Numerical Simulations of Stellar Convection and Overshoot

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Summary

Based on detailed 2D numerical radiation hydrodynamics calculations of time-dependent compressible convection, we have studied the dynamics and thermal structure of the convective surface layers of some **main sequence stars** with effective temperatures between 5000 K and 9500 K (K1 V to A0 V) and of several **DA white dwarfs** with effective temperatures between 11400 and 14200 K. We display example results of these simulations in comparison with conventional mixing length models. In contrast to the picture of mixing length theory (MLT), fast narrow downdrafts are the dominant feature in the hydrodynamical simulations. We find that merging of downdrafts produces **deep reaching plumes**, causing convective motions to extend well beyond the unstable layers. The resulting velocity field below the convection zone proper decays exponentially with depth, as expected for unstable g-modes. As an example, we demonstrate the consequences of this kind of **overshoot** for a normal A-type star ($T_{\rm eff} = 7943$ K, $\log g = 4.34$) for which we have calculated the photospheric abundances of selected elements resulting from the interplay between mixing due to overshoot and element diffusion below the surface convection zone. We find that ignoring the effects of overshoot can lead to substantial errors in the derived abundances.
We demonstrate that the results of our hydrodynamical convection models can be used to **determine the value of the mixing length parameter** α which gives the correct structure of stellar convection zones when used with a conventional stellar evolution code. We find that α (≈ 1.65) is approximately constant during the main sequence evolution of the Sun. Future investigations will provide the 'right' α to be used for **stellar evolution calculations** in different regions in the HR-diagram.

Motivation for investigating surface convection

Problems:

- A significant part of the photosphere can be convective.
- The photospheric temperature stratification can depend sensitively on the adopted mixing length parameter α.
- This results in uncertainties in the determination of effective temperatures.
- The depth of a model convection zone (and therefore its ability to mix underlying material into surface layers) depends on α.
- The upper layer of a convection zone determines its total depth depth (e.g. in the Sun).
- Overshoot.
- α depends on $T_{\rm eff}$, $\log g$, abundances.
- There is an interaction of convection and waves.

Commonly local mixing length theory (MLT) is used. But there are problems:

- Free parameter
- Unrealistic modeling of boundaries

One way to tackle the problem:

- Parameter free (computer) simulations based on the constitutive equations

The radiation hydrodynamics code

- Equations of 2D compressible hydrodynamics in Cartesian geometry
- Realistic equation of state
- Energy equation includes nonlocal radiation transport.
- Realistic opacities (grey or non-grey scheme)
- Computational box covers only a tiny fraction of the star
  but more than ≈ 10 pressure scale heights.
- No rotation, no magnetic fields

Grid of stellar models

- Surface layers of solar type convection zones with
  $5000\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 6000\,\mathrm{K}$, $\log g = 4.44$;
  $T_{\mathrm{eff}} = 5770\,\mathrm{K}$, $3.54 \leq \log g \leq 4.74$
- Main sequence A stars with $7500\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 9500\,\mathrm{K}$; $\log g = 4.4$
- DA white dwarfs with $11400\,\mathrm{K} \leq T_{\mathrm{eff}} \leq 14200\,\mathrm{K}$; $\log g = 8.0$
- DA white dwarfs with $T_{\mathrm{eff}} = 12200\,\mathrm{K}$; $\log g = 7.7, 8.0, 8.3$

See the selection of snapshots of convective flow fields below.

Overshoot and the 'boundaries' of a convection zone

Possible criteria for a convection zone:

- $\frac{ds}{dz} < 0$, convective instability
- $F_h > 0$, positive enthalpy flux
- $F_h \neq 0$, nonvanishing enthalpy flux
- $v_{\rm conv} \neq 0$, nonvanishing convective velocities

Local MLTs: all criteria coincide.
Radiation hydrodynamics: criteria define increasingly larger regions:

- $\frac{ds}{dz} < 0$, small disturbances grow, new jets are born
- $F_h > 0$, downward velocities are correlated with a temperature deficit, buoyancy driving
- $F_h < 0$, downward velocities are correlated with a temperature excess, buoyancy braking
- $v_{\rm conv} \neq 0, F_h \approx 0$, mass conservation, no net work, only pressure fluctuations

The vertical velocity in the convectively stable regions decays exponentially as expected for unstable linear g$^-$-modes .
$\Rightarrow$ The mass in the convectively mixed region may exceed the mass in the unstable zone by a factor of 10 for A-type stars (see Figures 1 and 2) or 100 for cool DA white dwarfs (see Figure 3).

