RT in the presence of magnetic field and ray tracing with Monte Carlo

Special environments where RT solvers discussed so far will fail

Magnetic fields and polarization

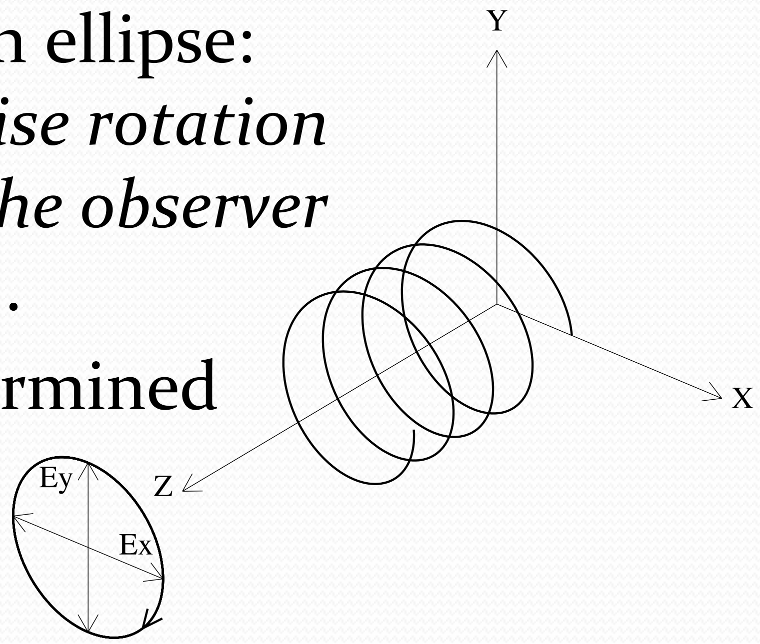
- In the presence of magnetic field the interaction of light and matter leads to polarization
- Polarization is characterized by the four Stokes parameters: I, Q, U and V (intensity, linear and circular polarization)
- These can be introduced by considering the electric field oscillation treating light as a wave propagating in *z*-direction:

$$E_x(t) = a_x \cdot e^{i(\phi_x - 2\pi tc/\lambda + 2\pi z/\lambda)}$$

$$E_y(t) = a_y \cdot e^{i(\phi_y - 2\pi tc/\lambda + 2\pi z/\lambda)}$$

Defining Stokes parameters

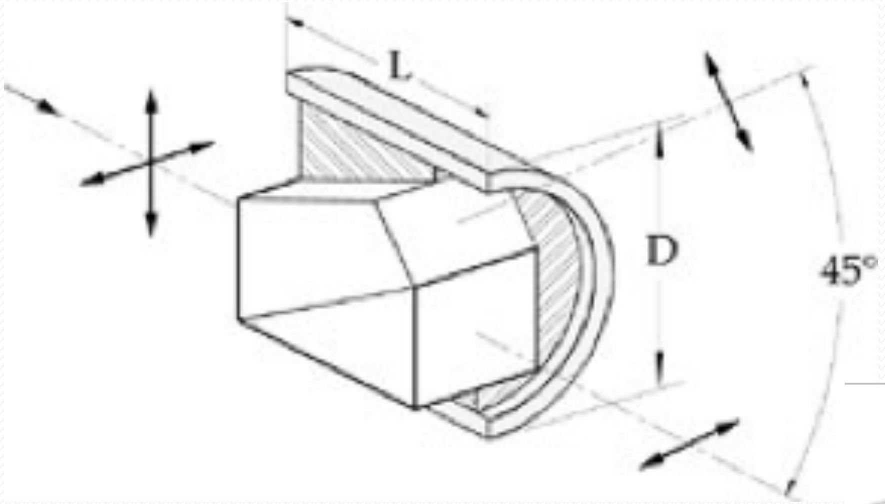
- *x* and *y* are two orthogonal directions in the plane perpendicular to the wave propagation (right-handed coordinate system).
- For a fixed point *z* the tip of the electric vector for a fully polarized wave will draw an ellipse: The convention is that *a clockwise rotation of the electric vector as seen by the observer is right-handed* (as in the figure).
- The direction of rotation is determined by the phase difference right-handed: $\delta \equiv \phi_x - \phi_y > 0$

