A calibration of the mixing-length for solar-type stars based on hydrodynamical models of stellar surface convection

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Abstract. We present a grid of numerical 2D radiation hydrodynamics (RHD) models of the solar-like surface convection zones of stars in a wide range of effective temperatures ($4300\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 7100\,\mathrm{K}$), gravities ($2.54 \leq \log g \leq 4.74$), and metallicities ([M/H]=0.0, -0.5, -1.0, -2.0).

We calibrate the free parameter α of the mixing-length theory (MLT) and of two variants of the Canuto-Mazzitelli theory (CMT) to give the same convective efficiency as the RHD models. None of these three 1D descriptions of convection is able to reproduce the results of the RHD simulations with the same parameter α for all combinations of $T_{\mathrm{eff}}/\log g$/[M/H] (Figs. 2 to 8). Nevertheless, the RHD results show no drastic deviations from the range of convective efficiencies assessed from the three simple convection theories.

1. Numerical simulations

Based on 2D numerical radiation hydrodynamics calculations of time-dependent compressible convection, a variety of stellar surface convection zones has been studied. The numerical models account for detailed microphysics and are characterized in the same way as classical stellar atmospheres by effective temperature, gravity, and metallicity (for details see Ludwig et al. 1994).

One result of a RHD simulation run is the entropy s_{env} of the deeper, adiabatically stratified layers. In Figs. 1 and 5 this entropy is plotted in the $T_{\mathrm{eff}}/\log g$ plane for every RHD model for the four metallicities considered, together with contour lines of an analytic fit function to these values. This entropy shows

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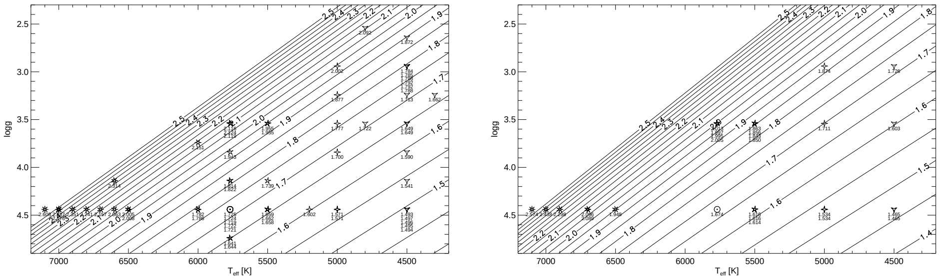

Figure 1. Entropy $s_{\rm env}$ of the adiabatic layers for [M/H]=0.0 (left) and -0.5 (right) as function of $T_{\rm eff}$ and log$g$. Each symbol indicates one RHD $T_{\rm eff}$/log$g$ parameter set. Every (tiny) number is the result of a RHD model. The Sun is marked by $\odot$.

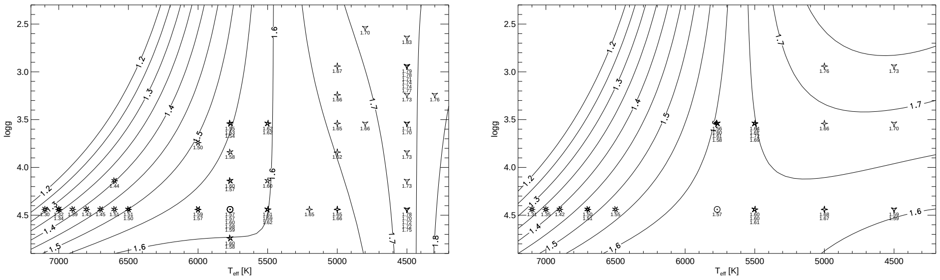

Figure 2. Calibrated parameter α for standard MLT for [M/H]=0.0 (left) and -0.5 (right) as function of $T_{\rm eff}$ and log$g$.