The 'optimum' mixing length parameter

By no choice of α can the local mixing length theory reproduce the velocity and flux profile computed with the radiation hydrodynamics code. So it was tried to determine a best fit from quantities as maximum enthalpy flux, maximum vertical velocity and photospheric temperature gradient for DA white dwarfs (Fig. 4) and main sequence A stars (Fig. 5).

Overshoot in diffusion calculations in A stars

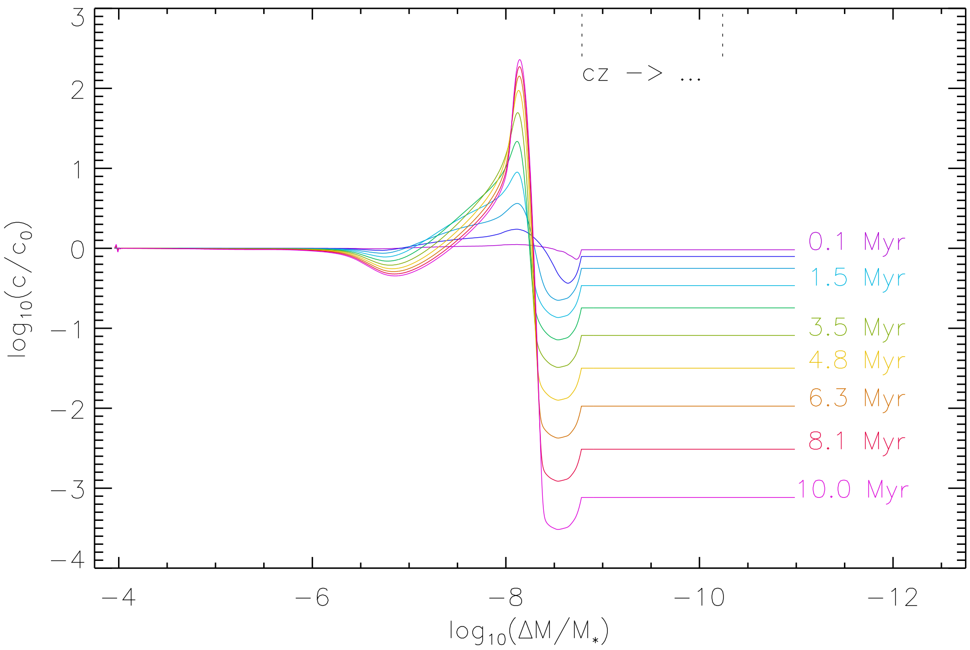

Figure 1: Relative titanium concentration in an A star model with $T_{\text{eff}} = 7943$ K and $\log g = 4.34$. The logarithm of the relative titanium concentration is given as a function of the relative stellar mass above a certain shell (surface at right) for 10 selected time steps. The lower boundaries of the two convection zones are given by the MLT (see the two short dashed lines at the top of the plot). There is no additional overshoot included. The calculations have been made with the modified Charbonneau code and this picture is more or less the same as in Turcotte & Charbonneau (1993, ApJ 413, 376). Beneath the convection zone there is a region where titanium becomes overabundant. But inside the convection zone and the photosphere titanium is depleted.

