Physics of planetary atmospheres

Lecture 2:
Introduction to
Radiative Energy Transport

Part I

Equation of Radiative Transport

- Radiation propagation has direction. Thus, we need to follow a ray.
- Photons in a ray carry energy (intensity) and interact with matter. As the result intensity can decrease (extinction) or increase (emissivity).
- For small geometrical step along the ray we can write the change of intensity as:

$$\Delta I_\lambda = -\kappa_\lambda I_\lambda \Delta x + j_\lambda \Delta x$$

- Decrease of intensity is proportional to intensity.
- Stimulated emission is also proportional to intensity and as such is treated as "negative" extinction.

Extinction

- Two processes can remove energy from a beam: absorption and scattering.
- Absorption destroys the photon. Photon is converted to internal energy (excitation) of the absorber (atom, molecule, dust particle) that is then released in inelastic collision or cascade emission.
- Scattering does not destroy a photon but redirects it. Absorption followed by re-emission involving the same two energy levels can be treated as scattering or absorption.

Extinction coefficient

- Extinction scales with the amount (density) of material along the ray.
- It is logical to separate this part, not connected to radiation.
- It can be done in several ways:

$$\kappa_\lambda \Delta x = a_\lambda \rho \Delta x \equiv a_\lambda \Delta m = \sigma_\lambda n \Delta x$$

where a_λ is mass extinction, ρ is mass density σ_λ is absorption cross-section and n is absorber number density.

Optical path (or depth)

- Various definitions of extinction connect in the previous slide define new quantity – optical path: $\Delta\tau = \kappa_\lambda \Delta x$
- The first thing we can do is re-writing the RT equation:

$$\frac{dI_\lambda}{d\tau_\lambda} = -I_\lambda + S_\lambda$$

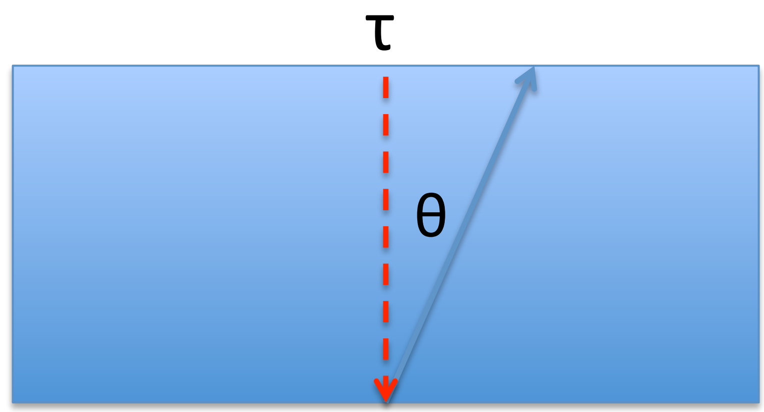
where: $S_\lambda = j_\lambda / \kappa_\lambda$ is the source function.

Optical path

- τ-x relation measures transparency of matter (surface for $\tau >> 1$).
- For incoming radiation it shows how deep radiation will penetrated before it is fully absorbed.
- For emission of matter it shows the depth of layer from which the emission originates.

Slang and traditions

- Optical path is measured along the ray.
- In 1D medium (all changes can be described as function of a single variable) we can use optical depth.
- In this case, optical path along the ray is: $\tau / \cos\theta = \tau / \mu$



- and RT equation changes to (note sign change):

$$\mu \frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

Examples: emission

Example: absorption

Emissivity

- Emissivity is the process of kinetic energy conversion to radiation.
- It can go as collisional excitation (inelastic collision) followed by radiative de-excitation
- Or it can be scattering from other rays.

Solving RT

- No emissivity (source function is zero)

$$I = I_0 \cdot e^{\tau/\mu}$$

- Surface intensity (streaming out, $\mu>0$) exponentially grows up the stream with e-folding length $\tau/\mu=1$.
- The formal solution of RT equation is:

$$I_1 = I_0 \cdot e^{-(\tau_1 - \tau_0)/\mu} + \frac{1}{\mu}\int_{\tau_0}^{\tau_1} S(t) \cdot e^{-(t-\tau_0)/\mu} dt$$

- Home work: derive the formal solution.

