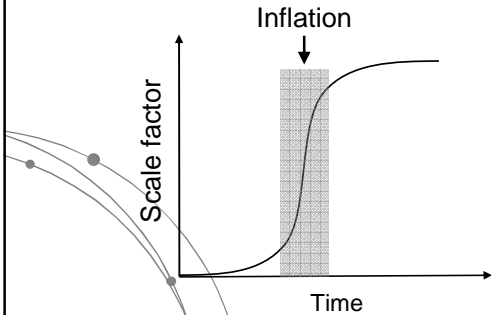


Cosmology AS7009, 2009 Lecture 9

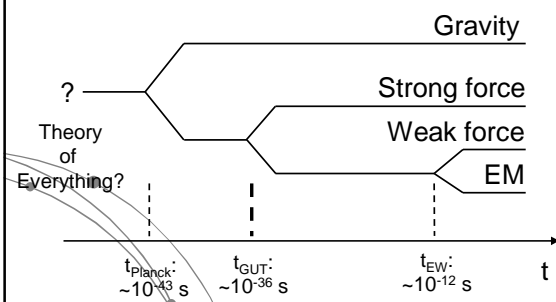


Outline

- Grand Unified Theories and phase transitions
- Problems with a non-inflationary Big Bang
- Inflation
 - Inflaton field
 - Slow-roll
 - Reheating
 - Seeds for structure formation
- Inflation as a solution to the flatness, horizon and magnetic monopole problems
- Eternal inflation
- Primordial black holes

Covers chapter 11 in Ryden + extra stuff

Grand Unification



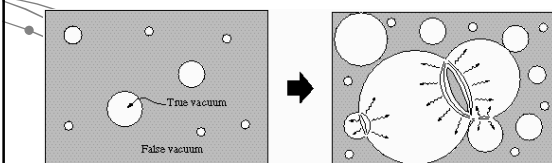
Grand Unification II

- Electroweak unification experimentally confirmed in late 1970s → Nobel prize in physics to Maxwell, Weinberg, Salam & Glashow for electroweak theory
- GUT happens at $E_{\text{GUT}} \sim 10^{12}$ TeV
- LHC reaches ~ 10 TeV → Experimental confirmation of GUT is not gonna happen soon...

Phase Transitions

The Universe underwent a phase transition when the temperature dropped below $T_{\text{GUT}} \sim 10^{28}$ K
Symmetry between strong and electroweak force lost → Topological defects!

Many GUT scenarios produce magnetic monopoles with $mc^2 \sim 10^{12}$ TeV



Why do we need inflation?

- To solve:
 - Flatness problem
 - Horizon problem
 - Magnetic monopole problem
- To seed structure formation

The flatness problem I

Observationally :

$$|1 - \Omega_0| \leq 0.1$$

One can show that this implies, at the Planck time :

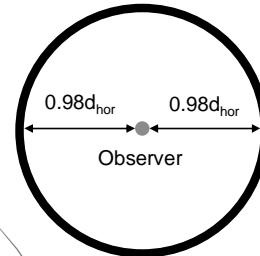
$$|1 - \Omega_{\text{Planck}}| \leq 10^{-60}$$

Hence, if the Universe is close to flat now,
it was extremely close to flat in the past.

Why is the Universe so close to flat?
If this is a coincidence, it very, very improbable!

The horizon problem I

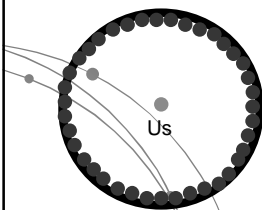
Last scattering surface



How can the CMBR be almost completely isotropic, when opposite sides of the sky have never been in casual contact?

The horizon problem II

$$\theta_{\text{hor}} = \frac{d_{\text{hor}}(t_{\text{ls}})}{d_A} \approx \frac{0.4 \text{Mpc}}{13 \text{Mpc}} \approx 2 \text{ degrees}$$



● Regions in causal contact
~20000 patches in the CMBR sky

The magnetic monopole problem I

Magnetic monopoles: zero-dimensional objects which act as isolated north or south poles of a magnet

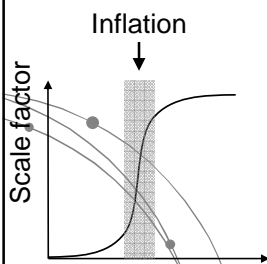
Many GUT models predict huge numbers of these!
While subdominant at creation, they would soon come to dominate the energy density of the Universe

Problem: No such objects have ever been observed!
Where are the magnetic monopoles?

Inflation

What is inflation?