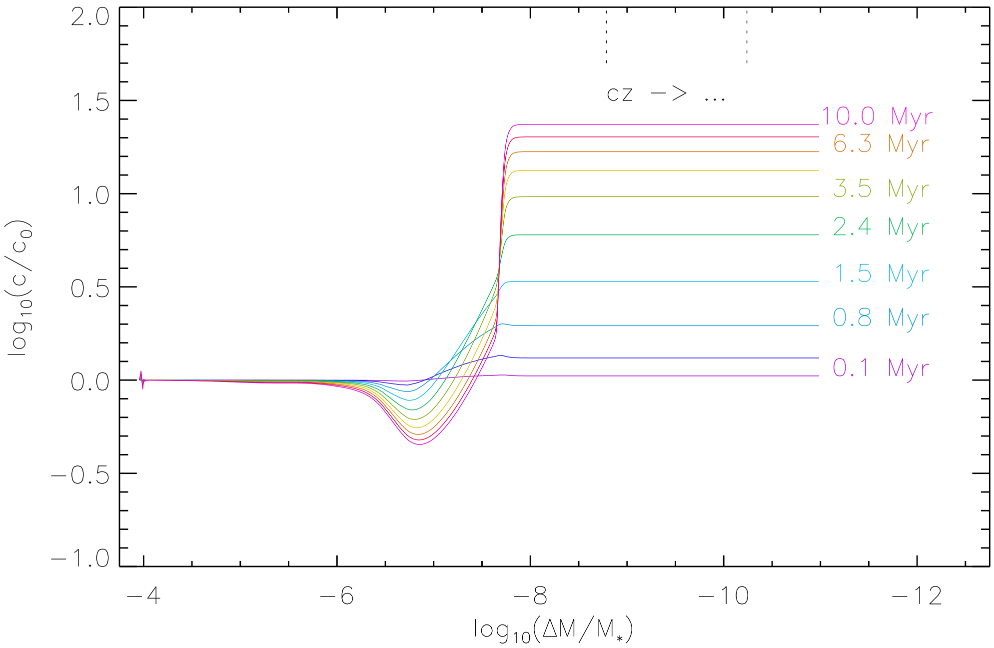

Figure 2: Relative titanium concentration in an A star model with $T_{\text{eff}} = 7943$ K and $\log g = 4.34$. The times are the same as in Figure 1. The lower boundaries of the two convection zones are again given by the MLT. But now there is additional overshoot included according to the formula $D_{\text{overshoot}}(z) = 10^{14} \exp((z - z_{\text{cz}})/(\frac{1}{8} H_{\text{p,cz}}))\,\text{cm}^2\text{s}^{-1}$. The concentration of titanium in the photosphere changes dramatically from underabundance in Figure 1 to overabundance because overshoot mixes the material from the small layer with titanium excess into the convection zone and photosphere.

