Physics of planetary atmospheres

Lecture 4: Temperature-Pressure Profiles
(Heng Ch. 4)

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Why do we need temperature and pressure?

In which way does the temperature in the atmosphere play a role for radiative transfer (cf. equations in previous lectures)?

Role of temperature and pressure

So far, we have been talking about transport of radiation through the atmosphere, describing all quantities as a function of optical depth, which is expressed as (Ch. 2):

$$\tau_\lambda = \int n\sigma_\lambda dx = \int \kappa_\lambda \rho dx$$

σ_λ ... cross section (units of cm^2)

κ_λ ... opacity (units of cm^{2}g^{-1}),
depends on temperature and pressure!

In some equations the Planck function appears, which depends on temperature.

How do we obtain temperature and pressure?

"For Earth and the Solar System bodies, the temperature-pressure profile may actually be measured via satellite or in-situ probes.
For exoplanets, this task is a lot harder and we need to resort to indirect methods."

Equilibrium temperature

Surface temperature T_{eq} of a planet with radius R and Bond albedo A_{B} due to heating by radiation from a star with radius $R_\star$ and effective temperature $T_\star$ at a distance a, without effects of an atmosphere:

Absorbed power: $\mathcal{P}_a = \dfrac{4\pi R_\star^2 \sigma_{\mathrm{SB}} T_\star^4}{4\pi a^2} \cdot \pi R^2 \left(1 - A_{\mathrm{B}}\right)$

Emitted power: $\mathcal{P}_e = 4\pi R^2 \cdot \sigma_{\mathrm{SB}} T_{\mathrm{eq}}^4$

$$\mathcal{P}_a = \mathcal{P}_e \Rightarrow T_{\mathrm{eq}} = T_\star \left(\frac{R_\star}{2a}\right)^{1/2} \left(1 - A_{\mathrm{B}}\right)^{1/4}$$

Equilibrium vs observed temperature

Object	Bond Albedo	T_{eq} (K)	T_{s} (K) measured	ΔT (K) warming
Mercury	0.06	440	440	0
Venus	0.75	240	740	$\sim$500
Earth	0.33	250	290	40
Mars	0.17	220	230	10
Titan	$\sim$0.2	85	90	5

Atmospheric effects

Reasons for differences between T_{eq} and measured T_{s} for planets with significant atmospheres:

- Cooling due to scattering of starlight
- Warming due to greenhouse effect:
  absorption or back-scattering of infrared radiation from planet by e.g. water, carbon dioxide, methane
- Cooling and heating by aerosols or clouds

This lecture: investigate analytical models to develop an understanding of these effects, before using numerical simulations in last part of course

Dual-band (double-gray) approximation I

Wien's law – wavelength for maximum of blackbody intensity:

$$\lambda_{\text{peak}} \approx 0.5\mu\text{m} \left(\frac{T}{6000\text{K}} \right)^{-1}$$

Examples: $\approx$0.5 μm for Sun-like star, $\approx$10 μm for Earth, $\approx$2 μm for hot exoplanet

This means that we can calculate the transfer of radiation in two separate wavelength bands:

- *Shortwave* (S): range of wavelengths corresponding to stellar heating, typically in the visible, except for very cool stars ("M dwarfs" $\rightarrow$ near-infrared)
- *Longwave* (L): range of wavelengths associated with thermal emission of planetary atmosphere, in infrared

Dual-band (double-gray) approximation II

New quantities:
zeroth, first, and second moments of intensity, integrated over shortwave and longwave bands
$\rightarrow$ shortwave/longwave total intensity $J_{\mathrm{S/L}}$, net flux $F_{\mathrm{S/L}}$, and net K-integral $K_{\mathrm{S/L}}$

K-integral (Sect. 3.5.1, Eq. 3.51): multiplying intensity with μ^2 and integrating over all angles, for outgoing and incoming radiation
$K_{\mathrm{out/in}}/c$ is the radiation pressure
Net K-integral $K_{-}(\lambda) = K_{\mathrm{out}} - K_{\mathrm{in}}$

Radiative transfer with scattering I

- Start from "governing equations" for two-stream approximation with non-isotropic scattering, for wavelength-dependent net flux and net K-integral (from Ch. 3, Eqs. 3.46 and 3.52)
- Express single-scattering albedo in terms of the absorption and scattering opacities: $\omega_0 = \dfrac{\kappa_\mathrm{s}}{\kappa_\mathrm{a} + \kappa_\mathrm{s}}$
- Change the independent variable (depth coordinate) from optical depth τ to column mass $\tilde{m} = \int \rho dx$
- $\tilde{m} = 0$ at the top of the atmosphere,
  $\tilde{m} \to \infty$ in the deep interior,
  pressure $P = \tilde{m} g$
  (in hydrostatic equilibrium, from Newton's second law)

