Treatment of the superadiabatic convection in low–mass metal–poor stars from realistic hydrodynamics simulations: application to globular clusters isochrones

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Abstract. For a grid of 2D Radiation Hydrodynamics (RHD) models of the surface convection zones of metal-poor stars we determine the convective efficiency of each model. We calibrate the free parameter α of the mixing length theory (MLT) used in stellar evolution calculations to give the same convective efficiency. The hydro-calibrated α depends in a non-trivial way on the effective temperature, gravity and metallicity of the models. Nevertheless, the resulting isochrones for the relevant age range of galactic globular clusters have only small differences with respect to isochrones computed adopting a constant solar calibrated value of α. Accordingly, the age of globular clusters is reduced by 0.2 Gyr at most. However, while predicting a similar outcome our work is free of the systematic uncertainties assocated with the MLT.

1. Introduction

Recent re-determinations of the ages of galactic globular clusters resulted in a significant reduction of their mean age, shifting it toward a value in agreement with the age of the universe as deduced from the value of the Hubble constant (see, e.g., Salaris et al. 1997, Chaboyer et al. 1998). During this process our awareness of the errors still inherent in evolutionary calculations has increased. To reduce one of the largest sources of uncertainty – the lack of a rigorous description of convection – we calibrated the superadiabatic temperature gradient in the envelope of metal-poor low-mass stars according to the results from a grid of 2D hydrodynamical models, which cover the Main Sequence and the lower Red Giant Branch of globular cluster (GC) stars.

2. Numerical simulations

With a 2D numerical RHD code (see for details Ludwig et al. 1994) a variety of models of stellar surface convection zones has been computed (see Freytag, Ludwig, Steffen in these proceedings), covering the range $4300\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 7100\,\mathrm{K}$, $2.54 \leq \log(g) \leq 4.74$, $-2.0 \leq [\mathrm{M/H}] \leq 0.0$; from this grid of models one can extract the entropy of the deeper, adiabatic convective layers (s_{env}) as a function of T_{eff}, g, and [M/H] (see Ludwig et al. 1998). Once this relation is implemented in a stellar evolution code, it completely fixes the T_{eff} of the star as determined from the solution of the stellar structure equations. A way for implementing easily this dependence of s_{env} on T_{eff} and g into an evolutionary code makes use of the MLT formalism. As explained in detail by Ludwig et al. (1998), for each fixed metallicity one can compute a grid of hydrostatic one-dimensional stellar envelope models based on the MLT, covering the same range of g and T_{eff} spanned by the RHD computations, and using the same input physics. By employing as surface boundary condition the $T(\tau)$ relation derived from the hydro-models, one can calibrate an effective α (α_{eff}) that is able to reproduce the s_{env}-T_{eff} relation obtained from the RHD computations. In this way one can derive a function $\alpha_{\mathrm{eff}}=\mathrm{f}(T_{\mathrm{eff}},g)$ at each metallicity (see Ludwig, Freytag, Steffen in these proceedings) that is easy to use for computing stellar evolutionary models. The estimated error on the values of α_{eff} derived by means of this procedure is equal to ± 0.05.

3. Isochrones and globular clusters ages

We have used the α_{eff} values derived from the previously discussed RHD models for computing isochrones with typical GC metallicities $[\mathrm{M/H}]=-2.0$ and $[\mathrm{M/H}]=-1.0$, Y=0.23, age t ranging from 9 to 14 Gyr, using the code described in Salaris et al. (1997). The zero point of the α_{eff} calibration has been increased by a factor 1.06, which accounts for the difference between the solar α value ($\alpha_{\odot}=1.59$) obtained from the RHD models (which employ the Kurucz 1979 ATLAS6 low-temperature opacities) and $\alpha_{\odot}=1.69$ as obtained from the evolutionary computations using the updated Alexander & Ferguson (1994) molecular opacities. We have employed in the evolutionary models the same $T(\tau)$ relation and the same MLT formalism used in the calibration of α_{eff}. For the sake of comparison, we have computed isochrones for the same t and [M/H] values, but using $\alpha_{\odot}=1.69$.

Representative isochrones (9 and 13 Gyr) for the two considered metallicities, computed with α_{eff} and $\alpha_{\odot}$, are displayed in Fig. 1, panel a). The MS loci are coincident, the T_{eff} values of the TO points are very similar (notice the linear scale for the T_{eff} axis), the biggest difference being equal to $\approx$45 K for the 9 Gyr most metal-poor isochrone (an age possibly too young for the metal-poor galactic GC population, see e.g. Salaris & Weiss 1998). Along the RGB the isochrones computed by using α_{eff} are systematically hotter by only $\approx$50 K for $[\mathrm{M/H}]=-1.0$ and $\approx$40 K for $[\mathrm{M/H}]=-2.0$. To explain this behaviour it is useful to study the run of α_{eff} with respect to $\log(L/L_{\odot})$ along the same isochrones, as shown in panel b) of the same figure.

The differences between α_{eff} and $\alpha_{\odot}$ along the lower MS are hardly relevant, since the T_{eff} of these stars is insensitive to the choice of α (the entropy jump from the photosphere to the deep adiabatically stratified layers is small anyway), while around the TO they depend on the age of the isochrones. In general, in the youngest, most metal-poor isochrones α_{eff} shows the largest difference with respect to $\alpha_\odot$, but since stars in these phases are quite hot ($T_{\rm eff}\approx$7000 K) and their convection zones are relatively shallow, the sensitivity of $T_{\rm eff}$ to α is not very large. Along the RGB, where the $T_{\rm eff}$ of stellar models is most sensitive to α because of deeper superadiabatic regions, $\alpha_{\rm eff}$ is systematically higher than $\alpha_\odot$ by 0.10-0.15 for both metallicities. This difference causes a systematic shift by $\approx$50 K toward higher $T_{\rm eff}$ with respect to the case of $\alpha_\odot$, a quantity marginally significant since the error by $\pm$0.05 on $\alpha_{\rm eff}$ translates into an error by $\approx\pm$15-20 K on the RGB $T_{\rm eff}$.

