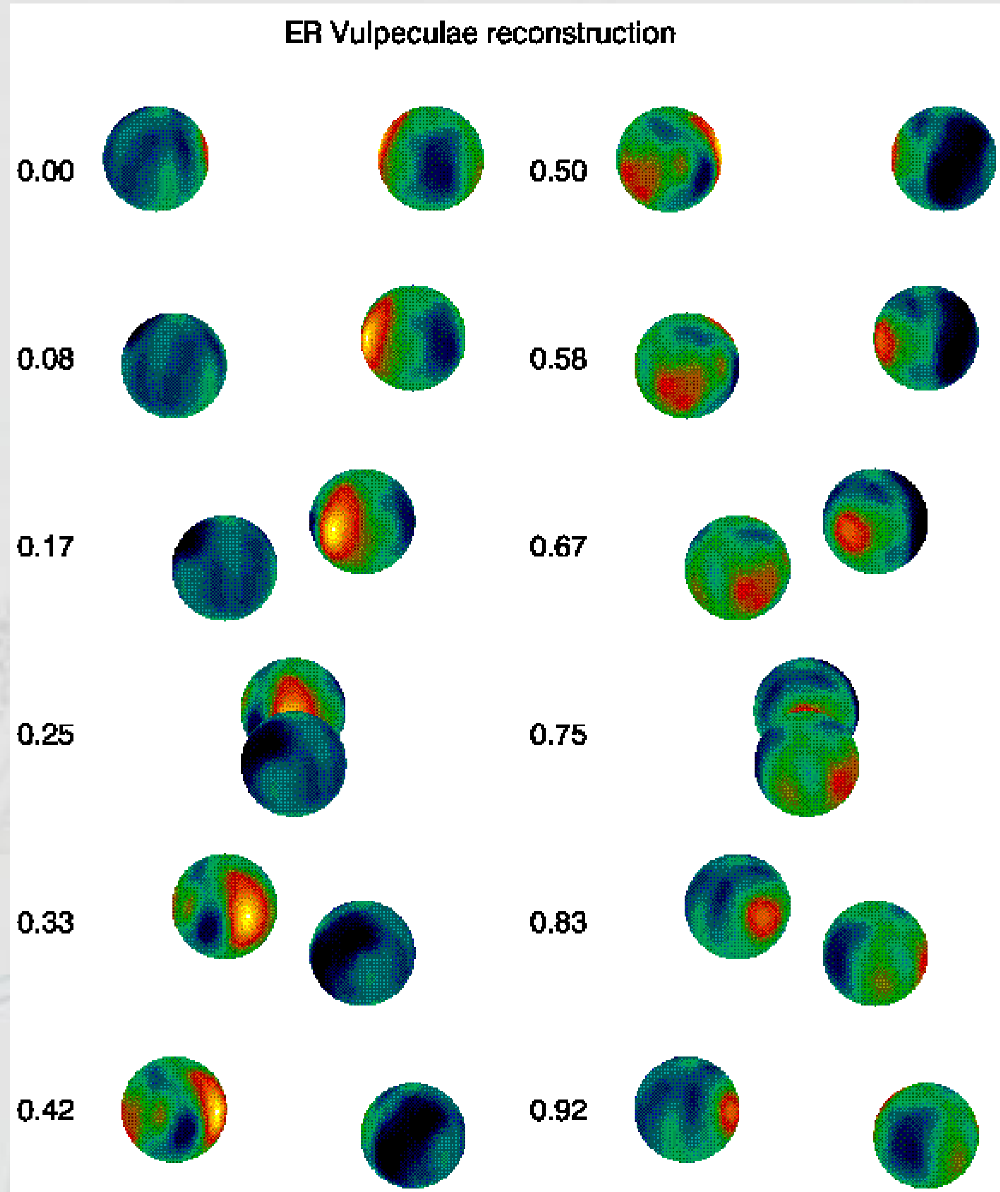
- The optimization problem now looks like this:

$$\sum_{\phi\lambda} \omega_\phi \cdot \left[F_{\phi\lambda}^{\text{obs}} - F_{\phi\lambda}^{\text{comp}}\left(T\right) \right]^2 + \Lambda \cdot \left| \overline{\nabla T} \right| = \min$$

