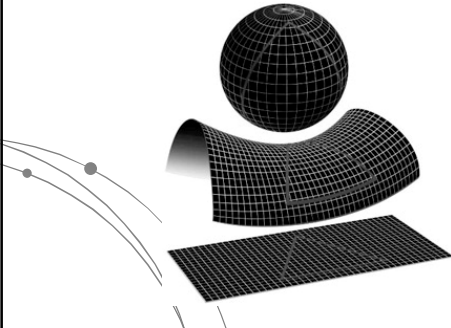


Cosmology AS7009, 2008 Lecture 2



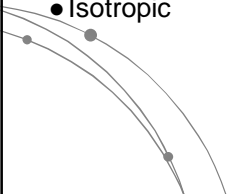
Outline

- The cosmological principle:
 - Isotropy
 - Homogeneity
- Big Bang vs. Steady State cosmology
- Redshift and Hubble's law
- Scale factor, Hubble time, Horizon distance
- Olbers' paradox: Why is the sky dark at night?
- Particles and forces
- Theories of gravity: Einstein vs. Newton
- Cosmic curvature

Covers chapter 2 + half of chapter 3 in Ryden

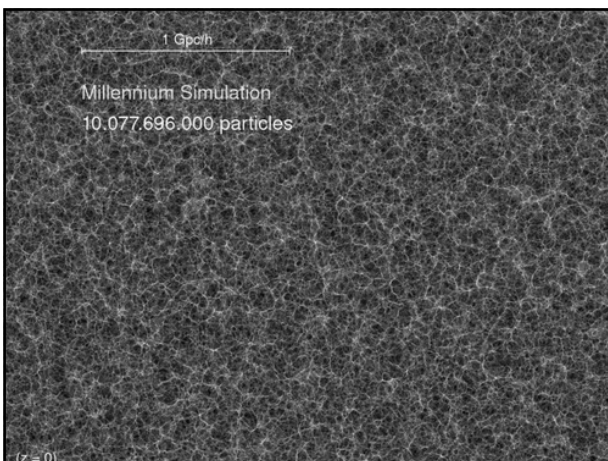
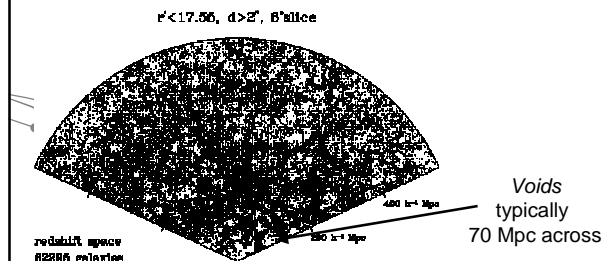
The Cosmological Principle I

- Modern cosmology is based on the assumption that the Universe is:
 - Homogeneous
 - Isotropic
- } The cosmological principle



The Cosmological Principle II

- These tenets *seem* to hold on large scales (>100 Mpc), but definitely not on small



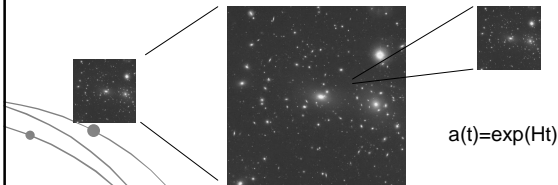
The Perfect Cosmological Principle

- In this case, one assumes that the Universe on large scales is:
 - Homogeneous
 - Isotropic
 - Non-evolving

This is incompatible with the Big Bang scenario, but the *Steady State model* (popular in the 1940-1960s) was based on this idea

Steady State Cosmology

Universe continuously expands, but due to continuous creation of matter, no dilution occurs → Steady State, no hot initial Big Bang and no initial singularity...



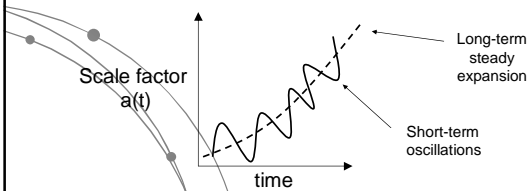
... But this cosmology fails to explain the CMBR, $T_{\text{CMBR}}(z)$, the production of light elements and the redshift evolution of galaxies & AGN

Suggestion for Literature Exercise: Quasi-Steady State Cosmology

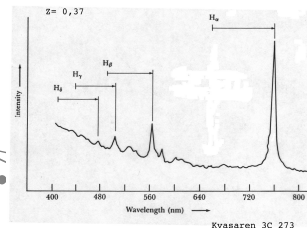
- Attempt in 1990s to resurrect Steady-State and explain the:
 - CMBR
 - production of light elements
 - dark matter
 - supernova type Ia data
 - Large-scale structure