Absorption and Scattering

- Looking closer at extinction we can separate the cross-section into the true absorption and scattering part:

$$\sigma_e = \sigma_a + \sigma_s$$

- We define the relative contribution of scattering as:

$$\omega_0 = \frac{\sigma_s}{\sigma_e} \equiv \frac{\sigma_s}{\sigma_a + \sigma_s}$$

- For light reflected from a surface this is albedo.

Splitting the source function

- Using the definition of ω_0 we can separate the source function into thermal emission and scattering parts:

$$S = \omega_0 \cdot \frac{J}{4\pi} + (1 - \omega_0) \cdot B$$

where *J*/4π is the mean intensity in a given point and *B* is the Planck function.

- The corresponding RT equation will be:

$$\mu \frac{dI_\lambda}{d\tau} = I_\lambda - \omega_0 \cdot \frac{J_\lambda}{4\pi} - (1 - \omega_0) \cdot B_\lambda$$

Transmission function (1)

- Scattering makes RT impossible to solve directly.
- Assuming no scattering and constant Planck function we can solve it for a planetary atmosphere.
- Let I_2 and I_1 be intensities at the bottom and the top of the atmospheres. Then integrating over μ of rewritten RT equation:

$$\mu I_2 \cdot e^{-(\tau_2-\tau_1)/\mu} - \mu I_1 = -B \int_{\tau_1}^{\tau_2} e^{-(t-\tau_1)/\mu} dt$$

Transmission function (2)

… for the up and down directions gives:

$$F_{\uparrow 1} = F_{\uparrow 2} T + \pi B(1 - T)$$

$$F_{\downarrow 2} = F_{\downarrow 1} T + \pi B(1 - T)$$

where arrows indicate the direction of we used the definition of flux:

$$F = \int_0^1 I \mu d\mu$$

Transmission function is defined as:

$$T \equiv \mu \int_0^1 \mu e^{-(\tau_2 - \tau_1)/\mu} d\mu$$

Transmission function (3)

- If T is 0 the atmosphere is totally opaque. The only light coming out is generated from the thermal energy.
- If T is 1 the atmosphere is totally transparent. What comes in streams out and no thermal emission is added.
- This is true in both directions.
- Of course, we ignored scattering and change of temperature along the way.

A note about units

- Intensity is the amount of energy per unit spectral bin that comes from a unit solid angle and crosses unit area per unit time along a given ray.
- Thus units are erg cm^{-2} st^{-1} s^{-1} $Å^{-1}$ or erg cm^{-2} st^{-1} s^{-1} Hz^{-1}.
- Emissivity has the same units as intensity.
- Flux is the total energy (from all directions) that crosses unit area, so no solid angle.
- Absorption is in cm^{-1} and is independent of spectral bin units.
- Optical path is unitless.

Special case: instantaneous transition to large optical depth

- Surfaces of solid objects.
- Geometrical albedo A_g is the comparison of the fraction of light reflected on one direction (to the observer) as compared with flat surface of the same shape that scatters light isotropically (Lambert disk).
- Reflected light from a planet is also affected by the geometry (phases). The corresponding Spherical albedo A_s is related to the A_g through phase function: $A_s = A_g \times P_g$.

More about albedo

- Geometric and spherical albedo are wavelength dependent.
- Mean or Bond albedo is a true measure of total fraction of reflected energy. It is computed as:

$$A_B = \frac{\int I_\lambda^\star A_s d\lambda}{\int I_\lambda^\star d\lambda}$$

What will be needed to solve RT?

- To solve the RT equation we need model atmosphere: T, P, partial number density of all absorbers, something that we can measure in the solar system but not elsewhere.
- Let’s assume we have a guess for vertical T-P structure and chemical composition.
- Then for each layer we need to compute the number of particles contributing to absorption and scattering across the whole wavelength range.

Solving RT

- Once we have opacities we need a robust method for solving RT (accounting for scattering).
- Assuming energy balance (what comes in from the host star is radiated out, albeit at different wavelengths) we can adjust temperature structure.
- New temperature will require solving the equation of hydrostatic equilibrium to get P. This is done iteratively as T affects partial pressures that affect mean density.
- This only works if we ignore atmospheric dynamics ☺
- Then the convergence is achieved if the total radiative flux through any layer is zero!

Home work

- 2.8.3. Greenhouse effect
- 2.8.6. Transit radius and optical depth