A short period of fast expansion, happening very early in the history of the Universe



One possible model:

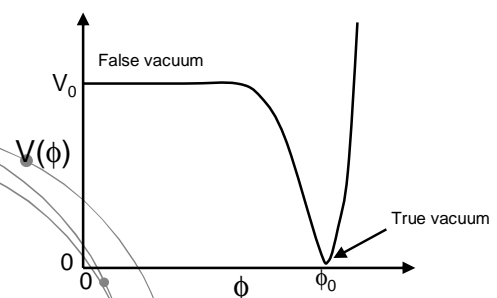
$$t_{\text{start}} \sim 10^{-36} \text{ s after the Big Bang}$$

$$t_{\text{stop}} \sim 10^{-34} \text{ s after the Big Bang}$$

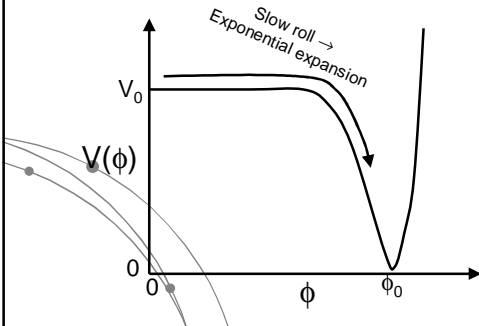
$$a(t_{\text{stop}})/a(t_{\text{start}}) \sim e^{100} \sim 10^{43}$$

The inflaton field I

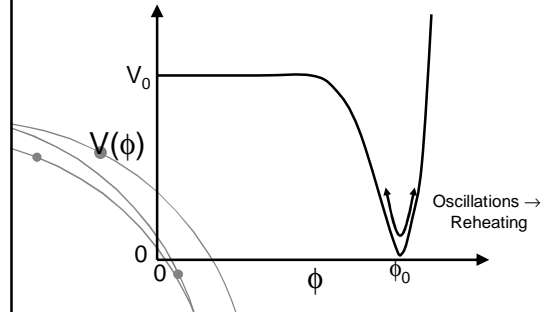
Consider a scalar field $\phi(r,t)$ with potential energy $V(\phi)$



The inflaton field II



The inflaton field III



Slow-roll

$$\epsilon_\phi = \frac{1}{2} \frac{1}{hc^3} \dot{\phi}^2 + V(\phi)$$

$$P_\phi = \frac{1}{2} \frac{1}{hc^3} \dot{\phi}^2 - V(\phi)$$

Slow roll:

$$\dot{\phi}^2 \ll hc^3 V(\phi) \Rightarrow$$

$$\epsilon_\phi \approx -P_\phi \approx V(\phi) \quad \begin{array}{l} \text{Negative pressure!} \\ \Lambda\text{-like expansion!} \\ \text{de Sitter phase!} \end{array}$$

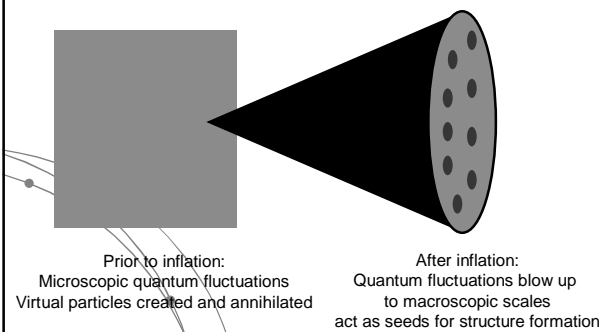
Reheating

If the Universe expands by a factor of $\sim e^{100} \rightarrow$
 Temperature drops by e^{-100}
 and the radiation energy density gets extremely small

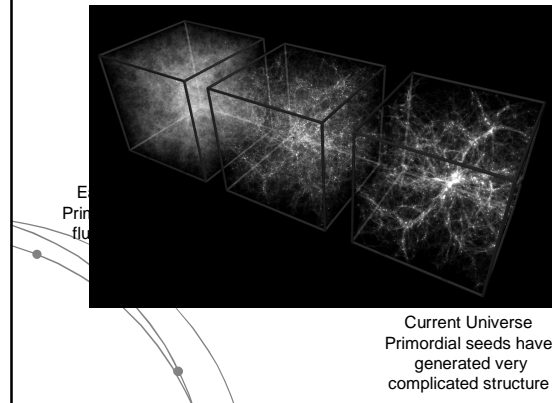
How come it's not small after inflation then?

Oscillations of ϕ around $\phi_0 \rightarrow$
 Some of the energy of the inflaton field are
 being carried away by radiation
 These photons *reheat* the Universe
 Hence, no shortage of photons after inflation!

Seeds for structure formation



Seeds for structure formation II



Inflation as a solution to the flatness problem I

The acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)$$

During inflation, the Universe is temporarily dominated by a component with $P < -\epsilon/3$ (i.e. $w < -1/3$), giving positive acceleration. One often assumes a *cosmological constant* $\Lambda_{\text{inflation}}$ to be responsible.