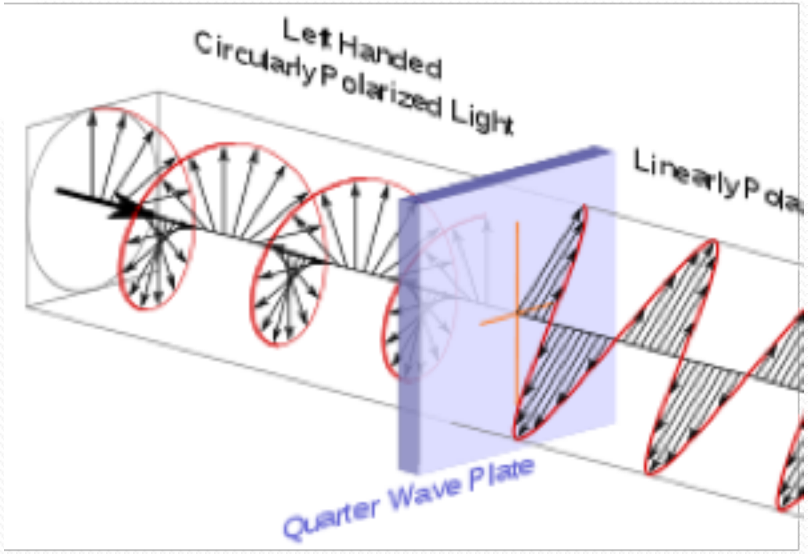


Defining Stokes parameters

- With optical tools we can (1) change the phase difference and (2) measure the amplitudes along *x* and *y* axes.
- Let's call $I_{\rightarrow}$, $I_{\uparrow}$ measurements taken along x and y and $I_{\nwarrow}$, $I_{\nearrow}$ along directions rotated by 45°. This will cover linear polarization.
- Shifting phase difference by 90 will convert circularly polarized to linear. The shift can be done along the *x*=y line resulting $I_{\circlearrowright}$ measured along *x* and $I_{\circlearrowleft}$- along *y* axis.

Beam splitter and retarder





Defining Stokes parameters

- The measured intensities are directly related to the Stokes parameters:

$$
\begin{aligned}
I &= I_{\rightarrow} + I_{\uparrow} = I_{\nwarrow} + I_{\nearrow} = I_{\circlearrowright} + I_{\circlearrowleft} \\
Q &= I_{\rightarrow} - I_{\uparrow} \\
U &= I_{\nwarrow} - I_{\nearrow} \\
V &= I_{\circlearrowright} - I_{\circlearrowleft}
\end{aligned}
$$

- We can also re-write this “instrumental” definition with the formal expressions for the amplitudes:

$$
\begin{aligned}
I &= a_x^2 + a_y^2 \\
Q &= a_x^2 - a_y^2 \\
U &= 2a_x a_y \cos\delta \\
V &= 2a_x a_y \sin\delta
\end{aligned}
$$

Properties of Stokes parameters

- Stokes parameters are additive:
  $(I, Q, U, V)_1 + (I, Q, U, V)_2 = (I, Q, U, V)_{1+2}$
- I and V are invariant against rotation around the propagation direction *z*
- Q and U are not invariant but transform in a simple fashion:

$$Q' = Q\cos 2\chi + U\sin 2\chi$$
$$U' = U\cos 2\chi - Q\sin 2\chi$$

RT equation for the Stokes vector

- Due to additive properties of the Stokes vector the equation looks similar to non-magnetic case:

$$\frac{d\mathbf{I}}{dz} = -\mathbb{K} \cdot (\mathbf{I} - \mathbf{S})$$

- The absorption is now a 4x4 matrix:

$\mathbb{K} = \kappa_c \mathbf{1} + \kappa_l \mathbf{\Phi}$ and in LTE $\mathbf{S} = (B_\lambda, 0, 0, 0)^\dagger$

$$\mathbf{\Phi} = \begin{pmatrix} \phi_I & \phi_Q & \phi_U & \phi_V \\ \phi_Q & \phi_I & \psi_V & -\psi_U \\ \phi_U & -\psi_V & \phi_I & \psi_Q \\ \phi_V & \psi_U & -\psi_Q & \phi_I \end{pmatrix}$$

Opacity matrix components

$$\phi_I = 1/2[\phi_p \sin^2\gamma + 1/2(\phi_r + \phi_b)(1 + \cos^2\gamma)]$$

$$\phi_Q = 1/2[\phi_p - 1/2(\phi_r + \phi_b)] \sin^2\gamma \cos 2\chi$$

$$\phi_U = 1/2[\phi_p - 1/2(\phi_r + \phi_b)] \sin^2\gamma \sin 2\chi$$

$$\phi_V = 1/2(\phi_r - \phi_b) \cos\gamma$$

$$\psi_Q = 1/2[\psi_p - 1/2(\psi_r + \psi_b)] \sin^2\gamma \cos 2\chi$$

$$\psi_U = 1/2[\psi_p - 1/2(\psi_r + \psi_b)] \sin^2\gamma \sin 2\chi$$

$$\psi_V = 1/2(\psi_r - \psi_b) \cos\gamma$$

Indices *b* and *r* refer to blue- and red-shifted σ components corresponding to transitions with Δm =+1 and -1. *p* indicates π components with Δm =0.

