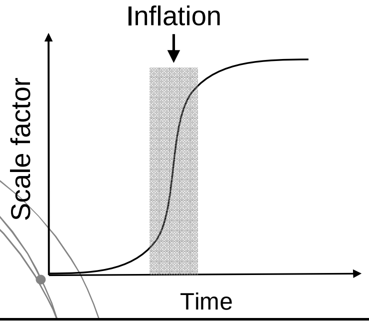
Cosmology AS7009, 2008
Lecture 9

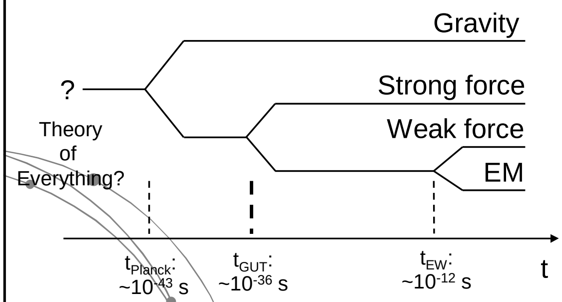


Outline

- Grand Unified Theories and phase transitions
- Problems with a non-inflationary Big Bang
- Inflation
  - Inflaton field
  - Slow-roll
  - Reheating
  - Seeds for structure formation
- Inflation as a solution to the flatness, horizon and magnetic monopole problems
- Eternal inflation
- Primordial black holes

Covers chapter 11 in Ryden + extra stuff

Grand Unification



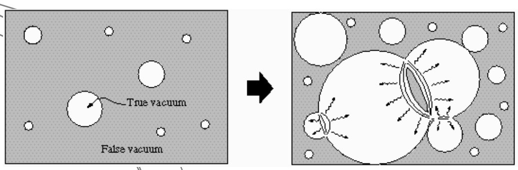
Grand Unification II

- Electroweak unification experimentally confirmed in late 1970s → Nobel prize in physics to Maxwell, Weinberg, Salam & Glashow for electroweak theory
- GUT happens at $E_{GUT}\sim 10^{12}$ TeV
- LHC reaches ~ 10 TeV → Experimental confirmation of GUT is not gonna happen soon…

Phase Transitions

The Universe underwent a phase transition when the temperature dropped below $T_{GUT}\sim 10^{28}$ K
Symmetry between strong and electroweak force lost → Topological defects!

Many GUT scenarios produce magnetic monopoles with $mc^2\sim 10^{12}$ TeV



Why do we need inflation?

- To solve:
  - Flatness problem
  - Horizon problem
  - Magnetic monopole problem
- To seed structure formation

The flatness problem I

Observationally:

$|1-\Omega_0| \leq 0.1$

One can show that this implies, at the Planck time:

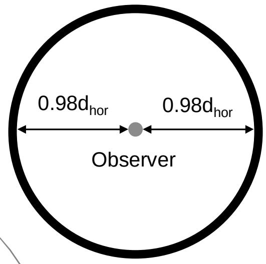
$|1-\Omega_{\rm Planck}| \leq 10^{-60}$

Hence, if the Universe is close to flat now, it was extremely close to flat in the past.

Why is the Universe so close to flat?
If this is a coincidence, it very, very improbable!

The horizon problem I

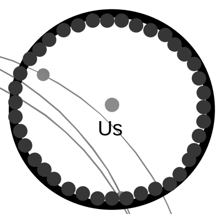
Last scattering surface



How can the CMBR be almost completely isotropic, when opposite sides of the sky have never been in casual contact?

The horizon problem II

$$\theta_{\rm hor} = \frac{d_{\rm hor}(t_{\rm lss})}{d_{\rm A}} \approx \frac{0.4{\rm Mpc}}{13{\rm Mpc}} \approx 2 \text{ degrees}$$



● Regions in causal contact
~20000 patches in the CMBR sky

The magnetic monopole problem I

Magnetic monopoles: zero-dimensional objects which act as isolated north or south poles of a magnet

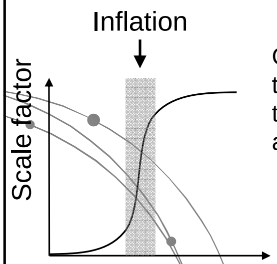
Many GUT models predict huge numbers of these! While subdominant at creation, they would soon come to dominate the energy density of the Universe

Problem: No such objects have ever been observed! Where are the magnetic monopoles?