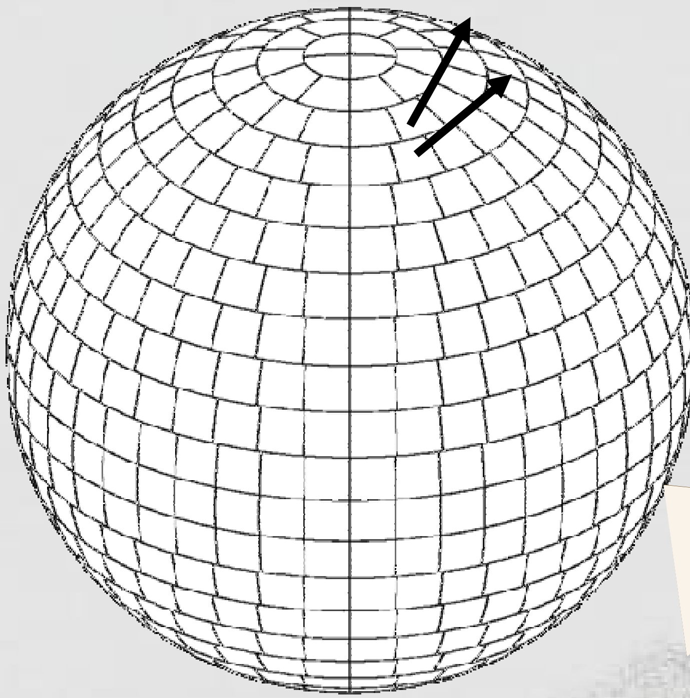
- $F_{\phi\lambda}$ here are emission intensities integrated over the visible part of the stellar disk
- The summation is carried out over all wavelengths and times of observations
- ω_ϕ are the relative weights selected according to the quality of the data

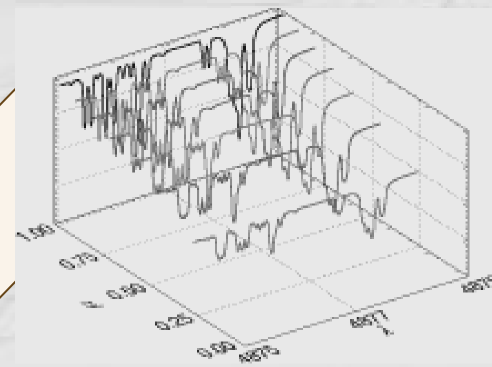
Results

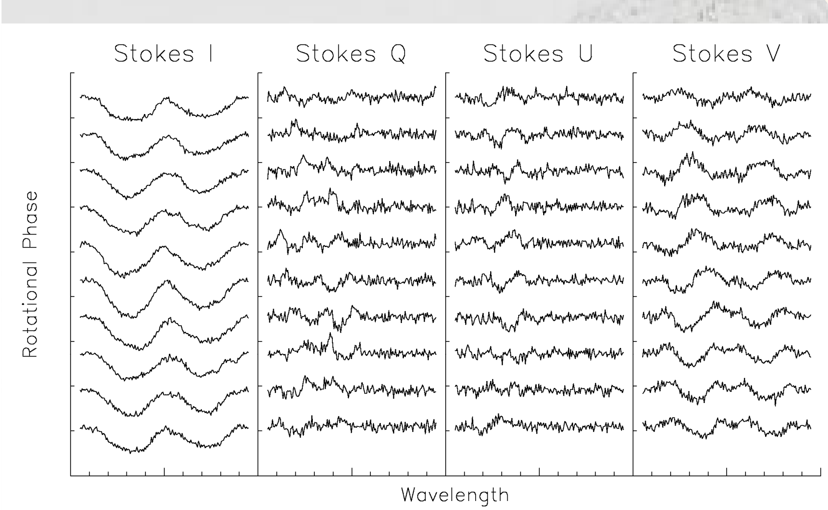
Surface temperature distribution in a close binary system:



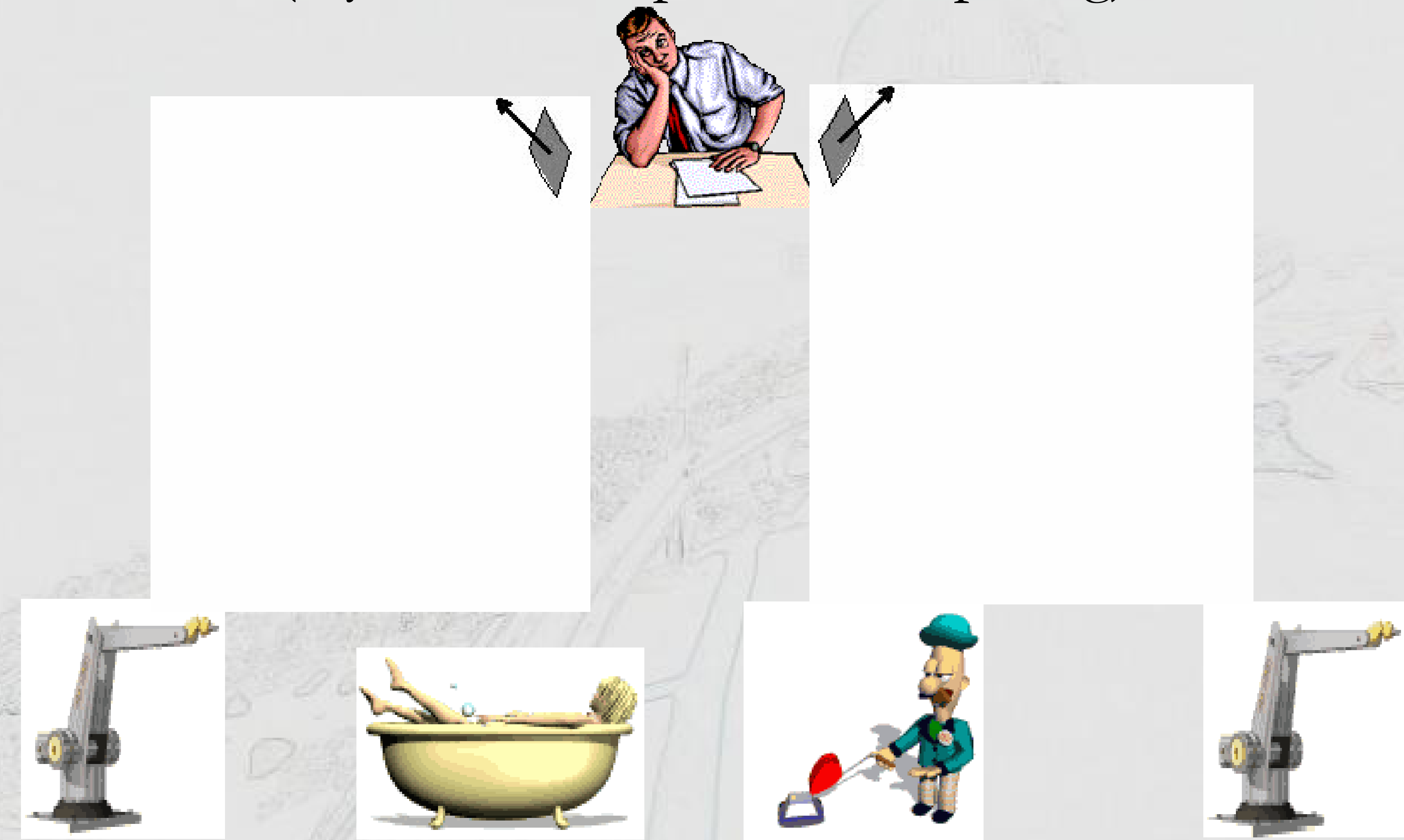
New Methods: MDI





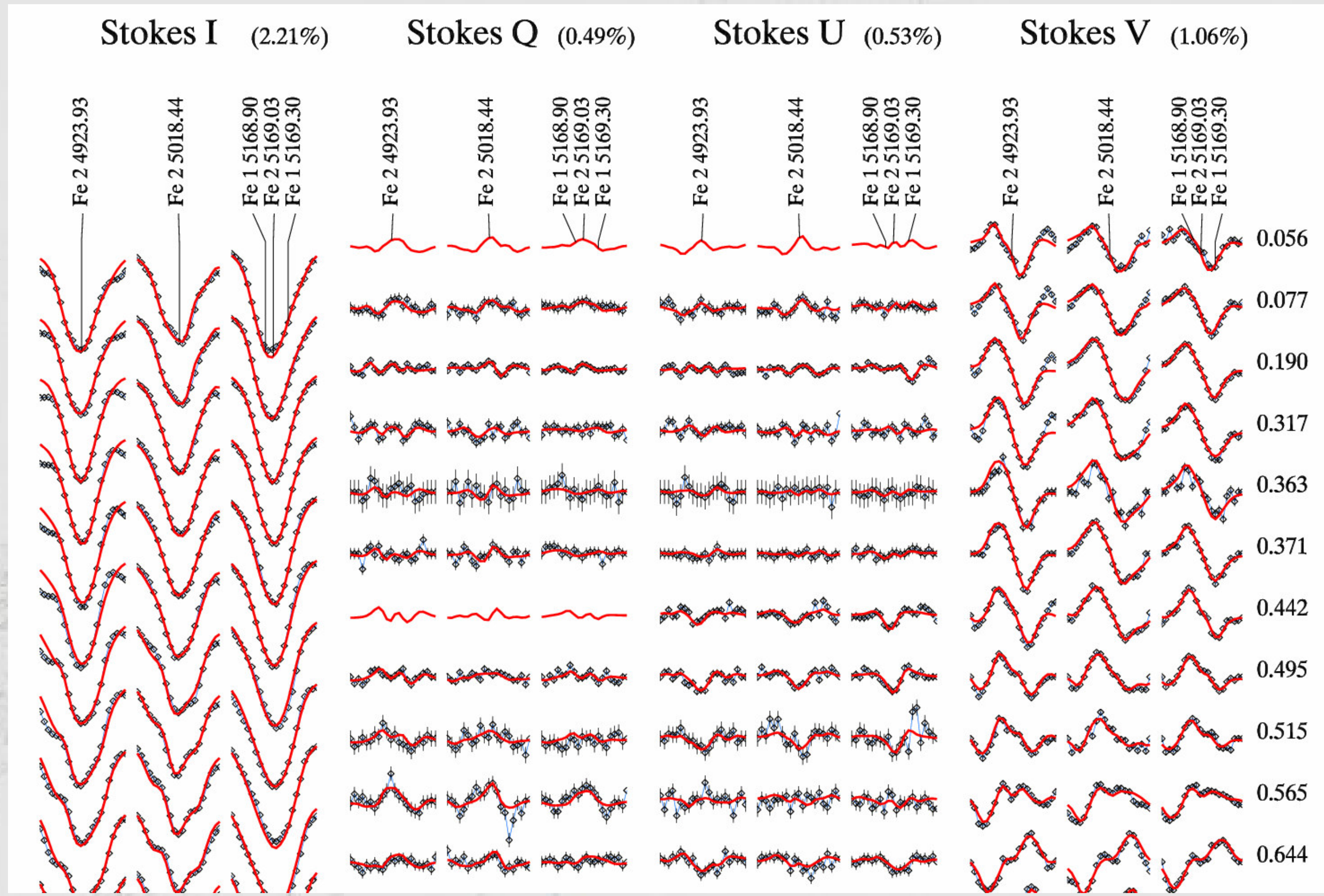


New computational scheme
(asynchronous parallel computing)



