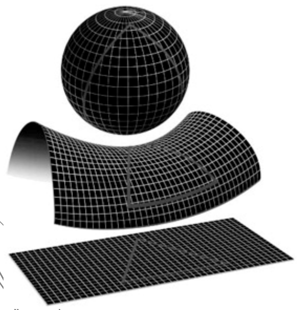
# Cosmology AS7009, 2010
# Lecture 2



## Outline

- The cosmological principle:
  - Isotropy
  - Homogeneity
- Big Bang vs. Steady State cosmology
- Redshift and Hubble's law
- Scale factor, Hubble time, Horizon distance
- Olbers' paradox: Why is the sky dark at night?
- Particles and forces
- Theories of gravity: Einstein vs. Newton
- Cosmic curvature

Covers chapter 2 + half of chapter 3 in Ryden

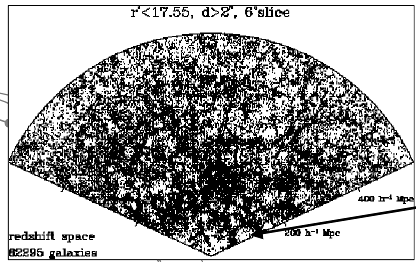
## The Cosmological Principle I

- Modern cosmology is based on the assumption that the Universe is:
  - Homogeneous
  - Isotropic

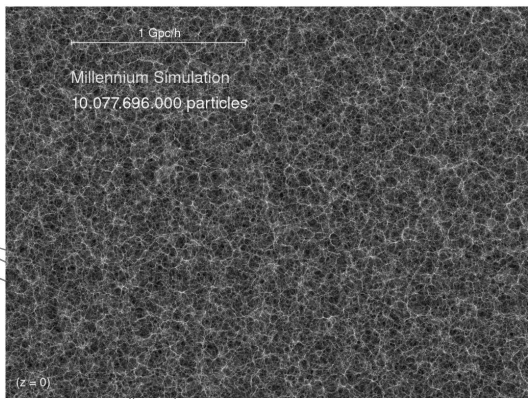
$\left.\begin{array}{l}\text{Homogeneous}\\ \text{Isotropic}\end{array}\right\}$ The cosmological principle

## The Cosmological Principle II

- These tenets *seem* to hold on large scales (>100 Mpc), but definitely not on small




*Voids* typically 70 Mpc across



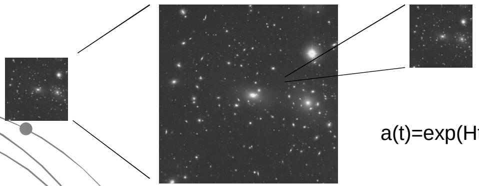

## The Perfect Cosmological Principle

- In this case, one assumes that the Universe on large scales is:
  - Homogeneous
  - Isotropic
  - *Non-evolving*

This is incompatible with the Big Bang scenario, but the *Steady State model* (popular in the 1940-1960s) was based on this idea

## Steady State Cosmology

Universe continuously expands, but due to continuous creation of matter, no dilution occurs → Steady State, no hot initial Big Bang and no initial singularity…



$a(t)=\exp(Ht)$

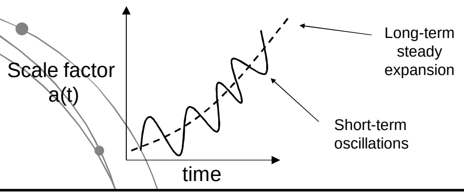
… But this cosmology fails to explain the CMBR, $T_{CMBR}(z)$, the production of light elements and the redshift evolution of galaxies & AGN

## Suggestion for Literature Exercise: Quasi-Steady State Cosmology

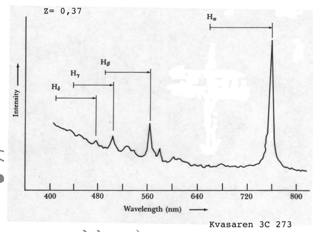
- Attempt in 1990s to resurrect Steady-State and explain the:
  - CMBR
  - production of light elements
  - dark matter
  - supernova type Ia data
  - Large-scale structure

