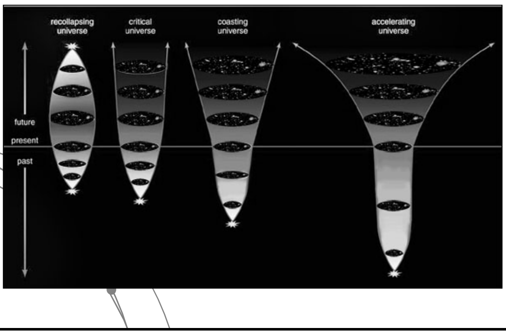
Cosmology AS7009, 2010
Lecture 4

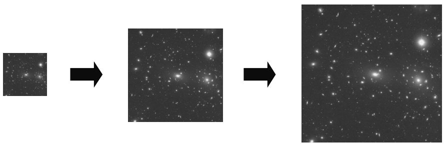


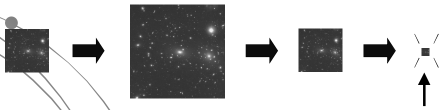
Outline

- Properties and fate of single-component Universes
  - Empty Universe
  - Flat Universe
  - Matter/Radiation/Lambda-dominated Universe
  - Einstein-de Sitter Universe
- Properties and fate of multiple-component Universes
  - Matter + curvature
  - Benchmark model
  - Other dark energy scenarios

Covers chapters 5 & 6 in Ryden

What the dynamics of the Universe can tell you


Past
Now
Future


Past
Now
Future
The Big Crunch

The fate of the Universe

Density evolution (general)

- Friedmann equation + fluid equation + equation of state → a(t), ε(t) or t(a), ε(a)
- Problem: There are many components in the Universe →
  - Evolution complicated
  - Different components dominate evolution at different times

---

Multiple energy components

$$\varepsilon_{\text{tot}} = \sum_w \varepsilon_w$$

Often, one just writes ε and Ω

$$\Omega_{\text{tot}} = \sum_w \frac{\varepsilon_w}{\varepsilon_{\text{c}}}$$

Remember:

$w = 0$ (non – relativistic matter)

$w = 1/3$ (radiation and relativistic matter)

$w = -1$ (cosmological constant)

$w < -1/3$ (dark energy) → Positive acceleration

Density evolution

Fluid equation + eq. of state (see exercise session) →

$$\varepsilon_{\text{w}}(a) = \varepsilon_{w,0} a^{-3(1+w)}$$

$$\begin{cases} w_m = 0 \quad \Rightarrow \quad \varepsilon_{\text{m}}(a) = \varepsilon_{m,0} a^{-3} \\ w_r = 1/3 \quad \Rightarrow \quad \varepsilon_{\text{r}}(a) = \varepsilon_{r,0} a^{-4} \\ w_\Lambda = -1 \quad \Rightarrow \quad \varepsilon_\Lambda(a) = \varepsilon_{\Lambda,0} \end{cases}$$

The benchmark model

The currently favoured cosmological model:

$\Omega_M \approx 0.3$

$\Omega_M = \Omega_{\text{Non-baryons (CDM)}} + \Omega_{\text{Baryons}}$
$\Omega_{\text{Non-baryons (CDM)}} \approx 0.26$
$\Omega_{\text{Baryons}} \approx 0.04$

More on this in the dark matter lecture

$\Omega_\Lambda \approx 0.7$

$\Omega_R \approx 8.4 \times 10^{-5}$

$\Omega_R = \Omega_{\text{CMB}} + \Omega_\nu + \Omega_{\text{starlight}}$
$\Omega_{\text{CMB}} \approx 5.0 \times 10^{-5}$
$\Omega_\nu \approx 3.4 \times 10^{-5}$
$\Omega_{\text{starlight}} \approx 1.5 \times 10^{-6}$

$\Omega_{\text{tot}} = \Omega_M + \Omega_\Lambda + \Omega_R \approx 1.0$
→ Flat Universe ($\kappa = 0$)

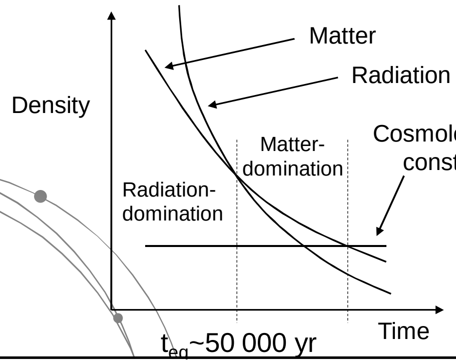
Why are single-component Universes relevant then?