Radiative transfer with scattering II

$$\omega_0 = \frac{\kappa_\mathrm{s}}{\kappa_\mathrm{a} + \kappa_\mathrm{s}}, \, \tilde{m} = \int \rho dx$$

$$d\tau = (\kappa_\mathrm{a} + \kappa_\mathrm{s})\rho dx = \frac{\kappa_\mathrm{a}}{1 - \omega_0} d\tilde{m}$$

$$\begin{aligned} \frac{dF_-}{d\tau} &= (1 - \omega_0)(J - 4\pi B) & \rightarrow \frac{dF_-}{d\tilde{m}} &= \kappa_\mathrm{a}(J - 4\pi B) \\ \frac{dK_-}{d\tau} &= F_+(1 - \omega_0 g_0) & \rightarrow \frac{dK_-}{d\tilde{m}} &= \kappa_\mathrm{a} F_+ \frac{1 - \omega_0 g_0}{1 - \omega_0} = \frac{\kappa_\mathrm{a} F_+}{\beta_0^2} \end{aligned}$$

$g_0 = \int_0^{4\pi} \mu'' \mathcal{P} d\Omega''$ … *scattering asymmetry factor*
(Ch. 3, Eq. 3.50), where $\mathcal{P}$ is the scattering phase function

β_0 … *scattering parameter*

$\rightarrow$ governing equations with absorption opacity and scattering parameter as input parameters

Scattering parameter

β_0 – combination of single-scattering albedo and scattering asymmetry factor

- Pure absorption: $\beta_0 = 1$
- Pure scattering: $\beta_0 = 0$
- Isotropic scattering: $g_0 = 0$
- Forward scattering: $g_0 > 0$
- Backward scattering: $g_0 < 0$

Towards a solution

A static atmosphere does not produce or consume any energy, i.e. we apply conservation of energy. This is ensured when the wavelength-integrated net flux is constant for each layer in the atmosphere (aka *radiative equilibrium*).

Integrate the governing equation for net flux over all wavelengths to obtain an equation for the wavelength-integrated net flux $\mathcal{F}_-$, where the absorption is divided into the shortwave and the longwave part, and the Stefan-Boltzmann law is used to integrate the Planck function:

$$\frac{d\mathcal{F}_-}{d\tilde{m}} = \kappa_{\mathrm{S}} J_{\mathrm{S}} + \kappa_{\mathrm{L}}(J_{\mathrm{L}} - 4\sigma_{\mathrm{SB}} T^4)$$

$\dfrac{d\mathcal{F}_-}{d\tilde{m}} = 0$ in radiative equilibrium

Inner boundary condition: net flux from deep interior is $\sigma_{\mathrm{SB}} T_{\mathrm{int}}^4$

Treatment of shortwave radiation I

"In the shortwave, stellar heating dominates and thermal emission from the exoplanet is negligible."

- Describe scattering in shortwave band by parameter β_{S_0} – a characteristic value of β_0
- Integrate governing equations for two-stream approximation over shortwave band:

$$
\begin{aligned}
\frac{dF_-}{d\tilde{m}} &= \kappa_{\mathrm{a}}(J - 4\pi B) \quad &\rightarrow \frac{dF_{\mathrm{S}}}{d\tilde{m}} &= \kappa_{\mathrm{S}} J_{\mathrm{S}} \\
\frac{dK_-}{d\tilde{m}} &= \frac{\kappa_{\mathrm{a}} F_+}{\beta_0^2} &\rightarrow \frac{dK_{\mathrm{S}}}{d\tilde{m}} &= \frac{\kappa'_{\mathrm{S}} F_{\mathrm{S}}}{\beta_{\mathrm{S}_0}^2}
\end{aligned}
$$

- κ_{S}, κ'_{S} … mean opacities weighted by intensity and flux

Treatment of shortwave radiation II

- Obtain a pair of second-order ordinary differential equations by taking the derivative of the previous equations and inserting the original equations back in 2 times (Eq. 4.22)
- Assumptions:
  - Use *shortwave closure* $K_\mathrm{S} = \mu^2 J_\mathrm{S}$ to eliminate one of three unknowns $J_\mathrm{S}, F_\mathrm{S}, K_\mathrm{S}$ (justified by allowing a solution reproducing Beer's extinction law)
  - $\kappa_\mathrm{S} = \kappa'_\mathrm{S}$
  - Power-law form of the shortwave opacity: $\kappa_\mathrm{S} = \kappa_{\mathrm{S}_0} \left(\dfrac{\tilde{m}}{\tilde{m}_0} \right)^{n_\mathrm{S}}$