## Suggestion for Literature Exercise: Quasi-Steady State Cosmology

- Cyclic creation events → Long-term steady expansion, but with short-term oscillations
- To meet observational constraints, QSS requires:
  - Strange intergalactic dust
  - Cyclic creation events ("little bangs") which cause local expansion of space



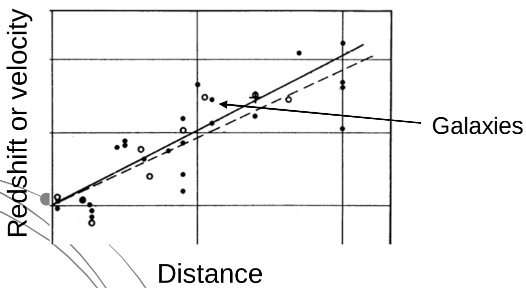

## Redshift




Definition of redshift:

$$\boxed{z=\frac{\lambda_{obs}-\lambda_{em}}{\lambda_{em}}}$$

## Hubble's law I




## Hubble's law II

Hubble's law:

The Hubble "constant"

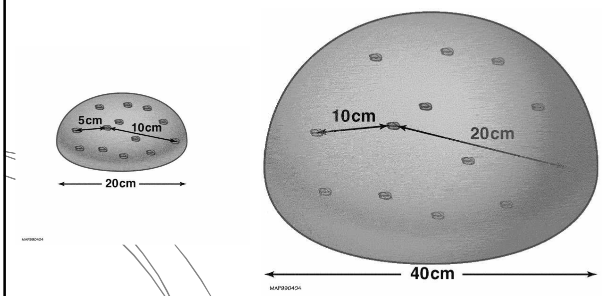
Luminosity distance

$$\boxed{z=\frac{H_0 d}{c}}$$

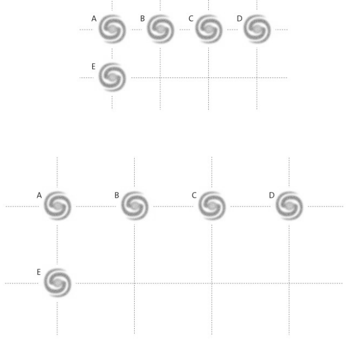
In observational astronomy, the term recession velocity, v, occurs frequently:

At low z: $\boxed{z\approx\frac{v}{c}}$ $\rightarrow$ $\boxed{v=H_0 d}$

## Expansion of the Universe I




## Expansion of the Universe II



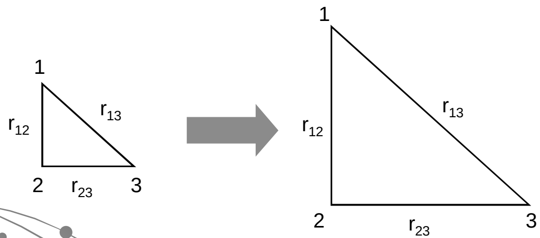

## Redshift and distance I

- Low redshift (z≈0) corresponds to:
  - Small distance (local Universe)
  - Present epoch in the history of the Universe
- High redshift corresponds to:
  - Large distance
  - Earlier epoch in the history of the Universe

## Redshift and distance II

- But beware:
  - At low redshift, Doppler components coming from peculiar motions may be substantial – must be corrected for before *d* is derived from *z* or *v*
  - The redshift coming from cosmic expansion is *not* a Doppler shift – don' t treat it like one!
  - The linear version of Hubble's law is only appropriate at z<0.15 (at 10% accuracy)

## Scale factor




Time t (earlier)
Scale factor a(t)

Time $t_0$ (now)
Scale factor $a(t_0)=1$

$$r_{12}(t) = a(t)/a(t_0)\, r_{12}(t_0) = a(t)\, r_{12}(t_0)$$
$$v_{12}(t) = a'/a\, r_{12}(t)$$

## Scale factor and redshift

Cosmic scale factor today (at $t_0$) — can be set to $a_0=1$

$$\boxed{1+z=\frac{a_0}{a}=\frac{1}{a}}$$

Cosmic scale factor when the Light was emitted (the epoch corresponding to the redshift z)

## The Hubble "constant"

$H_0 \approx 72 \pm 5$ km s$^{-1}$ Mpc$^{-1}$ [s$^{-1}$]

Today

Errorbars possibly underestimated…

Note: Sloppy astronomers often write km/s/Mpc…

In general:

$$H \equiv \frac{\dot{a}}{a}$$

Not a constant in our Universe!