$$w_m = 0 \quad \Rightarrow \quad \varepsilon_m(a) = \varepsilon_{m,0} a^{-3}$$
$$w_r = 1/3 \quad \Rightarrow \quad \varepsilon_r(a) = \varepsilon_{r,0} a^{-4}$$
$$w_\Lambda = -1 \quad \Rightarrow \quad \varepsilon_\Lambda(a) = \varepsilon_{\Lambda,0}$$

Values of Ω_M, Ω_Λ, Ω_R today + these relations →
$\varepsilon_M \geq \varepsilon_\Lambda$ at some point in the past
(matter-dominated Universe)
$\varepsilon_R \geq \varepsilon_M$ even further back
(radiation-dominated Universe)

The different epochs of the Universe can be *approximated* by single-component evolution

Different components dominate the sum at different times



The Milne Universe (empty)

Of pretty limited relevance for our Universe, but provides simple demonstration of how to derive current age of Universe, a(t) and t(z) from the Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$
$$\varepsilon(t) = 0 \quad \Rightarrow$$
$$H(t)^2 = -\frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2} \Rightarrow$$
$$\dot{a}(t)^2 = -\frac{\kappa c^2}{R_0^2}$$

The Milne Universe (empty) II

$$\dot{a}(t)^2 = -\frac{\kappa c^2}{R_0^2}$$

Empty, static Universe $\quad \kappa = 0 \quad \Rightarrow \quad \dot{a}(t) = 0$

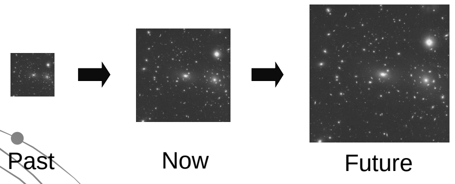
Empty, negatively curved $\quad \kappa = -1 \Rightarrow$

$$\dot{a}(t) = \frac{\mathrm{d}a}{\mathrm{d}t} = \pm\frac{c}{R_0}$$

If expanding (not contracting) $\Rightarrow$

Current age of Universe: $\quad t_0 = \frac{R_0}{c}, \quad a(t) = \frac{t}{t_0}, \quad t(z) = \frac{t_0}{1+z}$

Fate of the Universe in a Milne Universe



Eternal, constant expansion ('Big Chill')

Proper distance in a Milne Universe

Proper distance from us to object which emitted light at t_e :

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$a(t) = \frac{t}{t_0} \quad \Rightarrow d_p(t_0) = ct_0 \int_{t_e}^{t_0} \frac{dt}{t} = ct_0 \ln\left(\frac{t_0}{t_e}\right)$$

$$t_e = \frac{t_0}{1+z} \Rightarrow d_p(t_0) = ct_0 \ln(1+z) = \frac{c}{H_0} \ln(1+z)$$

Flat, single-component Universes

For w ≠ -1:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/(3+3w)}$$

$$\varepsilon(t) = \varepsilon_0 \left(\frac{t}{t_0}\right)^{-2}$$

Important types of flat, single-component Universes

Radiation-only Universe (radiation-dominated epoch)

$$t_0 = \frac{1}{2H_0}, \quad a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

Matter-only Universe (matter-dominated epoch)

$$t_0 = \frac{2}{3H_0}, \quad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Λ-only Universe I

Λ has w = -1 $\Rightarrow$

$$\dot{a}^2 = \frac{8\pi G \varepsilon_\Lambda}{3c^2} a^2$$

Rearrange:

$$\dot{a}^2 = H_0^{\,2} a^2 \quad \text{if}$$

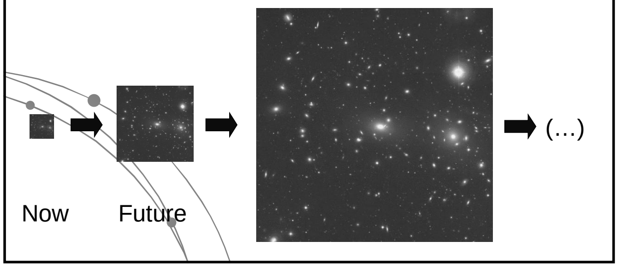
$$H_0 = \left(\frac{8\pi G \varepsilon_\Lambda}{3c^2}\right)^{1/2}$$

