Conclusions

- By no choice of α the local mixing length theory can reproduce the velocity and flux profile computed with the radiation hydrodynamics code.
- The 'optimum' α varies with $T_{\rm eff}$, $\log g$, abundances.
- MLT with individually calibrated α can be adequate for modelling certain aspects of convection (e.g. photospheric temperature stratification, depth of unstable zone, typical velocities)
- Especially the extended overshoot region with an exponentially decaying velocity below the classical convection zone may be important for diffusion/mixing problems.
- The the mass of the convectively mixed region exceeds the mass of the unstable zone by a factor of 10 (A type stars) or factor of 100 (cool DA White Dwarfs).
- The convection theory of the next generation must appreciate the full nonlocality of the problem (momentum equation, equation of continuity) to improve local MLT particularly at the boundaries of the convection zone.

Optimum mixing length parameter

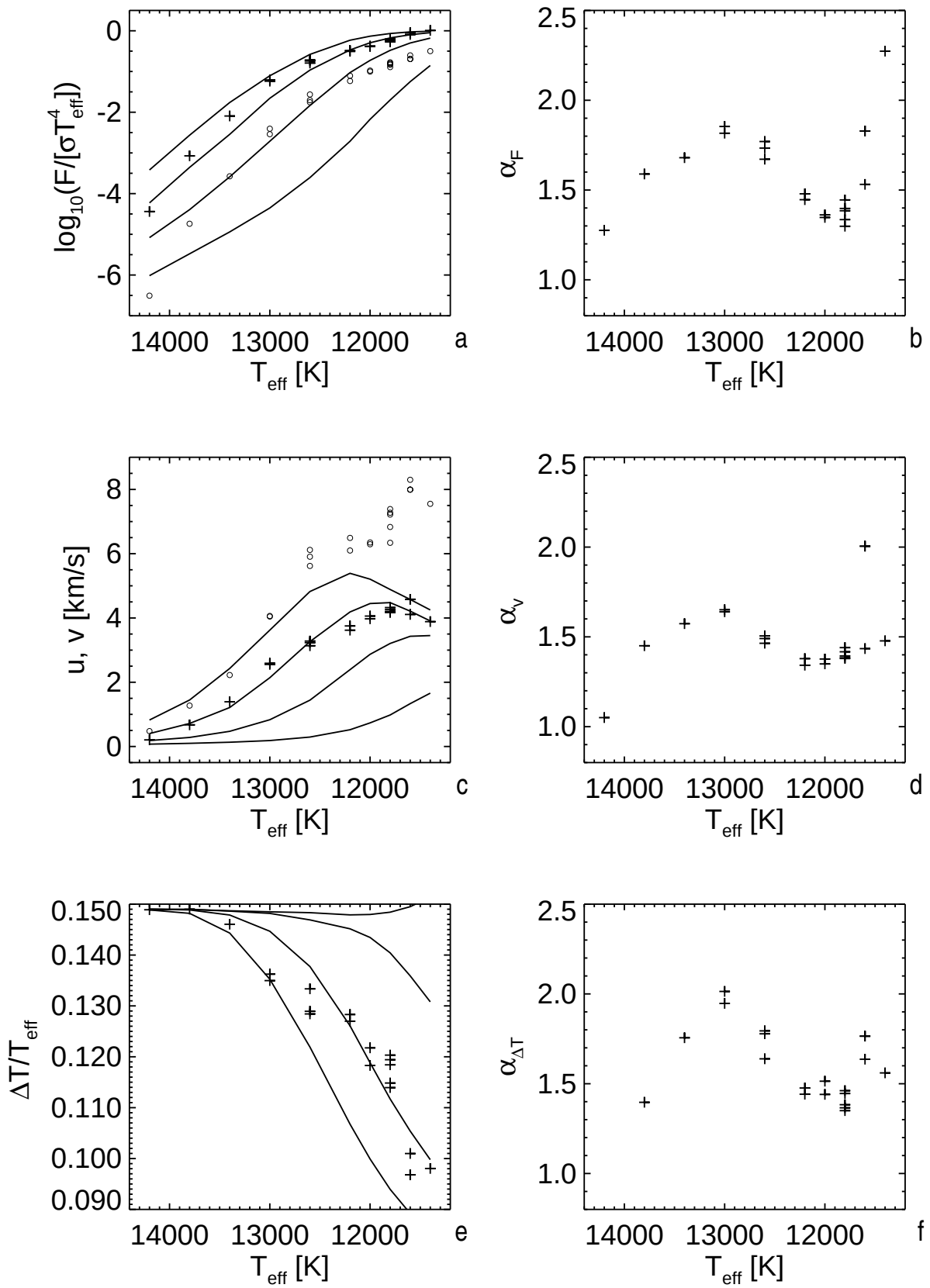

Figure 4: Derivation of an effective mixing length parameter α as function of effective temperature $T_{\rm eff}$ for DA white dwarfs with $\log g = 8.0$. Each plus sign ('+') or circle ('o') represents one radiation hydrodynamical model: a): '+': $F_{\rm h,max}$; 'o': $F_{\rm kin,min}$; lines: $F_{\rm MLT,max}$ for $\alpha = (0.5, 1.0, 1.5, 2.0)$ from bottom to top, b) from $F_{\rm h,max}$ derived optimum α, c): '+': $v_{\rm rms,max}$; 'o': $u_{\rm rms,max}$; lines: $v_{\rm MLT,max}$ for $\alpha = (0.5, 1.0, 1.5, 2.0)$ from bottom to top, d) from $v_{\rm rms,max}$ derived optimum α, e): '+': $(T(\log_{10}\tau = 0.2) - T(\log_{10}\tau = -0.2))/T_{\rm eff}$ from RHD simulations, lines: same quantity from MLT calculations forr $\alpha = (0.5, 1.0, 1.5, 2.0)$ from top to bottom f) from $(T(\log_{10}\tau = 0.2) - T(\log_{10}\tau = -0.2))/T_{\rm eff}$ derived optimum α.