Relation to Zeeman splitting and line profiles

$$\phi_b = \sum_b A_b H(a, v - \Delta\lambda_b/\Delta\lambda_{Dop})$$

$$\phi_p = \sum_p A_p H(a, v - \Delta\lambda_p/\Delta\lambda_{Dop})$$

$$\phi_r = \sum_r A_r H(a, v - \Delta\lambda_r/\Delta\lambda_{Dop})$$

$$\psi_b = 2\sum_b A_b F(a, v - \Delta\lambda_b/\Delta\lambda_{Dop})$$

$$\psi_p = 2\sum_p A_p F(a, v - \Delta\lambda_p/\Delta\lambda_{Dop})$$

$$\psi_r = 2\sum_r A_r F(a, v - \Delta\lambda_r/\Delta\lambda_{Dop})$$

Voigt and Faradey-Voigt profiles

$$H(a,v) = \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(v-y)^2+a^2}dy$$

$$F(a,v) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(v-y)e^{-y^2}}{(v-y)^2+a^2}dy$$

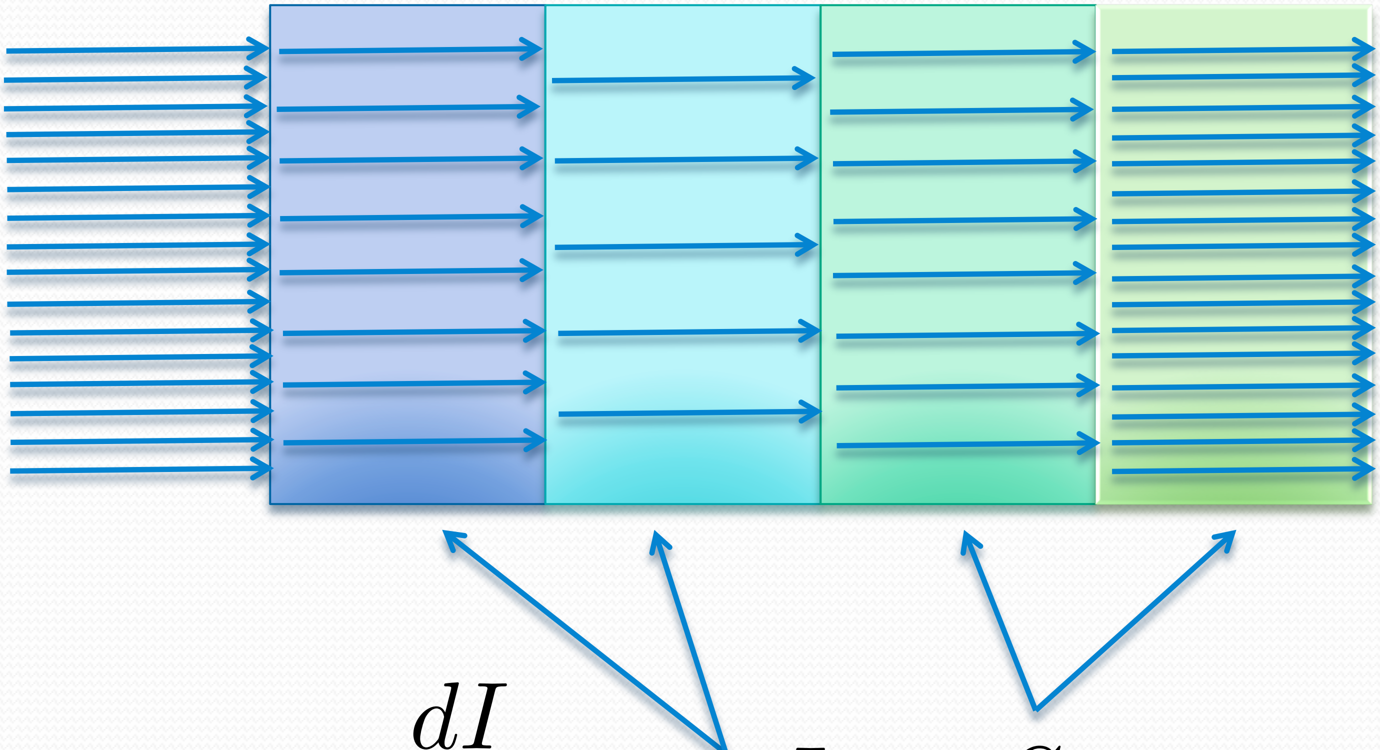
Jan Humlicek (1982, J.Q.S.R.T. 27, 437) came up with a great numerical approximations that generates both ***H*** and ***F*** in the same time.