Inflation

What is inflation?

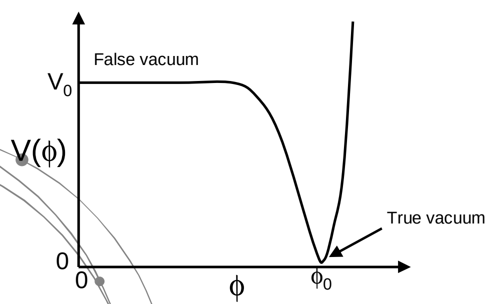
A short period of fast expansion, happening very early in the history of the Universe



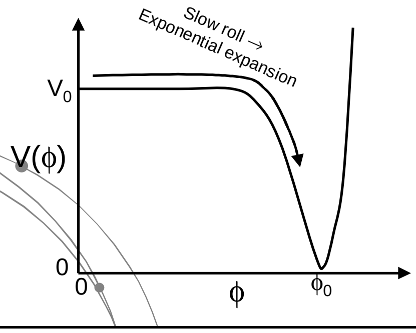
One possible model:
t_{start} ~ 10^{-36} s after the Big Bang
t_{stop} ~ 10^{-34} s after the Big Bang
$a(t_{stop})/a(t_{start})$ ~ e^{100} ~10^{43}

The inflaton field I

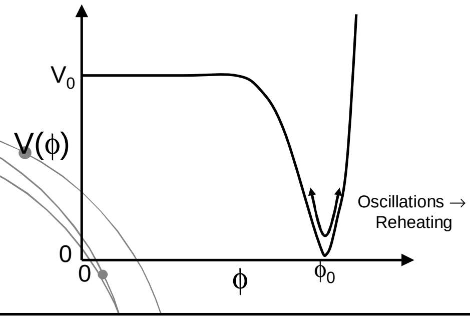
Consider a scalar field $\phi(r,t)$ with potential energy $V(\phi)$



The inflaton field II



The inflaton field III



Slow-roll

$$\varepsilon_\phi = \frac{1}{2}\frac{1}{hc^3}\dot{\phi}^2 + V(\phi)$$

$$P_\phi = \frac{1}{2}\frac{1}{hc^3}\dot{\phi}^2 - V(\phi)$$

Slow roll:

$$\dot{\phi}^2 << hc^3 V(\phi) \Rightarrow$$

$$\varepsilon_\phi \approx -P_\phi \approx V(\phi)$$

Negative pressure!
Λ-like expansion!
de Sitter phase!

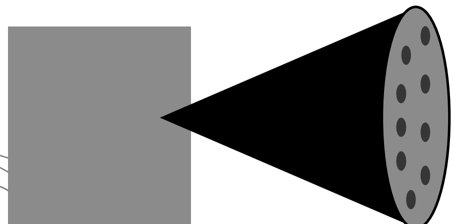
Reheating

If the Universe expands by a factor of ~e^{100} →
Temperature drops by e^{-100}
and the radiation energy denstiy gets extremely small

How come it's not small after inflation then?

Oscillations of ϕ around ϕ_0 →
Some of the energy of the inflaton field are
being carried away by radiation
These photons *reheat* the Universe
Hence, no shortage of photons after inflation!

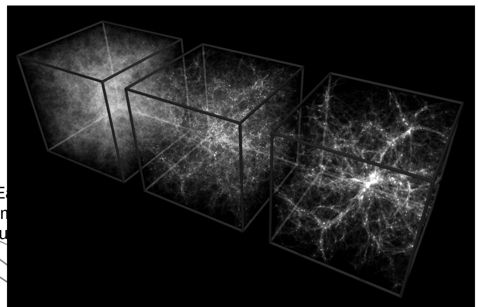
Seeds for structure formation



Prior to inflation:
Microscopic quantum fluctuations
Virtual particles created and annihilated

After inflation:
Quantum fluctuations blow up
to macroscopic scales
act as seeds for structure formation