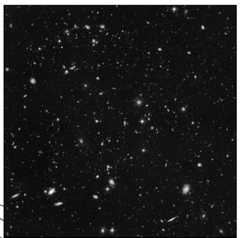
## Hubble time

In the case of constant expansion rate, the Hubble time gives the age of the Universe:

$$t_{\mathrm{H}} = \frac{1}{H_0} \approx 14 \text{ Gyr}$$

In more realistic scenarios, the expansion rate changes over time, but the currently favoured age of the Universe is still pretty close – around 13—14 Gyr.

## Olbers' paradox I



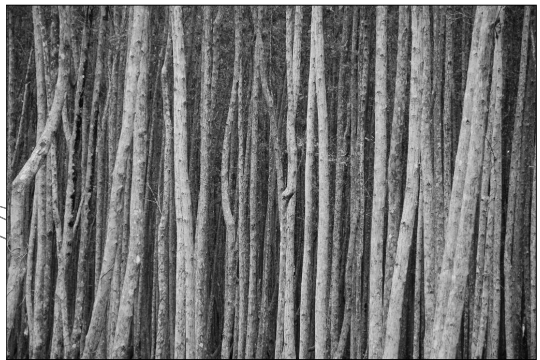
*"Why is the sky dark at night?"*
(Heinrich Olbers 1926)

If the Universe is:

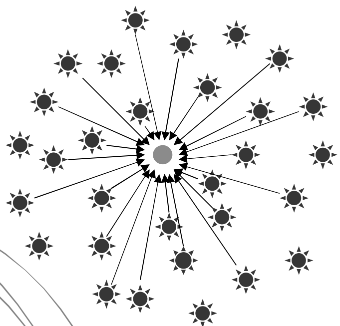
- Spatially infinite (i.e. infinite volume)
- Infinitely old and unevolving

- then the night sky should be bright!

## Olbers' paradox II

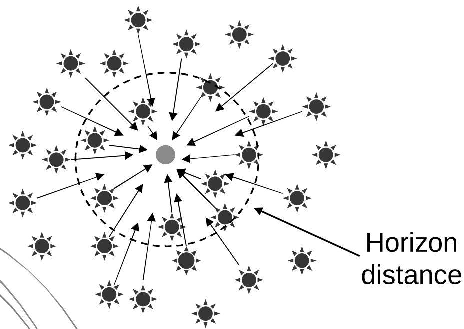


## Olbers' paradox III



Planet Earth surrounded by stars
in an infinte, unevolving Universe

## Olbers' paradox IV




Main solution: The Universe has finite age
The light from most stars have not had time to reach us!

## Horizon distance

- Horizon distance = Current distance to the most faraway region from which light has had time to reach us
- This delimits the causally connected part of the Universe an observer can see at any given time
- Horizon distance at time $t_1$:

$$\boxed{d_{\mathrm{hor}}(t_1) = c\int_{t=0}^{t_1} \frac{\mathrm{d}t}{a(t)}}$$

- Most realistic scenarios give:
  $d_{hor}(t_0)\sim c/H_0$ (the so-called Hubble radius)

## Particles and forces I

- The particles that make up the matter we encounter in everyday life:
  - Protons, p
    938.3 MeV
  - Neutrons, n
    939.5 MeV
  
  } Baryons (made of 3 quarks)
  - Electrons, $e^-$
    0.511 MeV
  
  } Lepton

Since most of the mass of 'ordinary matter' is contributed by protons and neutrons, such matter is often referred to as *baryonic*. Examples of mostly baryonic objects: Planets, stars, gas clouds (but not galaxies or galaxy clusters)

## Particles and forces II

- Other important particles (for this course):
  - Photon, $\gamma$
    Massless, velocity: c
  - Neutrinos, $\nu_e$ $\nu_\mu$ $\nu_\tau$
    ~eV (?), velocity close to c
    