Solving polarized RT

- Runge-Kutta (of course)
- Feautrier
- Attenuation operator

Ray tracing and Monte Carlo

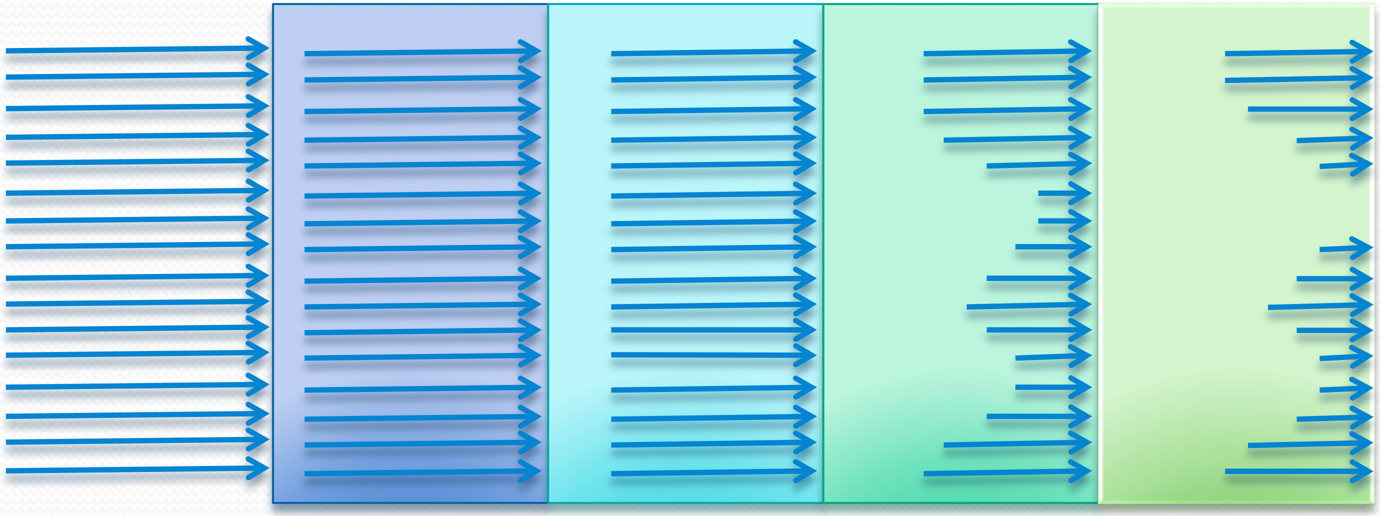
Sending bunch of photons evolving them according to the local conditions