Seeds for structure formation II



Current Universe
Primordial seeds have
generated very
complicated structure

Inflation as a solution to the flatness problem I

The acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$$

During inflation, the Universe is temporarily dominated by a component with $P < -\varepsilon/3$ (i.e. $w<-1/3$), giving positive acceleration. One often assumes a *cosmological constant* $\Lambda_{\text{inflation}}$ to be responsible. Note: This is a constant very different from the Λ driving the cosmic acceleration today. $\Lambda_{\text{inflation}} \sim 10^{107}\,\Lambda$…

Inflation as a solution to the flatness problem II

Hubble parameter and scale factor during inflation:

$$H_{\text{inflation}} = \left(\frac{\Lambda_{\text{inflation}}}{3}\right)^{1/2}$$

$$a(t) \propto \mathrm{e}^{H_{\text{inflation}} t}$$

Number of e-foldings during inflation:

$$N = H_{\text{inflation}}(t_{\text{stop}} - t_{\text{start}})$$

$$N \sim 100$$

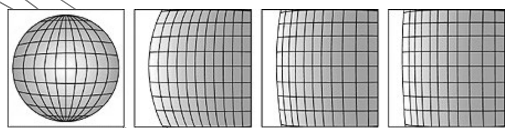
Inflation as a solution to the flatness problem III

$$|1-\Omega(t_{\text{stop}})| = \mathrm{e}^{-2N}|1-\Omega(t_{\text{start}})|$$

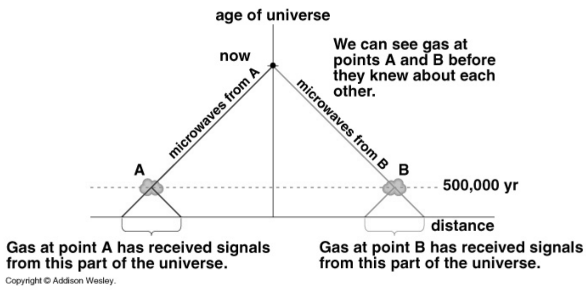
Example:

$$|1-\Omega(t_{\text{start}})| \approx 1 \Rightarrow |1-\Omega(t_{\text{stop}})| \approx 0$$

Inflation makes a curved Universe flat!

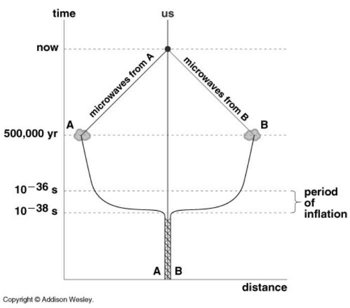


Inflation as a solution to the horizon problem I



The horizon problem

Inflation as a solution to the horizon problem II



The solution

Inflation as a solution to the horizon problem III

Horizon before and after inflation:

$$d_{\text{hor}}(t_2) = c\int_{t_1}^{t_2} \frac{\mathrm{d}t}{a(t)}$$

Before inflation:

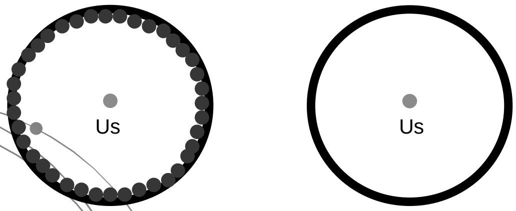
$$d_{\text{hor}} = 2ct_{\text{start}} \sim 6\times 10^{-28}\,\mathrm{m}$$

After inflation:

$$d_{\text{hor}} \approx \mathrm{e}^{N}\, 3ct_{\text{start}} \sim 2\times 10^{16}\,\mathrm{m}$$

Inflation as a solution to the horizon problem IV

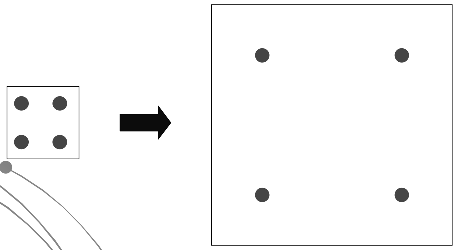
Without inflation | With inflation



Regions in causal contact
~20000 patches in the CMBR sky

Just one patch in the sky!