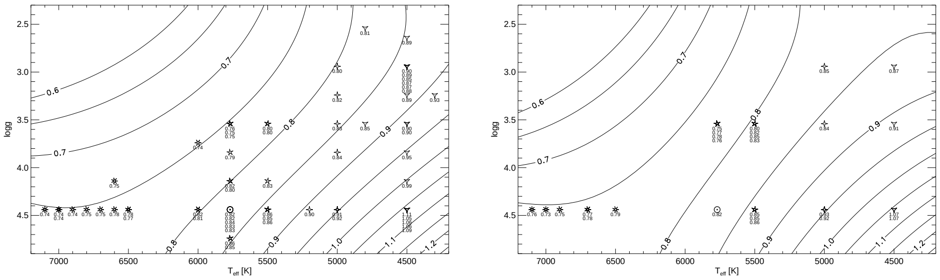

Figure 3. Calibrated parameter α for CMT(ℓ=αH_p) for [M/H]=0.0 (left) and -0.5 (right) as function of $T_{\rm eff}$ and log$g$.

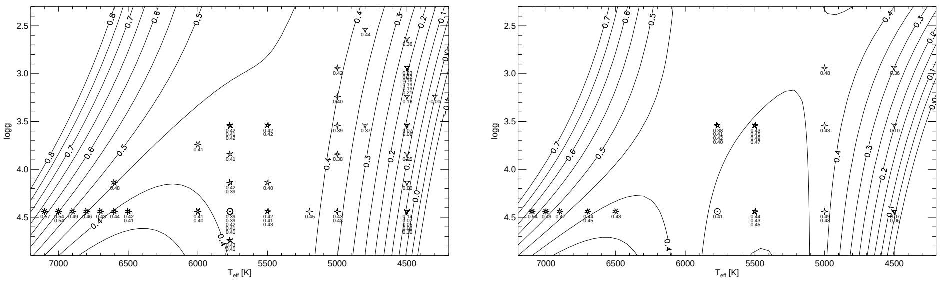

Figure 4. Calibrated parameter α for CMT(ℓ=z+$\alpha H_{p,{\rm top}}$) for [M/H]=0.0 (left) and -0.5 (right) as function of $T_{\rm eff}$ and log$g$.

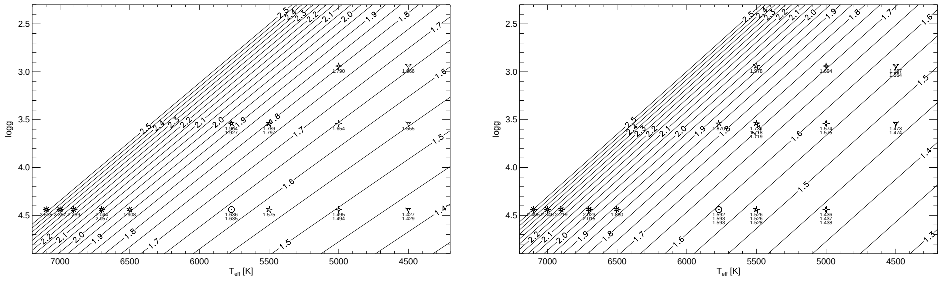

Figure 5. Entropy $s_{\rm env}$ of the adiabatic layers for [M/H]=-1.0 (left) and -2.0 (right) as function of $T_{\rm eff}$ and log$g$.

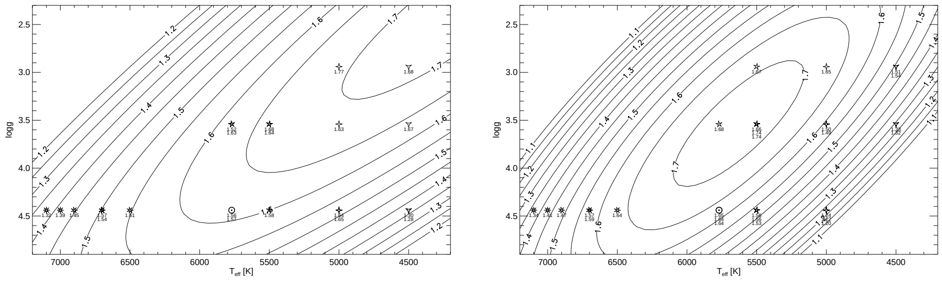

Figure 6. Calibrated parameter α for standard MLT for [M/H]=-1.0 (left) and -2.0 (right) as function of $T_{\rm eff}$ and log$g$.