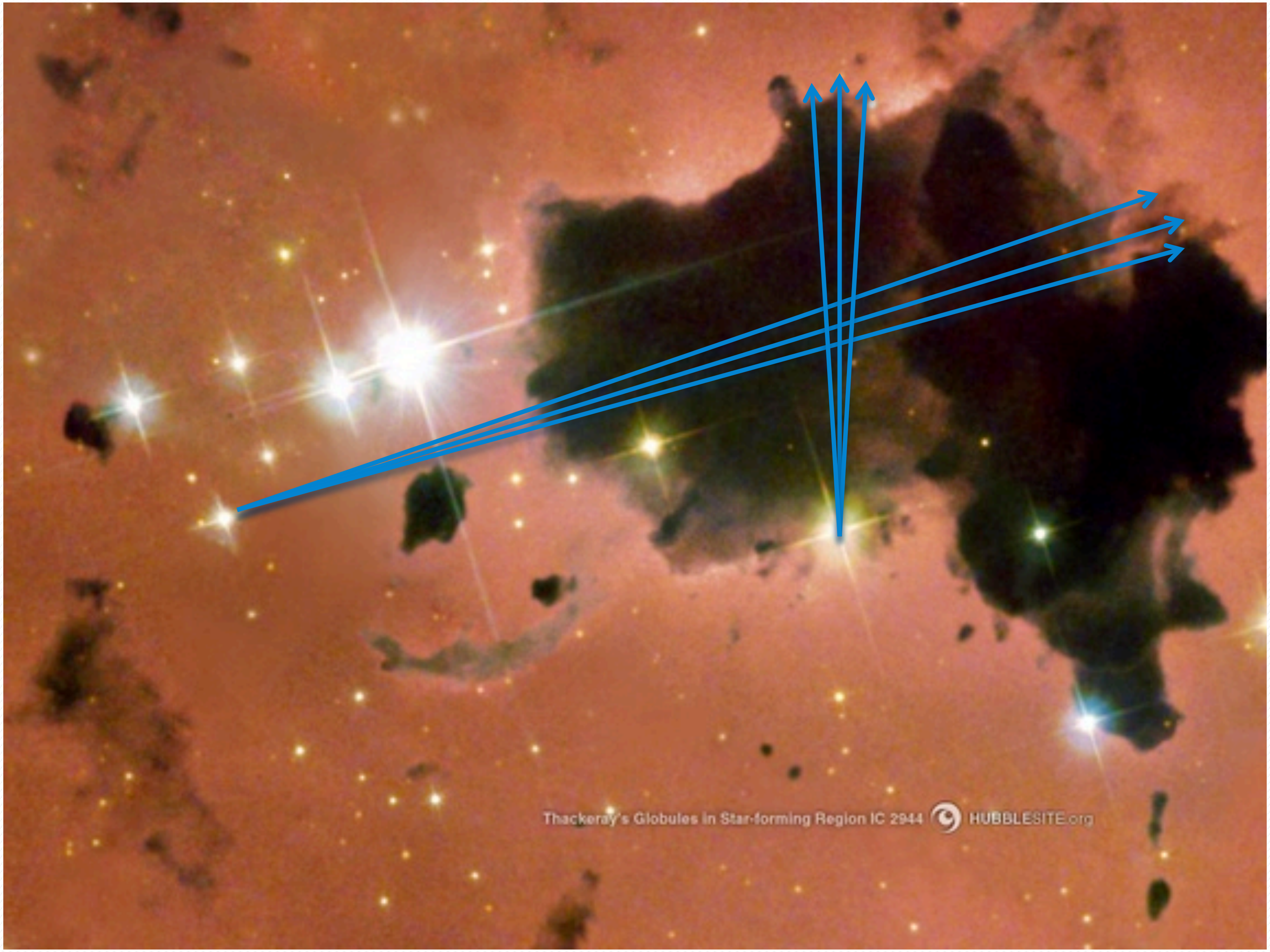
$$\frac{dI}{dz} = -\kappa I + \kappa S$$

Frequency dependent opacity

Sending bunch of photons with different frequencies (each arrow is a bunch)



Multi-dimensions



Interaction with matter

- Absorption:

$$n^{phot}(\lambda, \vec{z}) \to \kappa_\lambda \cdot N_{part} \to \tilde{n}^{phot}(\lambda, \vec{z})$$

- Source function:

$$S(\lambda, \vec{z}) = (\sigma_\lambda \cdot J_\lambda + k_\lambda B_\lambda)/\kappa_\lambda$$

- Level population:

$$\delta n_i^A = \sum_{j \neq i} n_j^A B_{ji} n_\lambda^{phot} + \sum_{j>i} n_j^A A_{ji}$$
$$- n_i^A \cdot \sum_{j \neq i} B_{ij} n_\lambda^{phot} - n_i^A \cdot \sum_{j<i} A_{ij}$$
$$+ \sum_{j \neq i} n_j^A C_{ji} - n_i^A \cdot \sum_{j \neq i} C_{ij}$$

How does MC work?

- Generate “super-photons” covering frequency, and directions for all sources. Each super-photon is assigned a weight indicating the actual number of photons.
- Calculate the propagation length by integrating a simple ODE: $$\frac{dl}{d\tau} = \frac{1}{\kappa(x)\rho}$$ to find geometrical length matching the limiting increment in optical depth.
- Compute changes in weights (absorption, emission) for the photons.
- Update mean intensity and source function.
- Compute changes in level population.

Summary of MC techniques (1)

- Easy, no differential equations to solve.
- Fast, for each ray.
- Can account for velocity field.
- Well fitted for 3D geometries, especially with point sources.
- Scattering is implicit: no ALI is needed.
- In fact, computing statistical equilibrium is straightforward.
- Easily combines radiation effect on matter, e.g. dust acceleration and sublimation.

Summary of MC techniques (2)

- Advanced random number generators can help describing complex boundary conditions (anisotropy, wavelength dependence etc.).
- Embarrassingly parallel, good for grid computing.
- Fails in optically thick regime (no photons is coming through).
- Difficult to partition the medium: step size selection can be done in the optical path. Integration is needed to get geometrical step.
- Hard to know when convergence is achieved.