‘Boundaries’ of convection zones in White Dwarfs

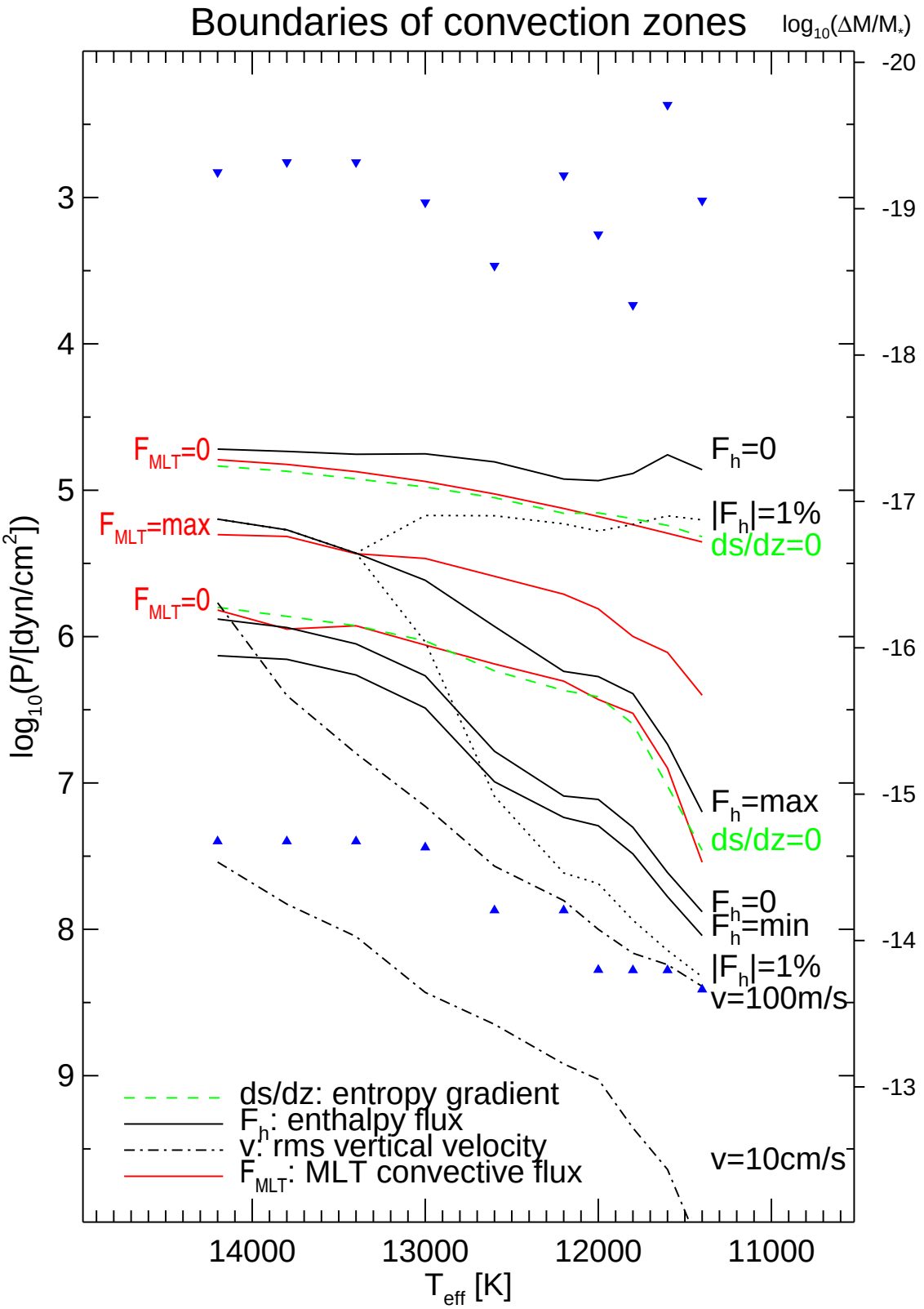

Figure 3: Possible boundaries of the convection zone versus $T_{\rm eff}$ derived from our radiation hydrodynamical models. Shown is the pressure and the logarithmic mass fraction. For 10 models with different effective temperatures the mean pressure at the top and at the bottom is indicated by blue triangles. The upper and lower boundaries of the unstable region (green lines) of the RHD models agree well with the convection zone according to MLT ATLAS models with a parameter $\alpha = 1.5$ (red lines). But the region with non-vanishing enthalpy flux is in the RHD models much larger than in the MLT models (black continuous lines). And the layers with non-vanishing velocities extend well beyond the actual boundaries of the RHD models (black dash-dotted lines).