Suggestion for Literature Exercise: Quasi-Steady State Cosmology

- Cyclic creation events → Long-term steady expansion, but with short-term oscillations
- To meet observational constraints, QSS requires:
 - Strange intergalactic dust
 - Cyclic creation events ("little bangs") which cause local expansion of space



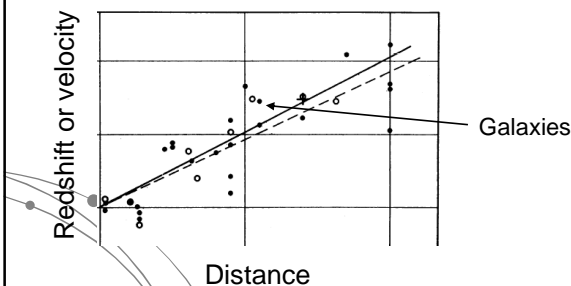
Redshift



Definition of redshift:

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$$

Hubble's law I



Hubble's law II

Hubble's law:

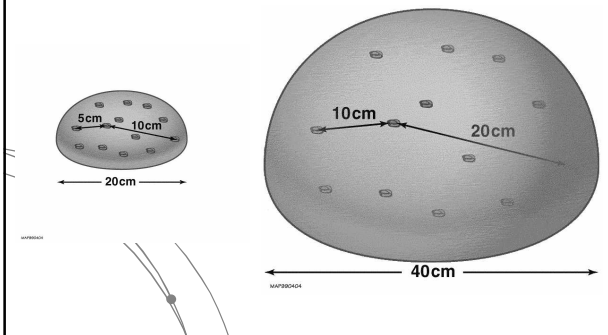
The Hubble "constant" → H_0 → Luminosity distance

$$z = \frac{H_0 d}{c}$$

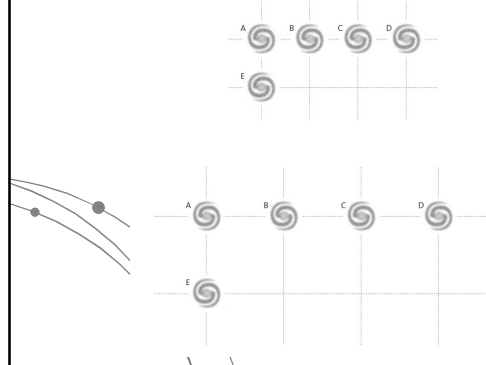
In observational astronomy, the term recession velocity, v , occurs frequently:

At low z : $z \approx \frac{v}{c} \rightarrow v = H_0 d$

Expansion of the Universe I



Expansion of the Universe II



Common Misconceptions about Modern Cosmology no. 1 & 2

- “Isn't it embarrassing that cosmologists have failed to pinpoint the site in the sky where the Big Bang explosion took place?”
- “The Big Bang was like a bomb going off in an previously empty space. The galaxies that we observe today represent the fragments of that explosion.”

No, no, no...

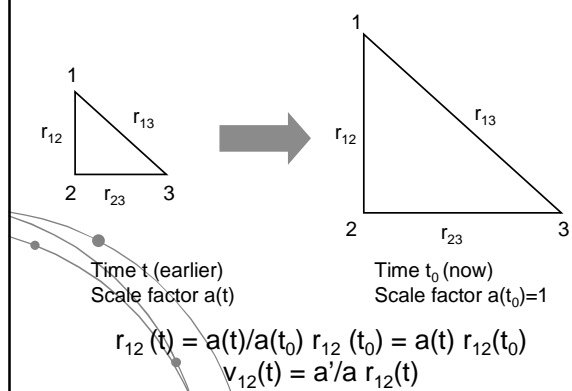
Redshift and distance I

- Low redshift ($z \approx 0$) corresponds to:
 - Small distance (local Universe)
 - Present epoch in the history of the Universe
- High redshift corresponds to:
 - Large distance
 - Earlier epoch in the history of the Universe

Redshift and distance II

- But beware:
 - At low redshift, Doppler components coming from peculiar motions may be substantial – must be corrected for before d is derived from z or v
 - The redshift coming from cosmic expansion is not a Doppler shift – don't treat it like one!
 - The linear version of Hubble's law is only appropriate at $z < 0.15$ (at 10% accuracy)

Scale factor



Scale factor and redshift

$$1+z = \frac{a_0}{a} = \frac{1}{a}$$

Cosmic scale factor today (at t_0)
— can be set to $a_0=1$

Cosmic scale factor when the Light was emitted (the epoch corresponding to the redshift z)

The Hubble “constant”

$$H_0 \approx 72 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1} [\text{s}^{-1}]$$

Today

Errorbars possibly underestimated...