‘Boundaries’ of convection zones in White Dwarfs

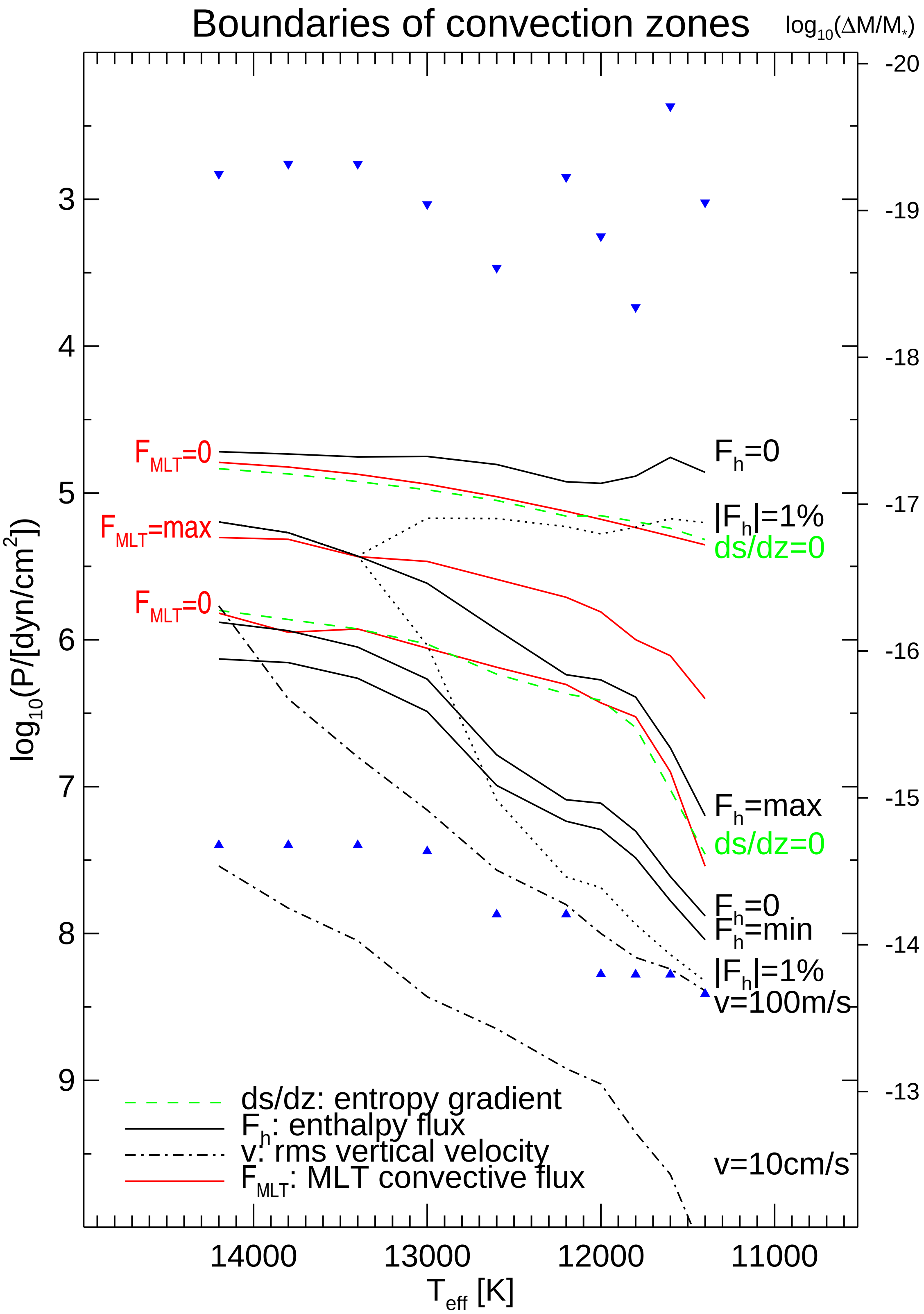

Figure 3: Possible boundaries of the convection zone versus T_{eff} derived from our radiation hydrodynamics models. Shown is the pressure and the logarithmic mass fraction. For 10 models with different effective temperatures the mean pressure at the top and at the bottom is indicated by blue triangles. The upper and lower boundaries of the unstable region (green lines) of the RHD models agree well with the convection zone according to MLT ATLAS models with a parameter $\alpha = 1.5$ (red lines). But the region with non-vanishing enthalpy flux is much larger in the RHD models than in the MLT models (black continuous lines). The layers with non-vanishing velocities extend well beyond the actual boundaries of the RHD models (black dash-dotted lines).