Overshoot in diffusion calculations in A stars

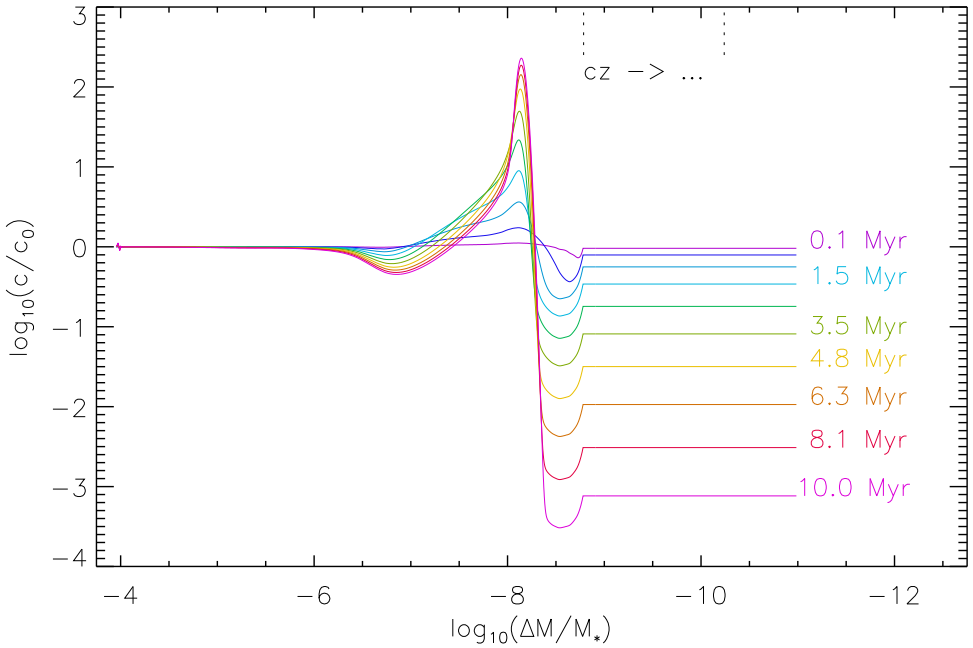

Figure 1: Relative titanium concentration in an A star model with $T_{\rm eff} = 7943$ K and $\log g = 4.34$. The logarithm of the relative titanium concentration is given as a function of the relative stellar mass above a certain shell (surface at right) for 10 selected time steps. The lower boundaries of the two convection zones are given by the MLT (see the two short dashed lines at the top of the plot). There is no additional overshoot included. The calculations have been made with the modified Charbonneau code and this picture is more or less the same as in Turcotte & Charbonneau (1993, ApJ 413, 376). Beneath the convection zone there is a region where titanium becomes overabundant. But inside the convection zone and the photosphere titanium is depleted.

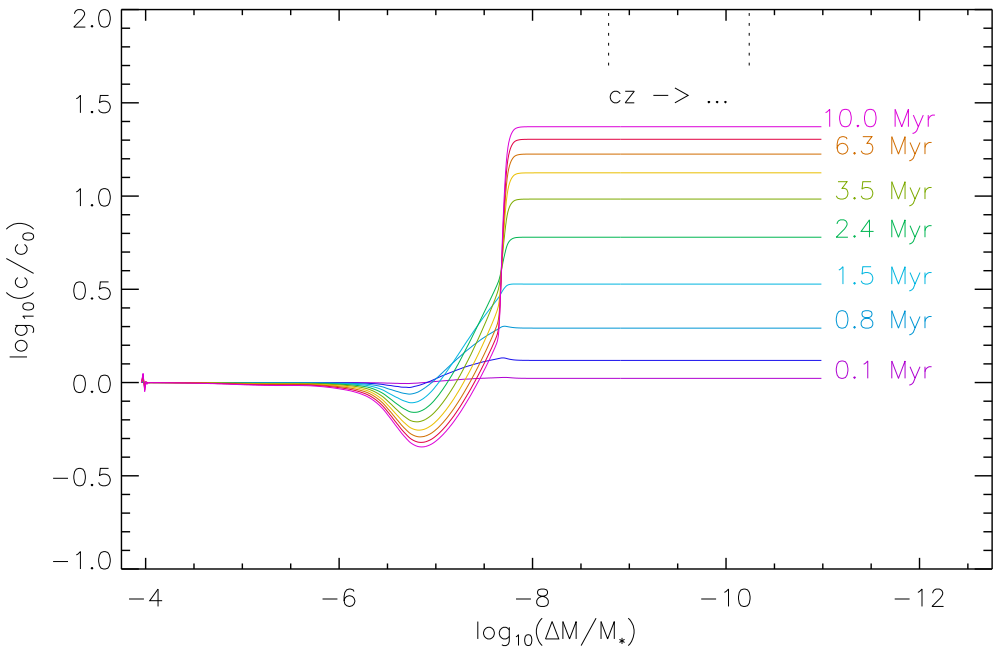

Figure 2: Relative titanium concentration in an A star model with $T_{\rm eff} = 7943$ K and $\log g = 4.34$. The times are the same as in **Figure 1**. The lower boundaries of the two convection zones are again given by the MLT. But now there is additional overshoot included according to the formula $D_{\rm overshoot}(z) = 10^{14}\exp((z-z_{\rm cz})/(\frac{1}{8}H_{\rm p,cz}))\mathrm{cm^2s^{-1}}$. The concentration of titanium in the photosphere changes dramatically from underabundance in **Figure 1** to overabundance because overshoot mixes the material from the small layer with titanium excess into the convection zone and photosphere.