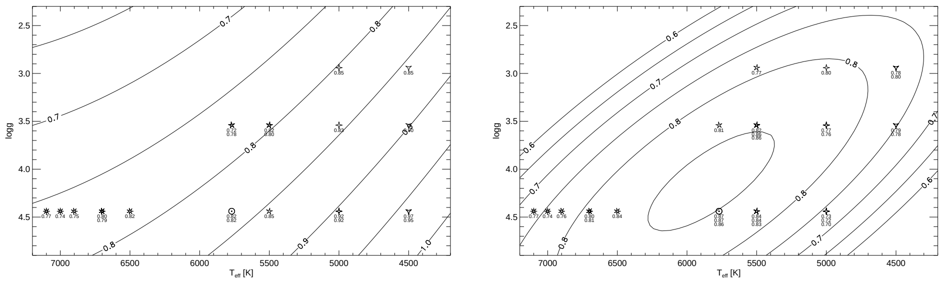

Figure 7. Calibrated parameter α for CMT(ℓ=αH_p) for [M/H]=-1.0 (left) and -2.0 (right) as function of $T_{\rm eff}$ and log$g$.

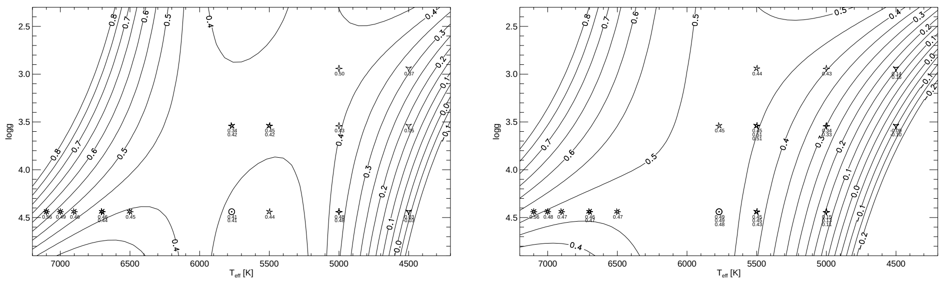

Figure 8. Calibrated parameter α for CMT(ℓ=z+$\alpha H_{p,\rm top}$) for [M/H]=-1.0 (left) and -2.0 (right) as function of $T_{\rm eff}$ and log$g$.

a rather regular behaviour with little qualitative differences between the four metallicities.

2. Calibrating mixing-length and Canuto-Mazitelli theory

The free parameter α fixing the mixing-length, ℓ=αH_p, is adjusted to produce MLT envelope models with an entropy of the adiabatic layers corresponding to the entropy $s_{\rm env}$ of the RHD models. In this way an effective mixing-length parameter α is derived (see Steffen 1993) for each $T_{\rm eff}$/log$g$/[M/H] combination and plotted in Figs. 2 and 6, together with a polynomial fit. For solar metallicity the parameter α shows an increase with falling effective temperature and only a slight dependence on gravity. In the main-sequence phase the effective α for the Sun is a constant close to 1.60. But it increases slightly during the evolution towards the red giant branch (upper right). For lower metallicities the mixing-length parameter shows a similar decrease towards higher effective temperatures and additionally a drop towards low effective temperatures and high gravities.

The characteristic length-scale of the Canuto-Mazzitelli theory can be calibrated in the same way as the mixing-length. With the parametrization ℓ=αH_p the parameter α shows a systematic dependence on $T_{\rm eff}$ and log$g$ (Figs. 3 and 7). For [M/H]=0.0, -0.5, -1.0 it increases monotonically with falling effective temperature and growing gravity. Its absolute value is smaller than the standard mixing-length parameter (roughly by a factor of 2), but its relative variation is larger. For the lowest metallicity it shows the same drop at low effective temperature and high gravity as in the MLT. With this parametrization the CMT shows a behaviour similar to that of the MLT.

One result of the CMT with the parametrization ℓ=$z + \alpha H_{p,\rm top}$ (with constant α) is that strong convection becomes even more and weak convection less efficient than in the MLT. For solar metallicity, the increase of the mixing-length parameter with falling effective temperature (cf. Figs. 2(left) and 4(left)) translates into a wide region in the solar neighbourhood of the $T_{\rm eff}$/log$g$ plane where the free parameter of the CMT is a constant close to 0.4. But at both bounds of our temperature range the compensation is too strong, leading to an increase of α at the high-temperature side and a strong drop at the low-temperature end. For [M/H]=-2.0 this drop is so strong that only *negative values* of α can account for the RHD results. For this metallicity the "plateau" of approximately constant α is rather narrow. Additionally, the "plateau" value of α increases with decreasing metallicity.

References

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