Magnetic Doppler Imaging of 53 Cam

Observations vs. model spectra



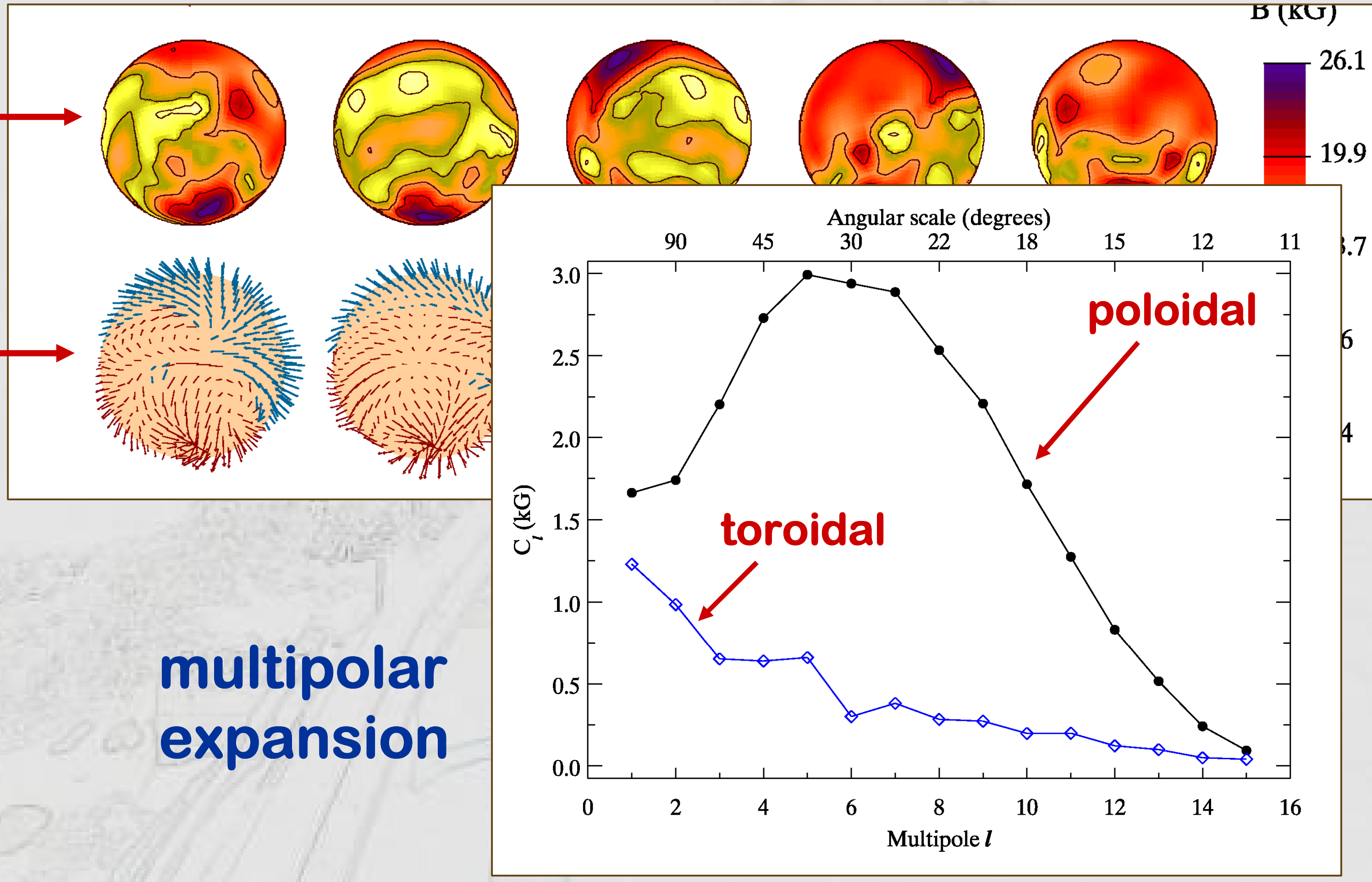
Magnetic Doppler Imaging of 53 Cam

field strength

field orientation

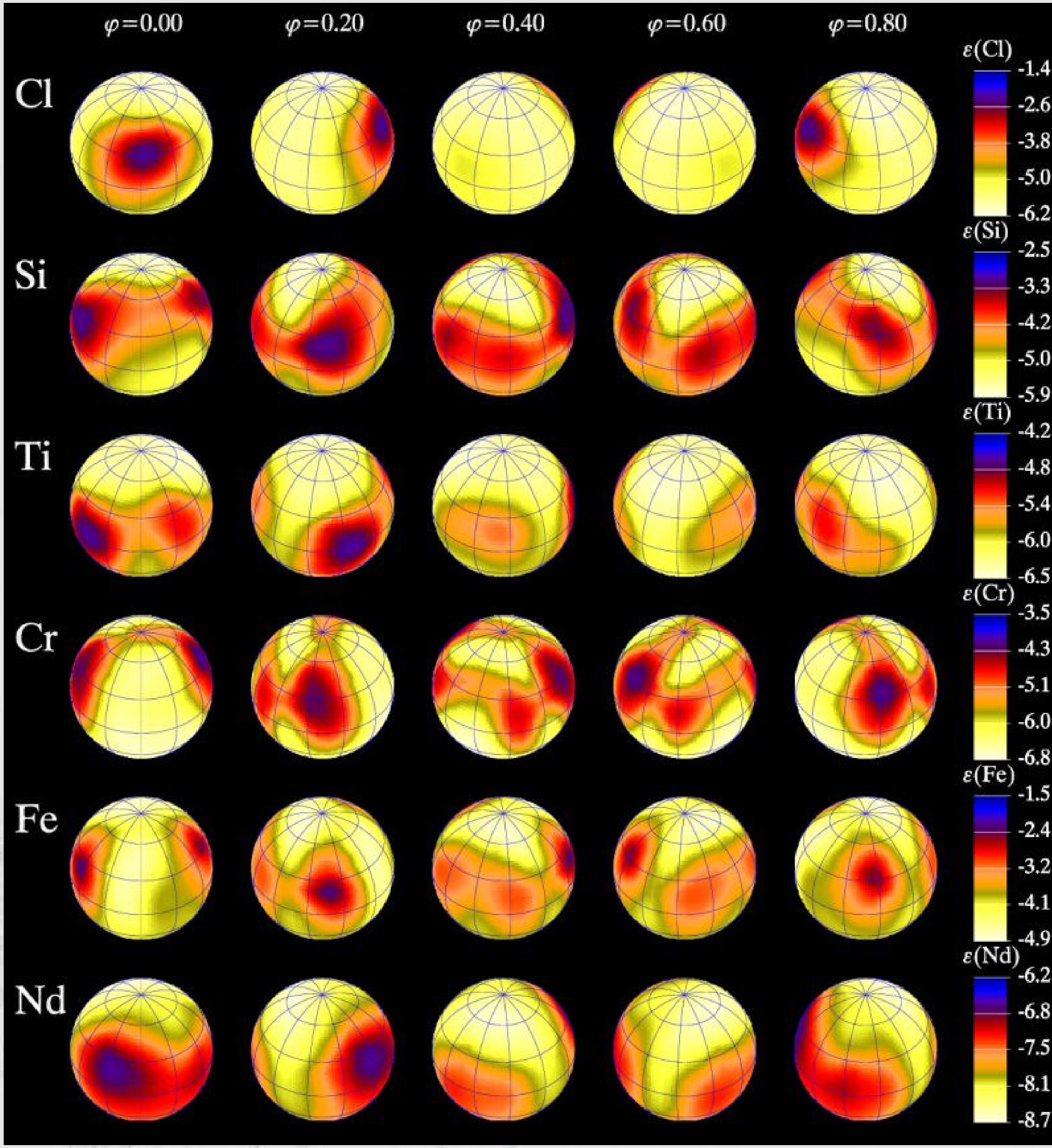
inward