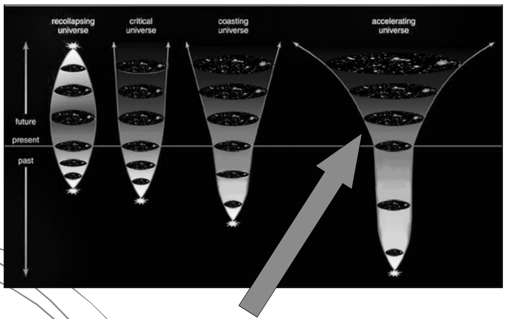
Λ-only Universe II

Solution:

$$a(t) = e^{H_0(t-t_0)}$$

de Sitter Universe (de Sitter phase)
Same growth as in Steady state cosmology





de Sitter Universe/phase
(after 'present')

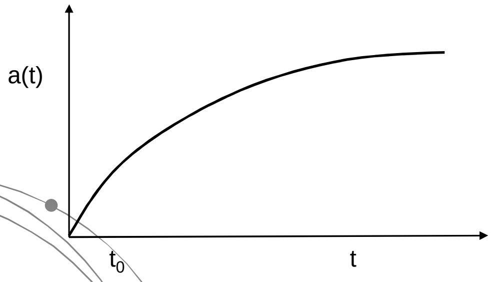
Einstein-de Sitter Universe I

This served as the benchmark model up until the mid-1990s

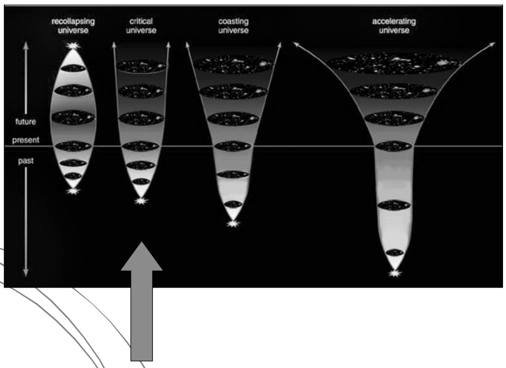
- Flat (i.e. critical-density), matter-dominated Universe
- $\Omega_M = 1.0$, $\Omega_{tot} = 1.0$, $\kappa = 0$

$$t_0 = \frac{2}{3H_0}, \quad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Einstein-de Sitter Universe II

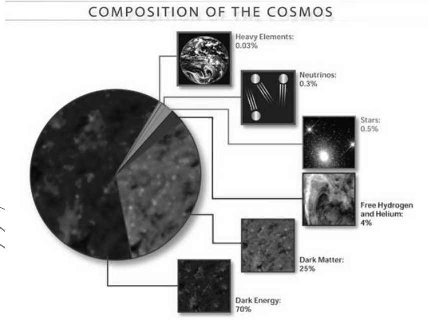


Asymptotically approaches zero expansion, but no recollapse



Einstein-de Sitter Universe

Multiple-component Universes I



Note: Valid for current time only!

Multiple-component Universes II

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

Recall:

$$\varepsilon(t) = \varepsilon_{\mathrm{M}}(t) + \varepsilon_{\mathrm{R}}(t) + \varepsilon_{\Lambda}(t) + \ldots$$

Multiple-component Universes III

For 3 components with arbitrary curvature, the FE may be rewritten:

$$\frac{H(t)^2}{H_0^2} = \underbrace{\frac{\Omega_{R,0}}{a(t)^4} + \frac{\Omega_{M,0}}{a(t)^3} + \Omega_{\Lambda,0}}_{\text{The 3 components}} + \underbrace{\frac{1-\Omega_{\mathrm{tot},0}}{a(t)^2}}_{\text{Curvature}}$$

More components can easily be added as additional terms, as long as you know their a(t)-dependence

Multiple-component Universes IV

Time (from Big Bang):

$$t(a) = \frac{1}{H_0}\int_0^a \frac{\mathrm{d}a}{\left[\Omega_{R,0}a^{-2} + \Omega_{M,0}a^{-1} + \Omega_{\Lambda,0}a + (1-\Omega_{\mathrm{tot},0})\right]^{1/2}}$$

Lookback-time = t_0-t(a)

Nasty integral! No analytical solution in general case – must be integrated numerically!