Optimum mixing length parameter: DA WDs

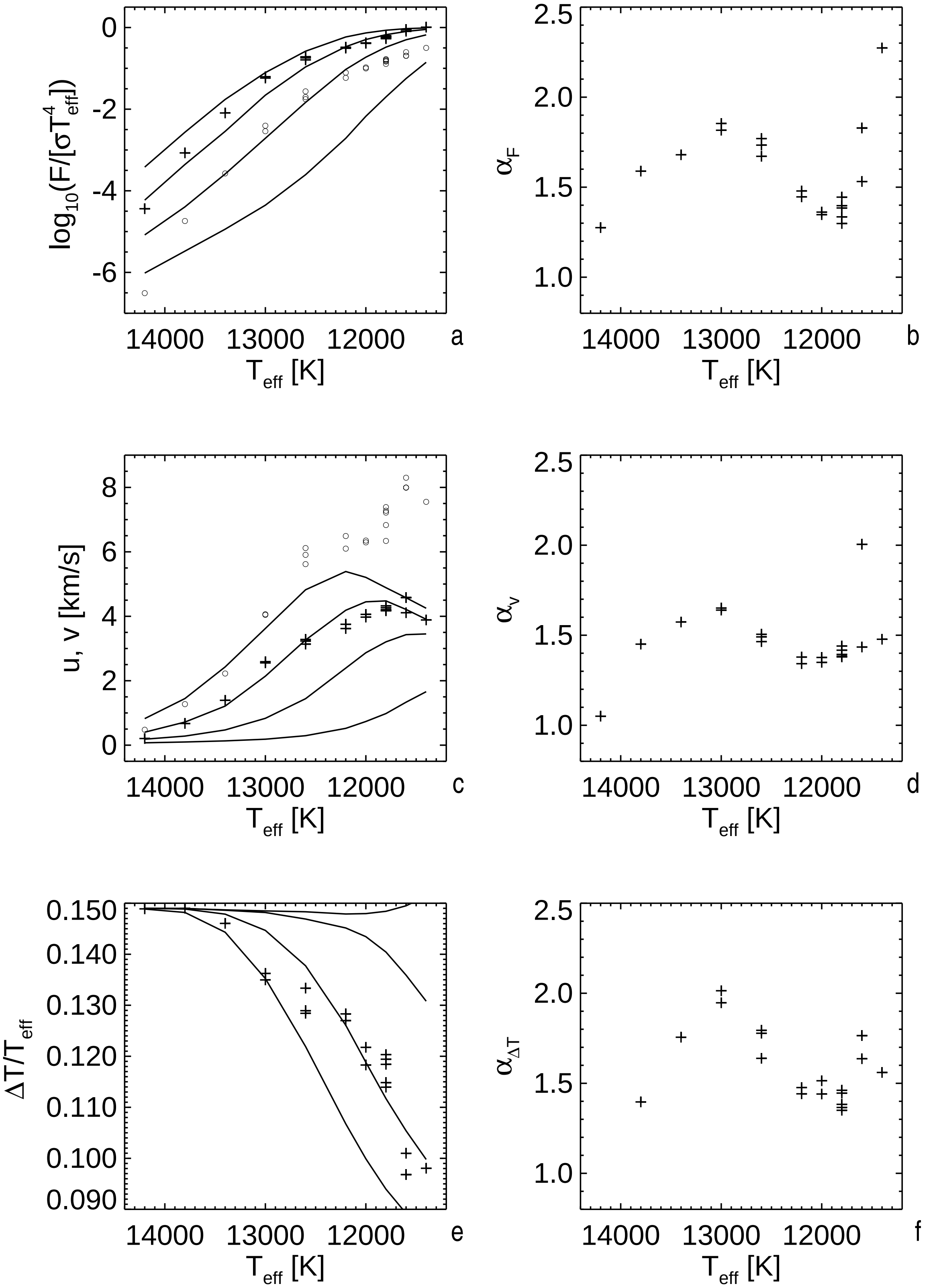

Figure 4: Derivation of an effective mixing length parameter α as function of effective temperature T_{eff} for DA white dwarfs with $\log g = 8.0$. Each plus sign ('+') or circle ('∘') represents one radiation hydrodynamical model: a): '+': $F_{\text{h,max}}$; '∘': $F_{\text{kin,min}}$; lines: $F_{\text{MLT,max}}$ for $\alpha = (0.5, 1.0, 1.5, 2.0)$ from bottom to top, b) optimum α derived from $F_{\text{h,max}}$, c): '+': $v_{\text{rms,max}}$; '∘': $u_{\text{rms,max}}$; lines: $v_{\text{MLT,max}}$ for $\alpha = (0.5, 1.0, 1.5, 2.0)$ from bottom to top, d) optimum α derived from $v_{\text{rms,max}}$, e): '+': $(T(\log_{10}\tau = 0.2) - T(\log_{10}\tau = -0.2))/T_{\text{eff}}$ from RHD simulations, lines: same quantity from MLT calculations for $\alpha = (0.5, 1.0, 1.5, 2.0)$ from top to bottom, f) optimum α derived from $(T(\log_{10}\tau = 0.2) - T(\log_{10}\tau = -0.2))/T_{\text{eff}}$. **Result:** $\alpha \approx \alpha_F \approx \alpha_V \approx \alpha_{\Delta T} \approx 1.5$ seems to be appropriate for all models. The variation with T_{eff} can be attributed to variations of the horizontal scale of convective elements.