    $\tilde{m}_0$ ... reference value of the column mass usually set to correspond to bottom of atmosphere

Treatment of shortwave radiation III

Analytical solutions for total intensity and net flux:

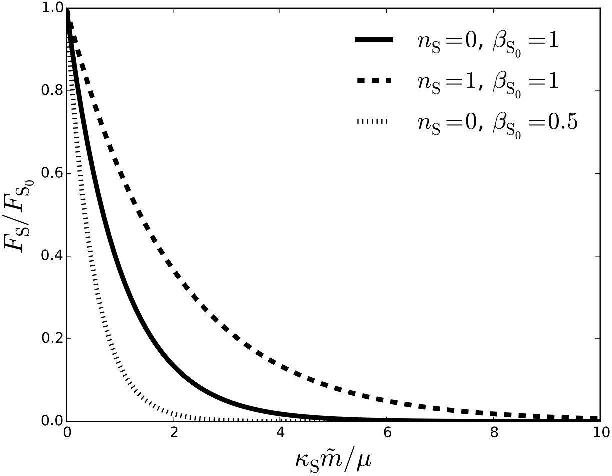
$J_S = J_S(\tilde{m} = 0)e^{\beta_S/\mu}$, $F_S = F_S(\tilde{m} = 0)e^{\beta_S/\mu}$,

where $\beta_S = \dfrac{\kappa_S \tilde{m}}{(n_S + 1)\beta_{S_0}}$

For constant κ_S ($n_S = 0$) and pure absorption ($\beta_{S_0} = 1$) this is Beer's extinction law ($\mu < 0!$).

Figure 4.2: Fractional decrease of incident stellar flux $F_S(\tilde{m} = 0)$ with depth in the atmosphere (towards larger column mass), for different values of n_S and β_{S_0}.

Treatment of shortwave radiation IV



Larger power-law index for opacity n_{S} causes less rapid decrease, presence of scattering (smaller β_{S_0}) causes more rapid decrease.

Treatment of shortwave radiation V

Outer boundary condition: shortwave flux at top of atmosphere is incident stellar flux $\mu\sigma_{\mathrm{SB}}T_{\mathrm{irr}}^4$ (for $0 \leq \phi \leq \pi$, otherwise 0)

$T_{\mathrm{irr}} = \sqrt{2}T_{\mathrm{eq}}$ ... *irradiation temperature*
– temperature at the substellar point of a planet,
where distance between atmosphere and star is shortest

Treatment of longwave radiation

- Describe scattering in the longwave band by parameter β_{L_0}
- Integrate governing equations for two-stream approximation over longwave band:

$$\frac{dF_-}{d\tilde{m}} = \kappa_{\mathrm{a}}(J - 4\pi B) \;\rightarrow \frac{dF_{\mathrm{L}}}{d\tilde{m}} = \kappa_{\mathrm{L}} J_{\mathrm{L}} - 4\kappa''_{\mathrm{L}} \sigma_{\mathrm{SB}} T^4$$

includes blackbody emission from planetary atmosphere!

$$\frac{dK_-}{d\tilde{m}} = \frac{\kappa_{\mathrm{a}} F_+}{\beta_0^2} \qquad \rightarrow \frac{dK_{\mathrm{L}}}{d\tilde{m}} = \frac{\kappa'_{\mathrm{L}} F_{\mathrm{L}}}{\beta_{\mathrm{L}_0}^2}$$

- κ_{L}, κ'_{L}, κ''_{L} ... mean opacities weighted by intensity, flux, and Planck function

Treatment of longwave radiation – assumptions

- To eliminate two of four unknowns $J_{\mathrm{L}}, F_{\mathrm{L}}, K_{\mathrm{L}}, T$ use two *longwave Eddington coefficients*: $\epsilon_{\mathrm{L}} = F_{\mathrm{L}}/J_{\mathrm{L}} = 3/8$ and $\epsilon_{\mathrm{L}_3} = K_{\mathrm{L}}/J_{\mathrm{L}} = 1/3$. These values are consistent with the other Eddington coefficients used in Ch. 3, and are derived in Problem 4.9.1.
- $\kappa_{\mathrm{L}} = \kappa'_{\mathrm{L}} = \kappa''_{\mathrm{L}}$