Overshoot and the 'boundaries' of a convection zone

Possible criteria for a convection zone:

- $\frac{ds}{dz} < 0$, convective instability
- $F_h > 0$, positive enthalpy flux
- $F_h \neq 0$, nonvanishing enthalpy flux
- $v_{\rm conv} \neq 0$, nonvanishing convective velocities

Local MLTs: all criteria coincide.
Radiation hydrodynamics: Each region comprises the predecessor:

- $\frac{ds}{dz} < 0$, small disturbances grow, new jets are born
- $F_h > 0$, downward velocities are correlated with a temperature deficit, buoyancy driving
- $F_h < 0$, downward velocities are correlated with a temperature excess ($F_h < 0$), buoyancy braking
- $v_{\rm conv} \neq 0, F_h \approx 0$, mass conservation, no net work, only pressure fluctuations

The vertical velocity in the convectively stable regions decays exponentially as expected for unstable linear g^--modes .
$\Rightarrow$ The mass in the convectively mixed region may exceed the mass in the unstable zone by a factor of 10 or 100 depending on effective temperature and type of star.

Dynamics of convection

- Start with radiative stratification + small random disturbances of the velocity field
- Linear growth of most unstable convective mode (up- and downflows symmetric)
- Nonlinear effects $\Rightarrow$ asymmetry between narrow downdrafts and broad upflow regions
- Downdrafts move and merge $\Rightarrow$ increased asymmetry, possibly production of sound waves, deep reaching plumes (and excitation of f-mode)
- Quiet regions (at least with small $v_{\rm vert}$) $\Rightarrow$ convective instability produces new downdrafts $\Rightarrow\uparrow$
- Amount of turbulence depends on efficiency of convection

- diameter of downdrafts $\approx$ half wavelength of most unstable mode, jet as 'wave packet'
- distance between downdrafts $\Leftrightarrow$ horizontal velocity in unstable region and growth rate of convective mode

The radiation hydrodynamics code

- Equations of 2D compressible hydrodynamics in Cartesian geometry
- Realistic equation of state
- Energy equation includes nonlocal radiation transport.
- Realistic opacities (grey or non-grey scheme)
- Computational box covers only a tiny fraction of the star but more than ≈ 10 pressure scale heights.
- No rotation, no magnetic fields

Grid of stellar models

- Surface layers of solar type convection zones with $5000\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 6000\,\mathrm{K}$, $\log g = 4.44$; $T_{\mathrm{eff}} = 5770\,\mathrm{K}$, $3.54 \leq \log g \leq 4.74$
- Main sequence A stars with $7500\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 9500\,\mathrm{K}$
- DA white dwarfs with $11400\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 14200\,\mathrm{K}$, $\log g = 8.0$
- DA white dwarfs with $T_{\mathrm{eff}} = 12200\,\mathrm{K}$, $\log g = 7.7, 8.0, 8.3$

Motivation for investigating surface convection

Problems:

- A significant part of the photosphere can be convective.
- Photospheric temperature stratification can depend sensitively on adopted mixing length parameter α.
- Uncertainties in the determination of effective temperatures.
- Depth of model convection zone (and therefore its ability to mix underlying material into surface layers) depends on α.
- Upper layer of convection zone determines depth (e.g. Sun).
- Overshoot.
- α depends on $T_{\rm eff}$, $\log g$, abundances.
- Interaction of convection with waves.

Commonly local mixing length theory (MLT) is used. But there are problems:

- Free parameter
- Unrealistic modelling of boundaries

One way to tackle the problem:

- Parameter free (computer) simulations based on the constitutive equations

Poster presented at the
32nd Liège International Astrophysical Colloquium, Liège, Belgium, 1995

Numerical Simulations of Stellar Convection and Overshoot

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Summary

- Motivation for investigating surface convection zones
- Features and approximations of the radiation hydrodynamics code
- Dynamics of convection
- Overshoot and the 'boundaries' of a convection zone
- Overshoot and mixing in A stars
- The 'optimum' mixing length parameter α
- Conclusions