Optimum mixing length parameter: A stars

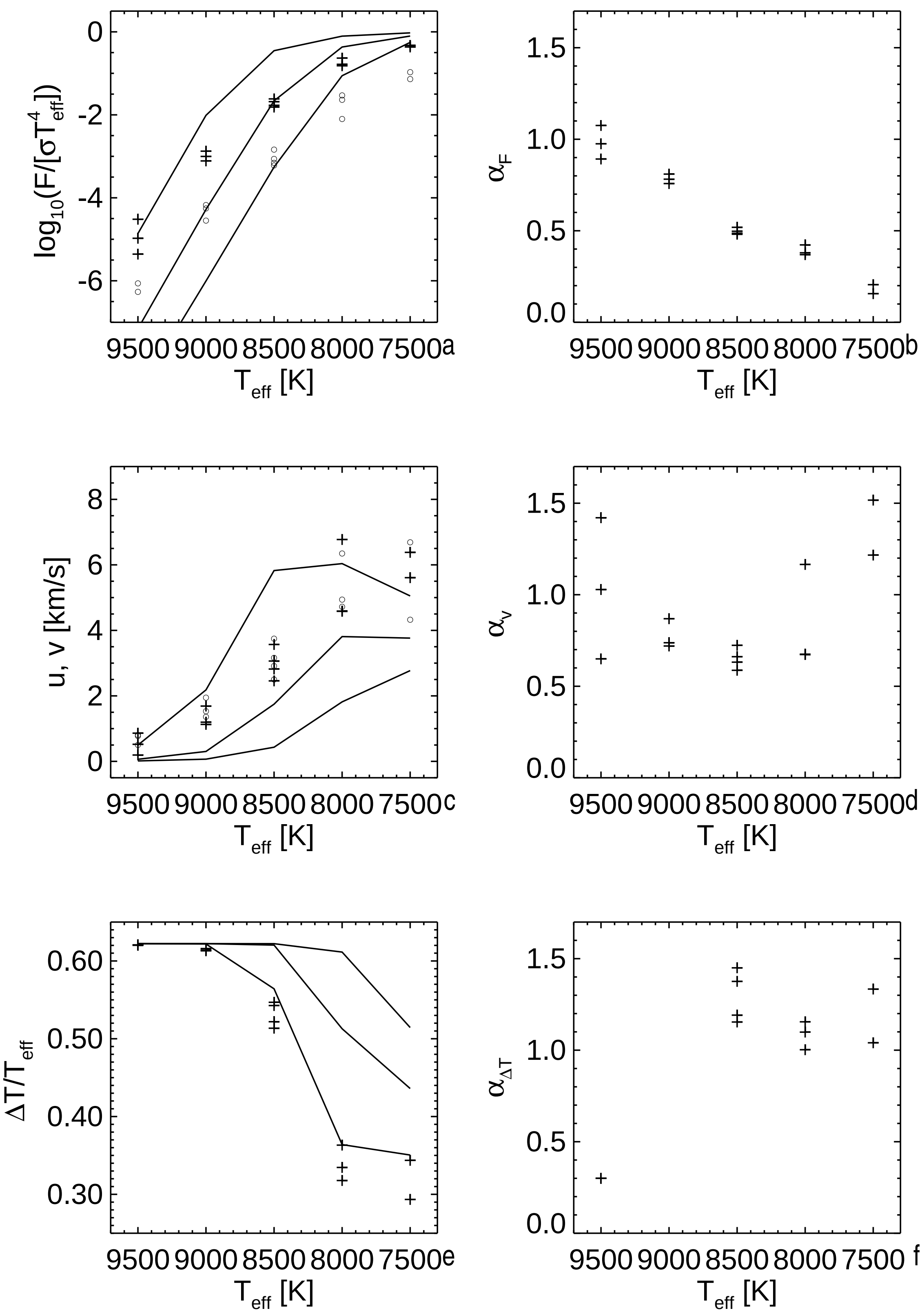

Figure 5: Derivation of an effective mixing length parameter α as function of effective temperature $T_{\rm eff}$ for A-type stars with $\log g = 4.4$. Each plus sign ('+') or circle ('$\circ$') represents one radiation hydrodynamical model: a): '+': $F_{\rm h,max}$; '$\circ$': $F_{\rm kin,min}$; lines: $F_{\rm MLT,max}$ for $\alpha = (0.3, 0.5, 1.0)$ from bottom to top, b) optimum α derived from $F_{\rm h,max}$, c): '+': $v_{\rm rms,max}$; '$\circ$': $u_{\rm rms,max}$; lines: $v_{\rm MLT,max}$ for $\alpha = (0.3, 0.5, 1.0)$ from bottom to top, d) optimum α derived from $v_{\rm rms,max}$, e): '+': $(T(\log_{10}\tau = 1.0) - T(\log_{10}\tau = 0.0))/T_{\rm eff}$ from RHD simulations, lines: same quantity from MLT calculations for $\alpha = (0.3, 0.5, 1.0)$ from top to bottom, f) optimum α derived from $(T(\log_{10}\tau = 1.0) - T(\log_{10}\tau = 0.0))/T_{\rm eff}$. **Result:** There is a clear disagreement between α_F, α_V, and $\alpha_{\Delta T}$. The values are generally lower than for white dwarfs.

The mixing-length parameter for solar-type convection zones inferred from hydrodynamical models of the surface layers

Standard evolutionary models of the Sun are constructed by calibrating the mixing-length parameter α and the initial helium content Y such that the models fit the current solar radius and luminosity. In this procedure α serves mainly as a technical parameter to calculate the entropy jump between the stellar surface and the deep, nearly adiabatic layers in the convective envelope. Under certain assumptions hydrodynamical models of stellar surface convection provide direct information about this entropy jump, offering the possibility of an independent determination of α.

The key quantity which is derived from the hydrodynamical models is the entropy s^* of the material which enters the simulation volume through the open lower boundary. Its value is determined by the requirement that the convective flow transports the correct energy flux given by the effective temperature of the model. The entropy s^* is interpreted as the entropy of the adiabat of the deep convective envelope. This entropy can be converted to an equivalent α by constructing an envelope model based on MLT matching this adiabat in the deep layers.

Is α a constant during the solar main sequence evolution?