    } Leptons
    
    Interacts via weak nuclear force only

## Particles and forces III

- The four forces of Nature:
  - Strong force
    - Very strong, but has short range (~$10^{-15}$ m)
    - Holds atomic nuclei together
  - Weak force
    - Weak and has short range
    - Responsible for radioactice decay and neutrino interactions
  - Electromagnetic force
    - Weak but long-range
    - Acts on matter carrying electric charge
  - Gravity
    - Weak, very long-range and always attratice

On the large scales involved in cosmology, gravity is by far the dominant one

## Newtonian gravity

- Space is Euclidian (i.e. flat)
- Planet are kept in their orbits because of the gravitational force:

$$\boxed{F = -\frac{GM_g m_g}{r^2}}$$

Gravitational mass

- The acceleration resulting from the gravitational force:

$$\boxed{F = m_i a}$$

Inertial mass

## Equivalence Principle

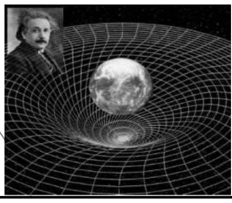
- Gravitational acceleration towards an object with mass $M_g$ is:

$$\boxed{a = -\frac{GM_g}{r^2}\left(\frac{m_g}{m_i}\right)}$$

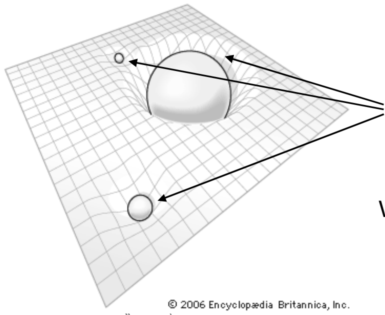
- Empirically $M_g$=$M_i$ (to very high precision)
- The equality of gravitational mass and inertial mass is called the equivalence principle
- In Newtonian gravity, $M_g$=$M_i$ is just a strange coincidence, but in General Relativity, this stems from the idea that masses cause curvature of space

# General Relativity

- 4D space-time
- Mass/energy curves space-time
- Gravity = curvature
- Pocket summary:
  - Mass/energy tells space-time how to curve
  - Curved space-time tells mass/energy how to move

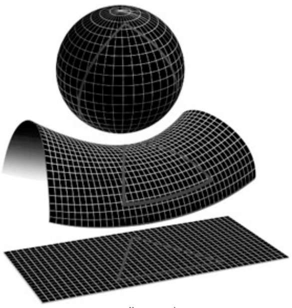


# Small-scale curvature




# Global Curvature I

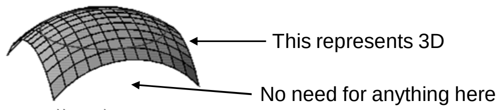
In the world models of general relativity, our Universe may have spatial curvature (on global scales)




# Global Curvature II

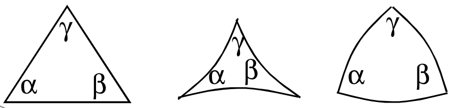
## Very tricky stuff…

- This an *intrinsic* curvature in 3D space
- Note: No need for encapsulating our 3D space in 4D space to make this work




# Global Curvature III

Angles in curved spaces




Flat: $\alpha+\beta+\gamma = 180°$

Negative: $\alpha+\beta+\gamma < 180°$

Positive: $\alpha+\beta+\gamma > 180°$

# Metrics I

- Metric: A description of the distance between two points
- Metric in 2 dimensional, flat space:

$$\boxed{ds^2 = dx^2 + dy^2}$$ (Pythagoras)

- Metric in 3 dimensional, flat space:

$$\boxed{ds^2 = dx^2 + dy^2 + dz^2}$$

# Metrics II

- Metric in 3 dimesions, flat space, polar coordinates:

$$ds^2 = dr^2 + r^2 d\Omega^2$$
$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

- Metric in 3 dimensions, arbitrary curvature:

$$ds^2 = \frac{dx^2}{1 - \kappa x^2 / R^2} + x^2 d\Omega^2$$

Curvature radius

$$\text{Flat}: \quad \kappa = 0,\ x = r$$
$$\text{Negative}: \quad \kappa = -1,\ x = R\sinh(r/R)$$
$$\text{Positive}: \quad \kappa = 1,\ x = R\sin(r/R)$$