General solution

- Global-mean temperature-pressure profile in radiative equilibrium:
  - $\bar{T}^4$ as a function of $\tilde{m}$ (Eq. 4.39)
  - input parameters $T_{\rm int}^4$, $T_{\rm irr}^4$, $\beta_{\rm S_0}$, $\beta_{\rm L_0}$
  - input functions $\kappa_{\rm S}$, $\kappa_{\rm L}$
  - contains also $\epsilon_{\rm L} = 3/8$, $\epsilon_{\rm L_3} = 1/3$, and $\beta_{\rm S} = \dfrac{\kappa_{\rm S}\tilde{m}}{(n_{\rm S}+1)\beta_{\rm S_0}}$ via $\mathcal{E}_i(\beta_{\rm S})$ (exponential integral $\mathcal{E}_i$ is defined in Eq. 2.32)
- Parametrized shortwave opacity $\kappa_{\rm S} = \kappa_{\rm S_0}\left(\dfrac{\tilde{m}}{\tilde{m}_0}\right)^{n_{\rm S}}$
- Parametrized longwave opacity $\kappa_{\rm L} = \kappa_0 + \kappa_{\rm CIA}\left(\dfrac{\tilde{m}}{\tilde{m}_0}\right)$

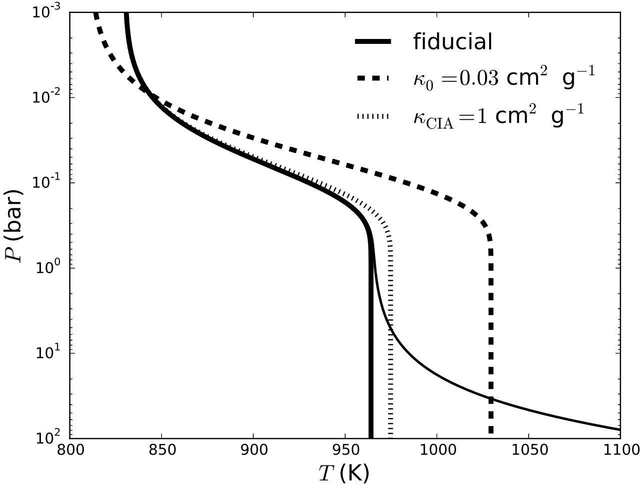
Atmospheric effects – greenhouse warming

- Fiducial model:
  $T_{\rm int} = 0/150$ K, $T_{\rm irr} = 1200$ K,
  $\beta_{\rm S_0} = 1$, $\beta_{\rm L_0} = 1$,
  $\kappa_{\rm S_0} = 0.01$ cm^{2}g^{-1}, $n_{\rm S} = 0$,
  $\kappa_0 = 0.02$ cm^{2}g^{-1}, $\kappa_{\rm CIA} = 0$, $g = 10^3$ cm s^{-2}
- Generic shape of temperature-pressure profile for irradiated atmospheres:
  temperature increases with pressure from the top downwards, then becomes isothermal;
  if $T_{\rm int} > 0$ then T increases again at greater depths
- Without atmosphere, $T_{\rm surface}$ would be $T_{\rm eq} \approx 850$ K, but $T_{\rm surface} \gtrsim 950$ K due to *greenhouse effect*: stellar shortwave radiation enters the atmosphere, heats the surface and is converted to longwave radiation, which stays in the atmosphere due to the higher longwave opacity

Atmospheric effects – collision-induced absorption (CIA)

- At high pressures, symmetric diatomic molecules (e.g. H_2 or N_2) may undergo inelastic collisions, during which they form a larger molecule which causes continuous absorption.
- Collisions need two particles $\rightarrow \tau = \int \kappa d\tilde{m}$ proportional to square of gas density $\rightarrow$ CIA opacity is proportional to $\tilde{m}$
- Absorption occurs mostly at longwave $\rightarrow$ including CIA warms the atmosphere

Greenhouse warming and collision-induced absorption

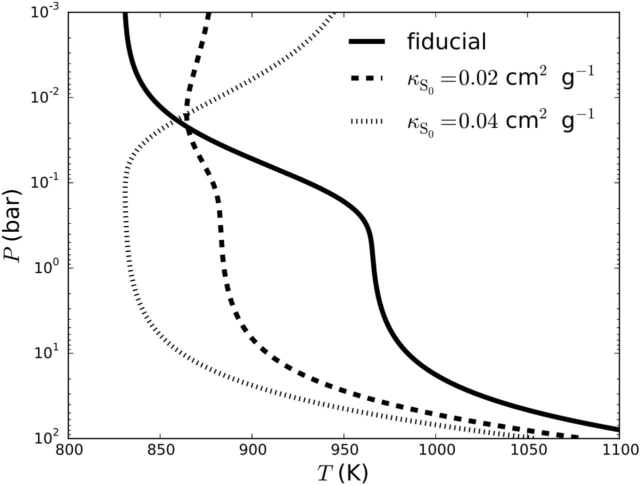


- Increasing the longwave opacity (κ_0) increases greenhouse warming
- Switching on CIA warms the atmosphere, in particular at high pressures