Figure 6 presents the results for the determination of α derived from the hydrodynamical models for the evolution of the Sun. Since in this study the radiative transfer was treated in the grey approximation, the absolute values of α are somewhat overestimated; only the *differential* changes of α are significant here. The uncertainty of the values amount to ± 0.02 and reflect the remaining temporal variations of s^* after the simulations reached a statistically stable state. The results are marginally consistent with the assumption of a constant α during the main sequence evolution of the Sun. We find a significant increase of α during the evolution towards the red giant branch.

Helioseismology, Hydrodynamics, and Stellar Evolution

Hydrodynamical models including frequency-dependent radiative transfer have been constructed for the present Sun. Since now all important physical processes are taken into account the derived entropy s^* and the corresponding mixing-length parameter should represent the actual values encountered in the solar convection zone. The results are summarized in the first row of Tab. 1. The uncertainties are larger than for the differential investigation and reflect the variations over all hydrodynamical models constructed so far. The second row of the table shows the results of a calibrated solar model from standard evolutionary calculations. In the evolutionary models OPAL opacities (Grevesse 1991 heavy element mixture; Rogers, F.J. & Iglesias, C.A., 1992, ApJ 401, 361) supplemented by low temperature opacities according Weiss et al. (Weiss, A., Keady, J.J., Magee, N.H., 1990, Atomic Data and Nuclear Data Tables 45, 209) are adopted. In low density regions (<0.01 g/ccm) the MHD, at higher densities a simpler equation of state has been used that accounts for partial ionization according to the Saha-Boltzmann equations and partial electron degeneracy. The envelope models from which the α corresponding to the hydrodynamical models is determined, are based on the same input physics so that all values of α are directly comparable in Tab. 1. The last row of the table gives the values of the surface abundance of helium and the depth of the convection zone as determined by Helioseismology.

Within the errors the theoretical models give a consistent picture while there is a clear discrepancy to the helioseismological measurements. Additional checks have shown that remaining uncertainties in the equation of state — primarily the neglect of the Coulomb interaction — are of minor importance here. Preliminary investigations have revealed that in the theoretical models the helium abundance has to be reduced as suggested by the seismologically determined value. While the surface abundance of helium is a free parameter in the hydrodynamical models, in the evolutionary models its value is fixed by the calibration procedure, so it can be changed only if additional physical effects are incorporated. Indeed, evolutionary models including the diffusion of helium show a significant depletion of helium in the solar envelope. Future investigations will show whether a consistent picture will emerge if helium diffusion is taken into account.