Note: This is a constant very different from the Λ driving the cosmic acceleration today.
 $\Lambda_{\text{inflation}} \sim 10^{107} \Lambda \dots$

Inflation as a solution to the flatness problem II

Hubble parameter and scale factor during inflation:

$$H_{\text{inflation}} = \left(\frac{\Lambda_{\text{inflation}}}{3} \right)^{1/2}$$

$$a(t) \propto e^{H_{\text{inflation}} t}$$

Number of e-foldings during inflation:

$$N = H_{\text{inflation}} (t_{\text{stop}} - t_{\text{start}})$$

$$N \sim 100$$

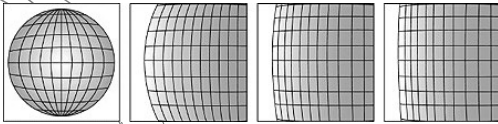
Inflation as a solution to the flatness problem III

$$|1 - \Omega(t_{\text{stop}})| = e^{-2N} |1 - \Omega(t_{\text{start}})|$$

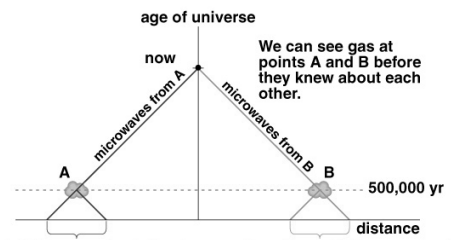
Example:

$$|1 - \Omega(t_{\text{start}})| \approx 1 \Rightarrow |1 - \Omega(t_{\text{stop}})| \approx 0$$

Inflation makes a curved Universe flat!



Inflation as a solution to the horizon problem I



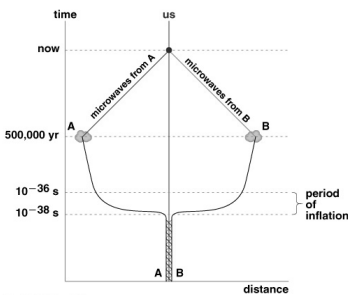
Gas at point A has received signals from this part of the universe.

Gas at point B has received signals from this part of the universe.

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The horizon problem

Inflation as a solution to the horizon problem II



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The solution

Inflation as a solution to the horizon problem III

Horizon before and after inflation:

$$d_{\text{hor}}(t_2) = c \int_{t_1}^{t_2} \frac{dt}{a(t)}$$

Before inflation:

$$d_{\text{hor}} = 2ct_{\text{start}} \sim 6 \times 10^{-28} \text{ m}$$

After inflation:

$$d_{\text{hor}} \approx e^N 3ct_{\text{start}} \sim 2 \times 10^{16} \text{ m}$$

Inflation as a solution to the horizon problem IV

Without inflation

Regions in causal contact
~20000 patches in the CMBR sky

With inflation

Just one patch in the sky!

Inflation as a solution to the magnetic monopole problem I

Expansion dilutes the number densities of objects, and inflation did this extremely efficiently

Inflation as a solution to the magnetic monopole problem II

At the end of inflation :

$$n_{\text{monopoles}}(t_{\text{stop}}) \sim e^{-300} n_{\text{monopoles}}(t_{\text{GUT}})$$

A realistic number density of monopoles at the GUT epoch would correspond to less than one monopole within the volume spanned by the last scattering surface

Eternal inflation I

Once the inflaton field has come to rest at ϕ_0 , inflation ends. But in some regions of space quantum fluctuations can make the inflation field move up the potential again \rightarrow Some regions keep inflating

Eternal inflation II

Eternal inflation III

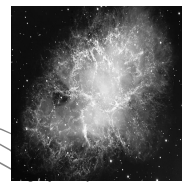
Differently inflated regions may end up with very different properties ('mutations') \rightarrow Multiverse
Good 'genes' may promote further expansion (self-reproducing Universes).

Eternal inflation IV

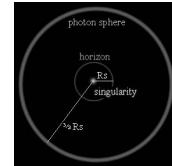
- Quantum fluctuations in $\phi \rightarrow$
Future-eternal inflation
Inflation will always continue (somewhere)
- Past-eternal inflation models also exist:
Revives the perfect cosmological principle!
The interior of each inflating bubble may be described by the Big Bang theory, but the multiverse as a whole has been around forever

Stellar Black Holes

Initial mass: $30 < M/M_{\text{solar}}$ Last stages: SN \rightarrow BH Remnant mass: $\sim 3\text{--}20 M_{\text{solar}}$



Supernova



Stellar Black Hole

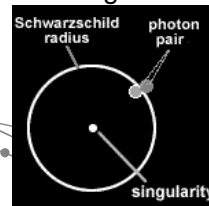
Since the progenitors of stellar black holes are baryonic, these black holes cannot contribute much to the matter density of the Universe

Primordial Black Holes

- High-density regions in the early Universe ($t \ll 1$ s) may collapse into primordial black holes
- PBHs could in principle form with masses from $M_{\text{Planck}} - 10^{15} M_{\text{solar}}$
- Remains a viable candidate for the cold dark matter: Ω_{PBH} could be $\sim 0.3!$
- Example:
 - $M_{\text{PBH}} \sim 10^{-8} M_{\text{solar}}$ (mass of the Moon) would have a size (event horizon) of $R \sim 0.1$ mm

Primordial Black Holes II

Hawking radiation:



- Observational constraints:
 - BBNS abundances
 - Gamma-ray background
 - CMBR

$$\tau_{\text{exp}} \sim 9 \cdot 10^{-18} M^3 \text{ s}$$

$$T_{\text{rad}} = \frac{hc^3}{16 \pi^2 kGM}$$

Objects with $M > 5 \times 10^{11}$ kg would still be around!

Unclear what happens at M_{Planck} . Relics may form!