Inflation as a solution to the magnetic monopole problem I



Expansion dilutes the number densities of objects, and inflation did this extremely efficiently

Inflation as a solution to the magnetic monopole problem II

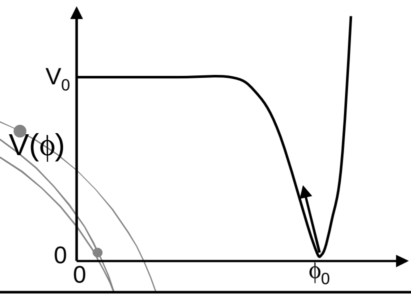
At the end of inflation:

$$n_{\text{monopoles}}(t_{\text{stop}}) \sim e^{-300} n_{\text{monopoles}}(t_{\text{GUT}})$$

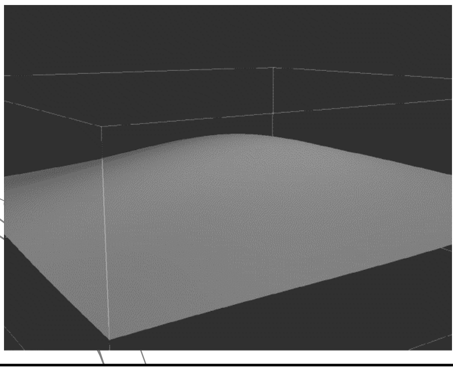
A realistic number density of monopoles at the GUT epoch would correspond to less than one monopole within the volume spanned by the last scattering surface

Eternal inflation I

Once the inflaton field has come to rest at ϕ_0, inflation ends. But in some regions of space equantum fluctuations can make the inflation field move up the potential again $\rightarrow$ Some regions keep inflating

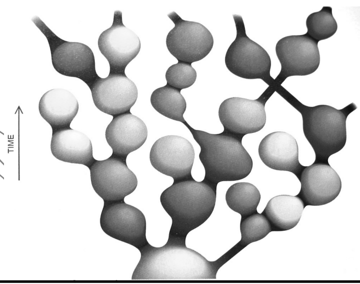


Eternal inflation II



Eternal inflation III

Differently inflated regions may end up with very different properties ('mutations') $\rightarrow$ Multiverse
Good 'genes' may promote further expansion (self-reproducing Universes).



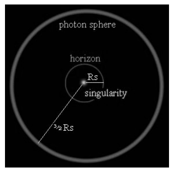
Eternal inflation IV

- Quantum fluctuations in $\phi \rightarrow$ *Future-eternal inflation*
  Inflation will always continue (somewhere)
- Past-eternal inflation models also exist:
  Revives the perfect cosmological principle!
  The interior of each inflating bubble may be described by the Big Bang theory, but the multiverse as a whole has been around forever

Stellar Black Holes

Initial mass:	Last stages:	Remnant mass:
$30 < M/M_{solar}$	SN $\rightarrow$ BH	~ 3—20 M_{solar}

$\rightarrow$



Supernova

Stellar Black Hole

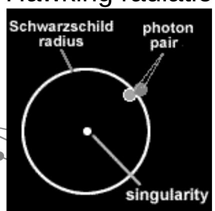
Since the progenitors of stellar black holes are baryonic, these black holes cannot contribute much to the matter density of the Universe

Primordial Black Holes

- High-density regions in the early Universe (t « 1 s) may collapse into primordial black holes
- PBHs could in principle form with masses from M_{Planck} - 10^{15} M_{solar}
- Remains a viable candidate for the cold dark matter: Ω_{PBH} could be ~0.3!
- Example:
  - M_{PBH}~10-8 M_{solar} (mass of the Moon) would have a size (event horizon) of R~0.1 mm

Primordial Black Holes II

Hawking radiation:



$$\tau_{evap} \sim 9 \cdot 10^{-18} M^3 s$$

$$T_{rad} = \frac{hc^3}{16\pi^2 kGM}$$

Objects with M > 5×10^{11} kg would still be around!

Unclear what happens at M_{Planck}. Relics may form!

·Observational constraints:
- BBNS abundances
- Gamma-ray background
- CMBR