Note: Sloppy astronomers often write km/s/Mpc...

In general:

$$H \equiv \frac{\dot{a}}{a}$$

Not a constant in our Universe!

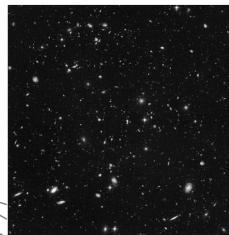
Hubble time

In the case of constant expansion rate, the Hubble time gives the age of the Universe:

$$t_H = \frac{1}{H_0} \approx 14 \text{ Gyr}$$

In more realistic scenarios, the expansion rate changes over time, but the currently favoured age of the Universe is still pretty close – around 13–14 Gyr.

Olbers' paradox I



"Why is the sky dark at night?"
(Heinrich Olbers 1926)

If the Universe is:

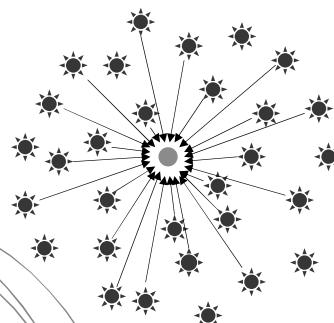
- Spatially infinite (i.e. infinite volume)
- Infinitely old and unevolving

- then the night sky should be bright!

Olbers' paradox II

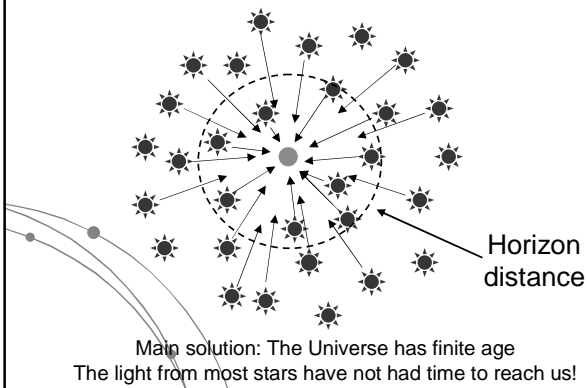


Olbers' paradox III



Planet Earth surrounded by stars in an infinite, unevolving Universe

Olbers' paradox IV



Horizon distance

- Horizon distance = Current distance to the most faraway region from which light has had time to reach us
- This delimits the causally connected part of the Universe an observer can see at any given time
- Horizon distance at time t_1 :

$$d_{\text{hor}}(t_1) = c \int_{t=0}^{t_1} \frac{dt}{a(t)}$$

- Most realistic scenarios give:
 $d_{\text{hor}}(t_0) \sim c/H_0$ (the so-called Hubble radius)

Particles and forces I

- The particles that make up the matter we encounter in everyday life:
 - Protons, p
938.3 MeV
 - Neutrons, n
939.5 MeV
 - Electrons, e^-
0.511 MeV

Baryons (made of 3 quarks)

Lepton

Since most of the mass of 'ordinary matter' is contributed by protons and neutrons, such matter is often referred to as *baryonic*.
Examples of mostly baryonic objects: Planets, stars, gas clouds (but not galaxies or galaxy clusters)

Particles and forces II

- Other important particles (for this course):
 - Photon, γ
Massless, velocity: c
 - Neutrinos, ν_e, ν_μ, ν_τ
eV (?), velocity close to c

Leptons

Interacts via weak nuclear force only

Particles and forces III

- The four forces of Nature:
 - Strong force
 - Very strong, but has short range ($\sim 10^{-15}$ m)
 - Holds atomic nuclei together
 - Weak force
 - Weak and has short range
 - Responsible for radioactive decay and neutrino interactions
 - Electromagnetic force
 - Weak but long-range
 - Acts on matter carrying electric charge
 - Gravity
 - Weak, very long-range and always attractive