Special case: Matter + Curvature I

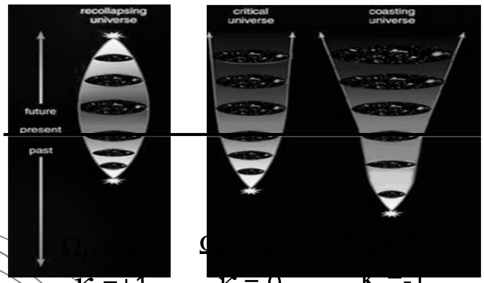
$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_{\mathrm{M},0}}{a(t)^3} + \frac{1-\Omega_{\mathrm{M},0}}{a(t)^2}$$

H(t)-evolution → Possible fates of Universe

This is one of the hand-in exercises!

(Note: Lots of help on page 85 in Ryden)

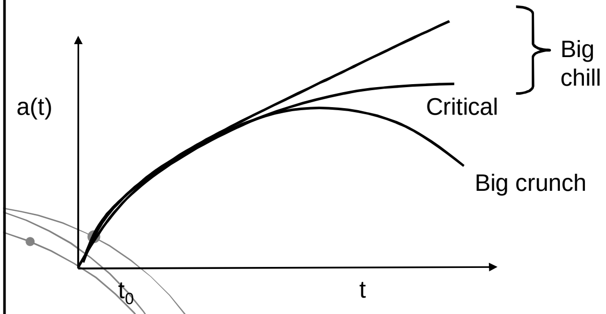
Special case: Matter + Curvature II



$\Omega_M > 1.0$	$\Omega_M = 1.0$	$\Omega_M < 1.0$
$\kappa = +1$	$\kappa = 0$	$\kappa = -1$
'Big Crunch'	'Big Chill'	'Big Chill'
Finite	Infinite*	Infinite*

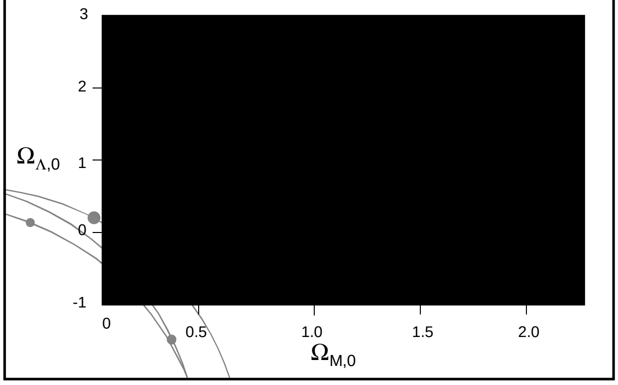
* Note: In the case of a simple topology

Special case: Matter + Curvature III



Up until the mid-90s, these were the only 3 possible fates of the Universe typically quoted in textbooks

Matter + curvature + Λ

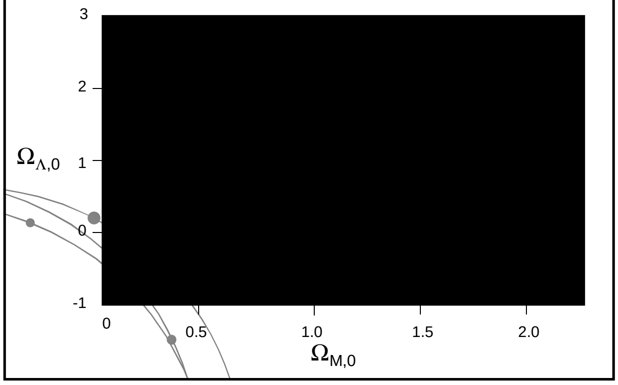


Properties of the benchmark model

- Matter-radiation equality:
  - $a \approx 2.8\times10^{-4}$
  - $t \approx 4.7\times10^4$ yr
- Matter-Λ equality:
  - $a \approx 0.75$
  - $t \approx 9.8$ Gyr
- Now:
  - a = 1
  - $t \approx 13.7$ Gyr

Hand-in exercise!

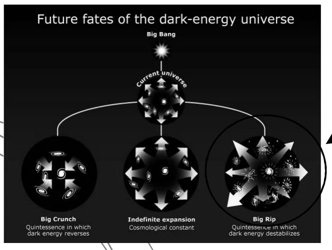
Properties of the benchmark model II



Other Dark Energy Models

What if dark energy is something other than Λ?

- Varying equation of state $w(t)$?
- Constant $w \neq -1$?



More on this in exercise session

Our ignorance about the nature of dark energy →
Ultimate fate of our Universe unclear