outward



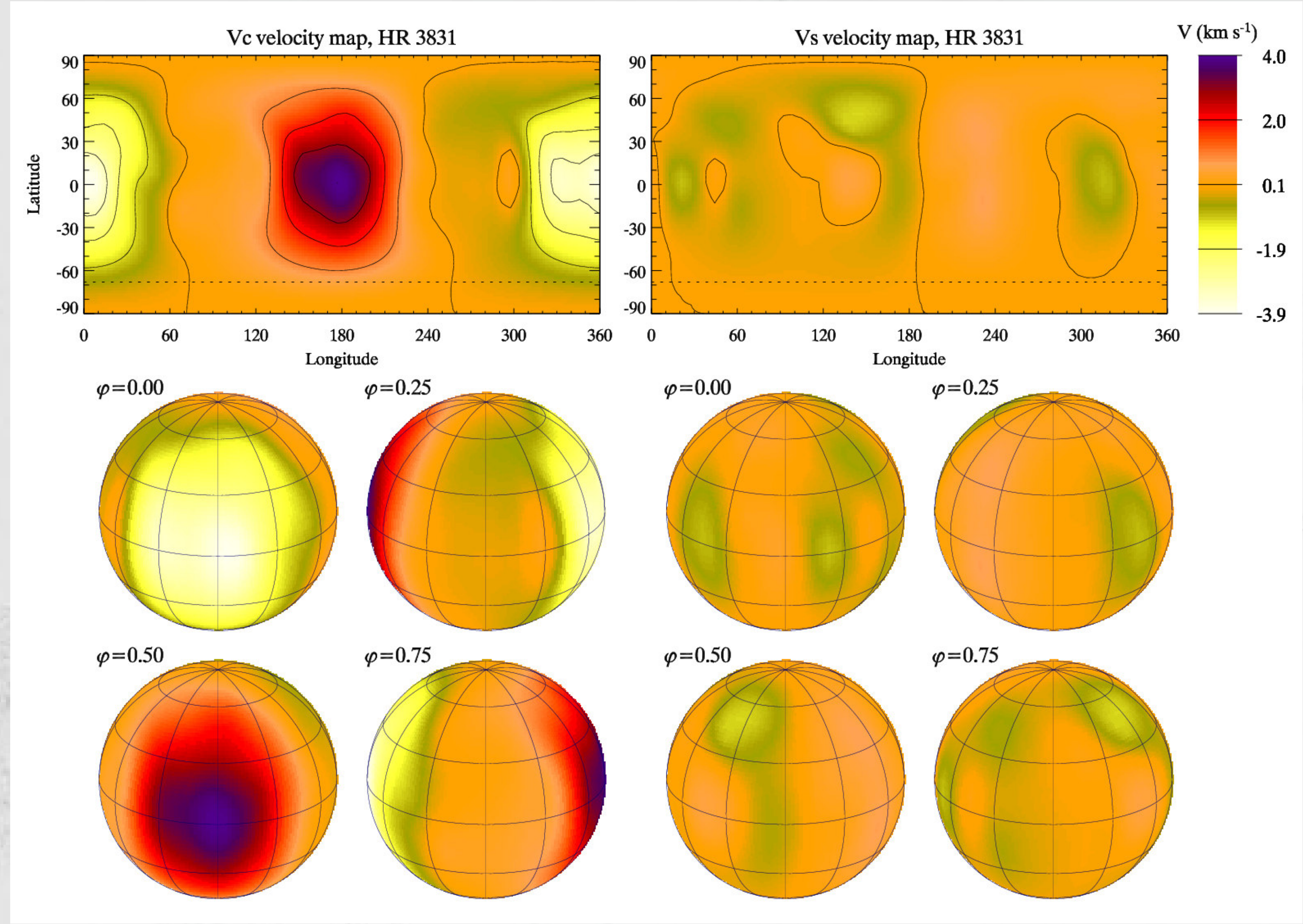
multipolar expansion

Chemical spots on α^2 CVn

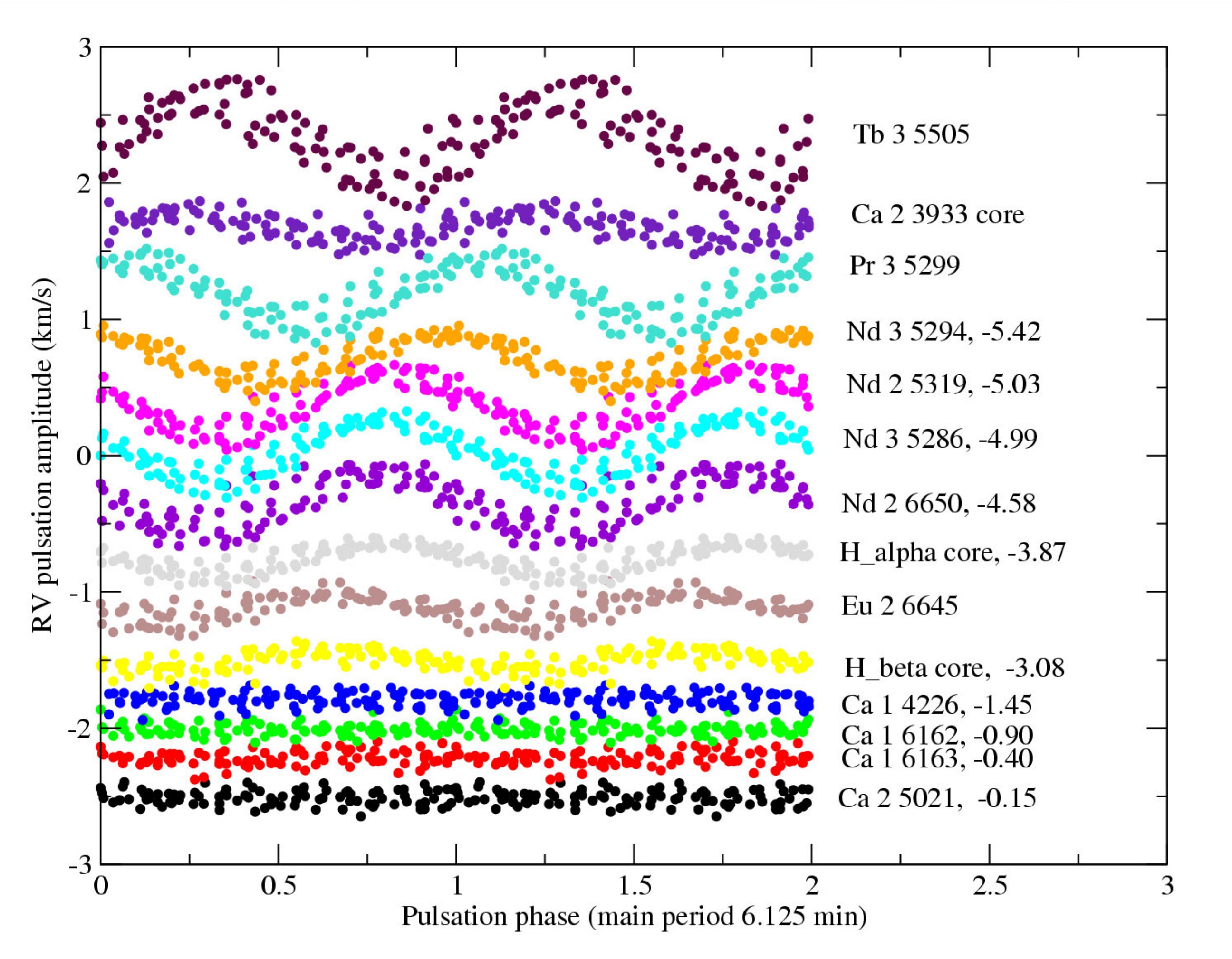


Pulsation Mode Identification

Pulsation velocities in HR 3831



Vertical Distribution of Pulsation Velocities



Conclusions

- We left behind the times when the theoreticians and observers were people from different planets. Not only they understand each other: no new model is seriously considered until it is confronted with observations.
- Models became highly sophisticated with lots of detailed microphysics and advanced numerical methods.
- These conclusions are relevant for many fields, not only to astrophysics

Discussion session

- ???