What are the implications of this hydro-calibration of α for the estimated GC ages? In panels c) and d) of Figure 1 we compare the TO position (brightness and colour) in the age range 9-14 Gyr for the two sets of isochrones with $\alpha_{\rm eff}$ and $\alpha_\odot$. We have transformed the isochrones to the observational V-(B-V) plane according to the colours and bolometric corrections used by Salaris & Weiss (1998), but the results of this comparison do not depend on the particular set of transformations used. As it is evident from the figure, the age differences as derived from the TO brightness (or colour) are basically negligible, amounting to 0.2 Gyr at most ($\alpha_{\rm eff}$ isochrones being younger).

4. Conclusions

The main results of this analysis show that the $T_{\rm eff}$ of GC isochrones computed employing the MLT formalism and $\alpha_\odot$, or the $\alpha_{\rm eff}$ calibration as derived from detailed RHD models are in good mutual agreement: the maximum deviation in the relevant age range for galactic GC amounts at most to a systematic shift by $\approx$ 50$\pm$20 K along the RGB. GC ages as derived from the TO brightness or colour would result (at most) only 0.2 Gyr younger than in the case of using $\alpha_\odot$ isochrones. Preliminary comparisons (Ludwig et al. 1998) between the adopted grid of 2D RHD models and a small sample of 3D ones show only a very small systematic shift of $\alpha_{\rm eff}$ as derived from the RHD models ($\approx$0.07 for the Sun), which does not affect our results appreciably.

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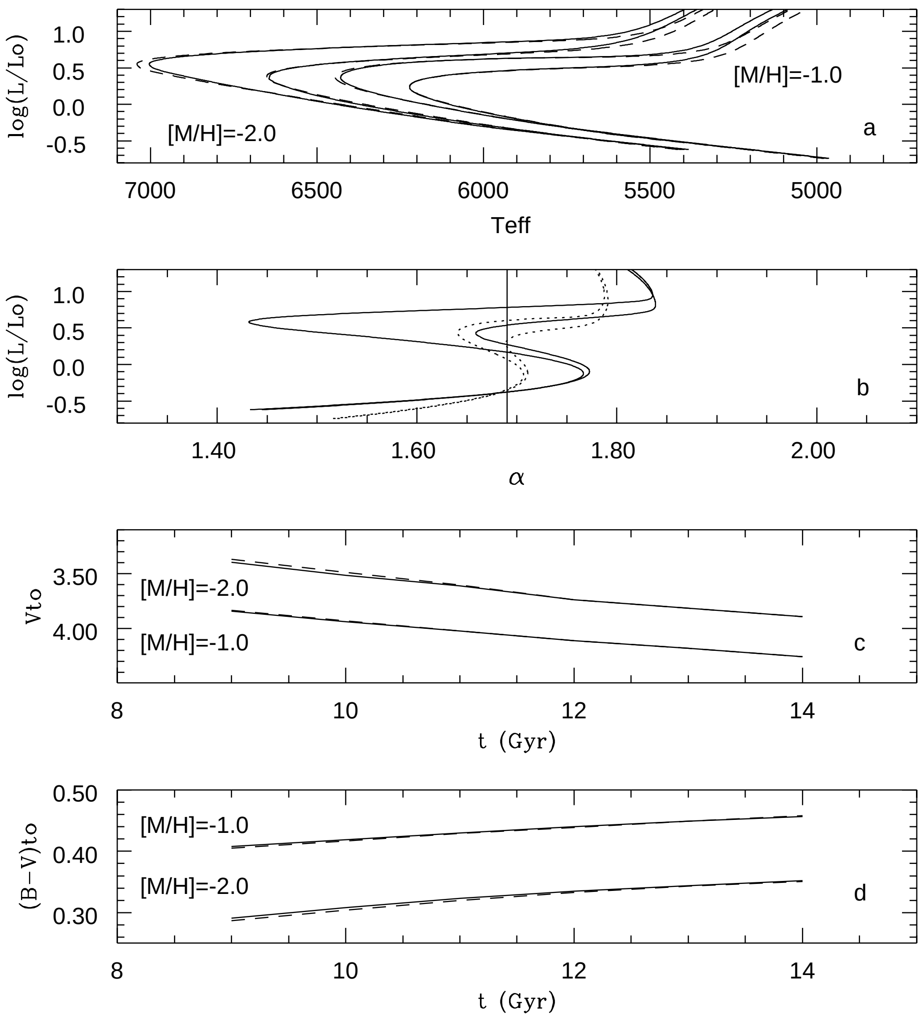

Figure 1. Panel a): isochrones for [M/H]=-2.0 and [M/H]=-1.0, t=9 and 13 Gyr, computed with $\alpha_{\rm eff}$ (solid lines) and $\alpha_\odot$ (dashed lines). Panel b): values of α along the isochrones (solid line [M/H]=-2.0, dotted line [M/H]=-1.0). The vertical line marks the value of $\alpha_\odot$. Panels c) and d): values of, respectively, the TO V magnitude and (B-V) colour as a function of the age ([M/H]=-2.0 and [M/H]=-1.0) for $\alpha_{\rm eff}$ (solid lines) and $\alpha_\odot$ (dashed lines) isochrones.