On the large scales involved in cosmology, gravity is by far the dominant one

Newtonian gravity

- Space is Euclidian (i.e. flat)
- Planet are kept in their orbits because of the gravitational force:

$$F = -\frac{GM_g m_g}{r^2}$$

Gravitational mass

- The acceleration resulting from the gravitational force:

$$F = m_i a$$

Inertial mass

Equivalence Principle

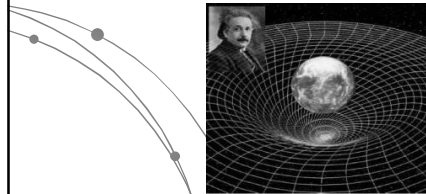
- Gravitational acceleration towards an object with mass M_g is:

$$a = -\frac{GM_g}{r^2} \left(\frac{m_g}{m_i} \right)$$

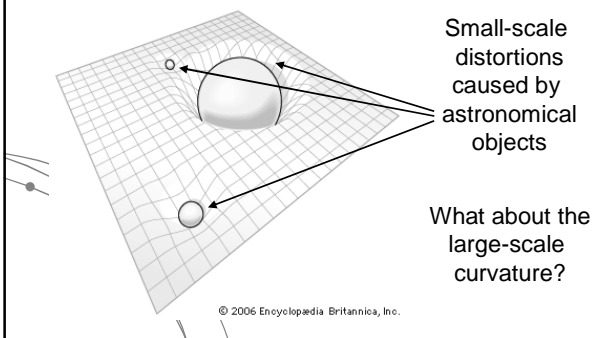
- Empirically $M_g = M_i$ (to very high precision)
- The equality of gravitational mass and inertial mass is called the equivalence principle
- In Newtonian gravity, $M_g = M_i$ is just a strange coincidence, but in General Relativity, this stems from the idea that masses cause curvature of space

General Relativity

- 4D space-time
- Mass/energy curves space-time
- Gravity = curvature
- Pocket summary:
 - Mass/energy tells space-time how to curve
 - Curved space-time tells mass/energy how to move

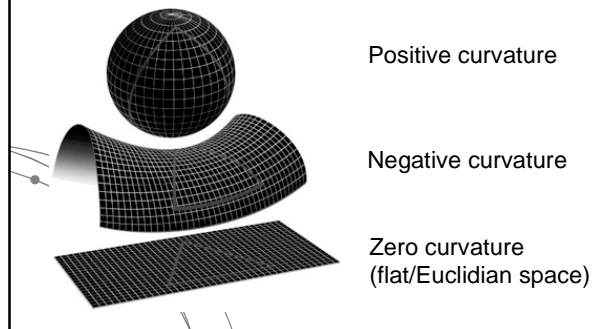


Small-scale curvature



Global Curvature I

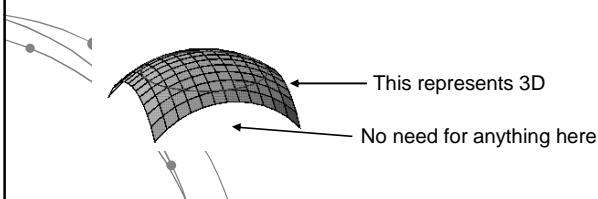
In the world models of general relativity, our Universe may have spatial curvature (on global scales)



Global Curvature II

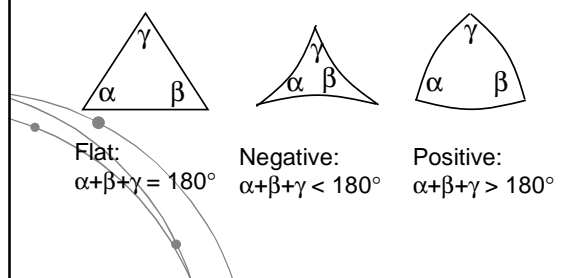
Very tricky stuff...

- This is an *intrinsic* curvature in 3D space
- Note: No need for encapsulating our 3D space in 4D space to make this work



Global Curvature III

Angles in curved spaces



Metrics I

- Metric: A description of the distance between two points
- Metric in 2 dimensional, flat space:

$$ds^2 = dx^2 + dy^2 \quad (\text{Pythagoras})$$

- Metric in 3 dimensional, flat space:

$$ds^2 = dx^2 + dy^2 + dz^2$$

Metrics II

- Metric in 3 dimesions, flat space, polar coordinates:

$$ds^2 = dr^2 + r^2 d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$$

- Metric in 3 dimensions, arbitrary curvature:

$$ds^2 = \frac{dx^2}{1 - \kappa x^2 / R^2} + x^2 d\Omega^2$$

Flat : $\kappa = 0, x = r$

Negative : $\kappa = -1, x = R \sinh(r / R)$

Positive : $\kappa = 1, x = R \sin(r / R)$

Curvature radius