Atmospheric effects – cooling and inversions

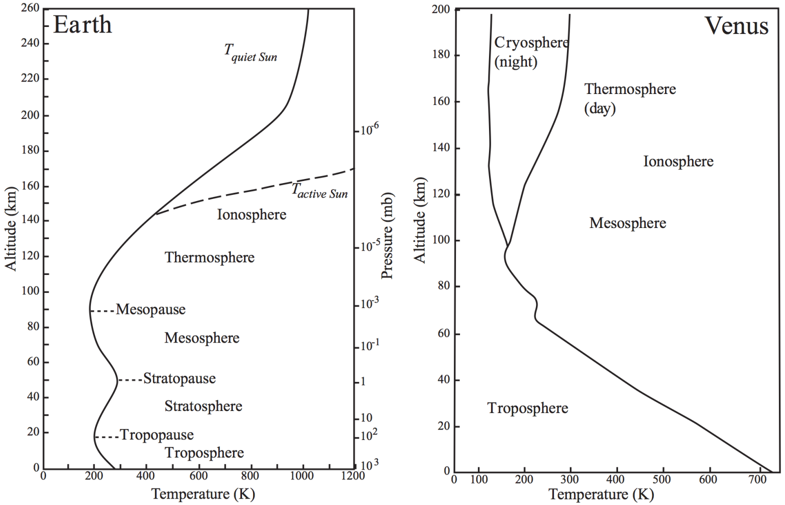
- When shortwave opacity is increased
  $\rightarrow$ *anti-greenhouse effect*: upper atmosphere is warmed up and lower atmosphere is cooled down
- If shortwave opacity is higher than longwave opacity
  $\rightarrow$ *temperature inversion* (temperature increases with altitude)
- Examples for extra absorbers of shortwave (stellar) radiation: ozone on Earth, possibly TiO on exoplanets
- Existence of temperature inversions in exoplanetary atmospheres is under debate

Anti-greenhouse cooling and temperature inversion



Effect of increasing the shortwave opacity to being equal to and 2 times the longwave opacity

Measured temperature profiles



Lissauer and de Pater (2013)

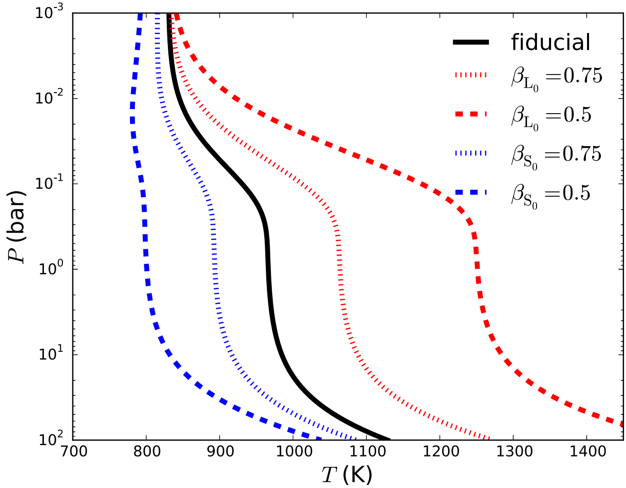
Atmospheric effects – albedo variations

- Bond albedo = fraction of incident starlight reflected (scattered) away from the atmosphere across all wavelengths
- Depends on amount of aerosols, clouds, and dust grains contained in the atmosphere
- Decreasing *shortwave* scattering parameter corresponds to increasing Bond albedo, e.g. $\beta_{S_0} = 0.75 \rightarrow A_B = 14\%$, $\beta_{S_0} = 0.5 \rightarrow A_B = 33\%$
- Effect: entire temperature-profile shifts to lower temperatures, total energy content of the atmosphere is reduced by a factor of $(1 - A_B)$

Scattering greenhouse effect

- Effect of decreasing *longwave* scattering parameter: entire temperature-profile shifts to higher temperatures
- Thermal emission from planetary atmosphere attempts to escape, but some of it is scattered back into the planet

Increased shortwave and longwave scattering



Shortwave/optical scattering cools ($\beta_{S_0} < 1$),
longwave/infrared scattering warms ($\beta_{L_0} < 1$) the entire atmosphere

Home work

- 4.9.1 The longwave Eddington coefficients (cf. Chapter 3)
- 4.9.3 Non-constant shortwave opacity (cf. Appendix E)