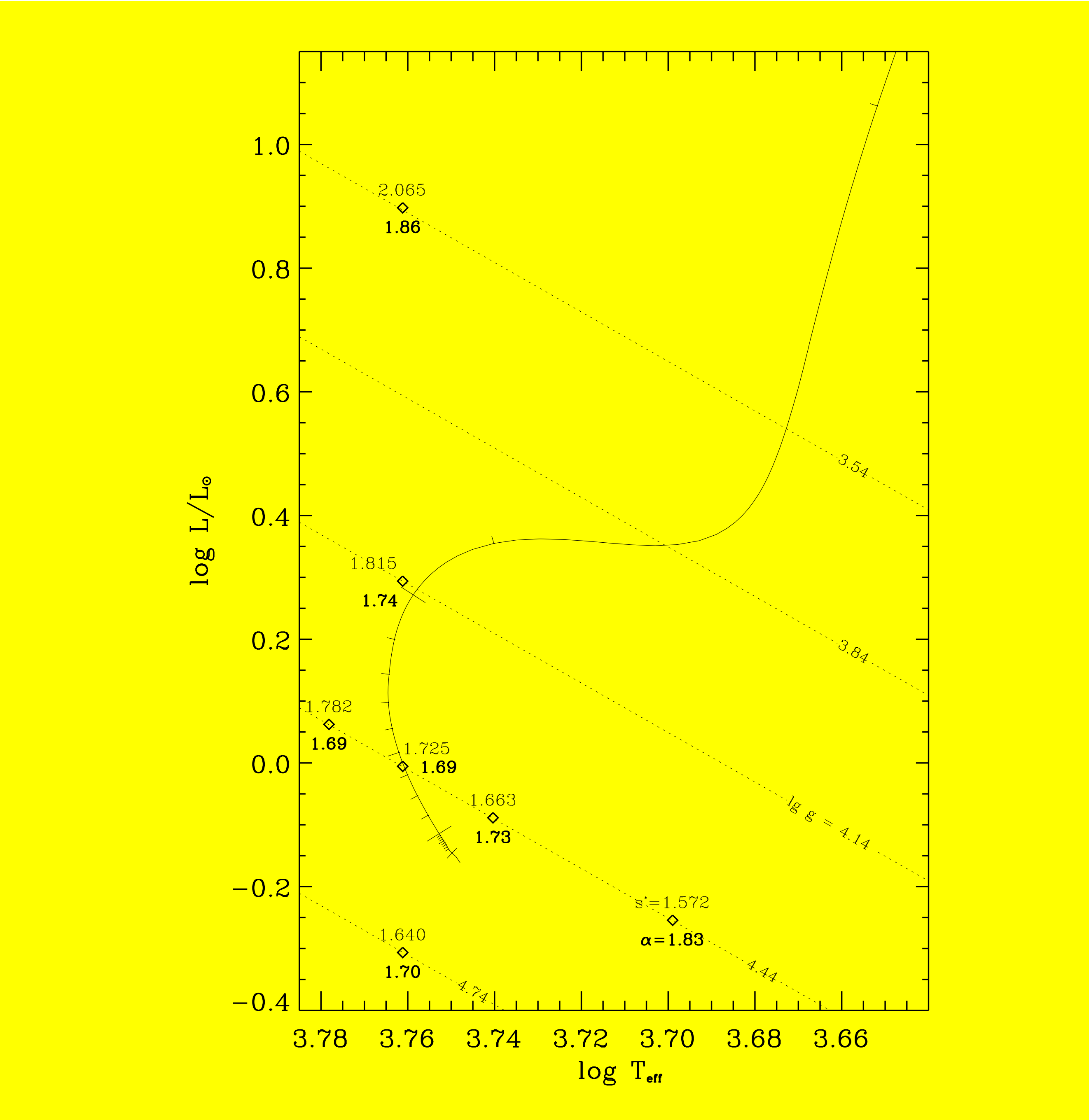

Figure 6: Mixing-length parameter α derived from hydrodynamical model atmospheres in the $\log T_{\rm eff} - \log L$ plane. For each model a diamond indicates its position together with the derived values of the entropy of the adiabat (s^* in 10^9 erg/g/K, upper number) and the corresponding mixing-length parameter (α, lower number). The uncertainty of the determination of α amounts to ± 0.02. Lines of constant $\log g$ are shown for a one solar mass star. An evolutionary track of the Sun is plotted with tick marks indicating the age from the ZAMS; the first mark corresponds to 0.1 Ga followed by marks in steps of 0.1 Ga. The first long mark corresponds to 1.0 Ga followed by marks in steps of 1.0 Ga.

	X	Y	$s^*[10^9$ erg/g/K$]$	α	$R_{\rm cz}/R_\odot$
Hydrodynamics	0.7035	0.2795	1.735 ± 0.01	1.645 ± 0.06	0.729 ± 0.006
Evolution	0.7015	0.2798	-	1.62	0.731
Helioseismology	-	0.242 ± 0.003	-	-	0.713 ± 0.003

Table 1: Results from hydrodynamical and evolutionary model calculations as well as helioseismological measurements. X: hydrogen abundance, Y: helium abundance, s^* entropy of adiabat, α: mixing-length parameter, $R_{\rm cz}/R_\odot$: location of the base of the convective envelope in solar radii.

Conclusions

- By no choice of α can the local mixing length theory reproduce the velocity and flux profile computed with the radiation hydrodynamics code.
- MLT with individually calibrated α can be adequate for modeling certain aspects of convection (e.g. photospheric temperature stratification, depth of unstable zone, typical velocities)
- The 'optimum' α varies with $T_{\rm eff}$, $\log g$, and abundances:
  cool DA white dwarfs: $\alpha \approx 1.5$,
  main sequence A-type stars: $\alpha \approx 0.25 - 1.5$ (MLT as in ATLAS).
- Hydrodynamical models of convective stellar surface layers can provide the "correct" value of α to be used with conventional stellar evolution calculations. We find that during the main sequence evolution of the Sun $\alpha \approx 1.65 \approx$ constant (MLT as in the evolutionary code).
- Extended overshoot regions found in hydrodynamical convection models are characterized by an exponentially decaying velocity field below the classical convection zone which has to be taken into account for diffusion/mixing problems.
- The the mass of the convectively mixed region exceeds the mass of the unstable zone by a factor of 10 (A-type stars) or a factor of 100 (cool DA White Dwarfs).
- The convection theory of the next generation must appreciate the full non-locality of the problem (momentum equation, equation of continuity) to improve local MLT particularly at the boundaries of the convection zone.