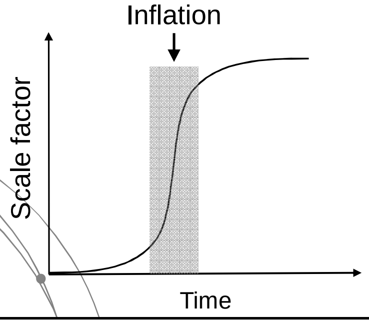
# Cosmology AS7009, 2010
# Lecture 9

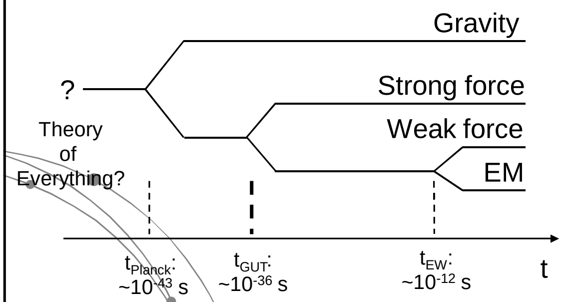



## Outline

- Grand Unified Theories and phase transitions
- Problems with a non-inflationary Big Bang
- Inflation
  - Inflaton field
  - Slow-roll
  - Reheating
  - Seeds for structure formation
- Inflation as a solution to the flatness, horizon and magnetic monopole problems
- Eternal inflation
- Primordial black holes

Covers chapter 11 in Ryden + extra stuff

## Grand Unification



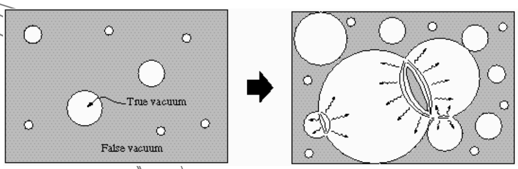

## Grand Unification II

- Electroweak unification experimentally confirmed in late 1970s → Nobel prize in physics to Maxwell, Weinberg, Salam & Glashow for electroweak theory
- GUT happens at $E_{GUT}$~$10^{12}$ TeV
- LHC reaches ~ 10 TeV → Experimental confirmation of GUT is not gonna happen soon…

## Phase Transitions

The Universe underwent a phase transition when the temperature dropped below $T_{GUT}$~$10^{28}$ K
Symmetry between strong and electroweak force lost → Topological defects!

Many GUT scenarios produce magnetic monopoles with $mc^2$~$10^{12}$ TeV




## Why do we need inflation?

- To solve:
  - Flatness problem
  - Horizon problem
  - Magnetic monopole problem
- To seed structure formation

# The flatness problem I

Observationally:

$|1-\Omega_0| \le 0.1$

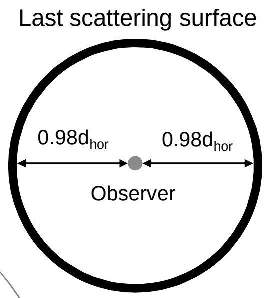
One can show that this implies, at the Planck time:

$|1-\Omega_{\text{Planck}}| \le 10^{-60}$

Hence, if the Universe is close to flat now, it was extremely close to flat in the past.

Why is the Universe so close to flat?
If this is a coincidence, it very, very improbable!

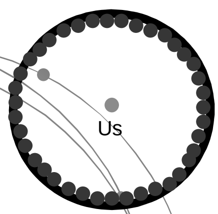
# The horizon problem I




How can the CMBR be almost completely isotropic, when opposite sides of the sky have never been in casual contact?

# The horizon problem II

$$\theta_{\text{hor}} = \frac{d_{\text{hor}}(t_{\text{lss}})}{d_A} \approx \frac{0.4\text{Mpc}}{13\text{Mpc}} \approx 2 \text{ degrees}$$




● Regions in causal contact
~20000 patches in the CMBR sky

# The magnetic monopole problem I

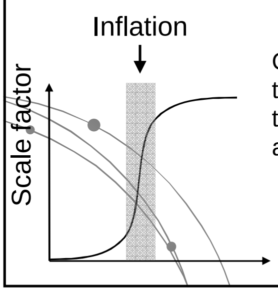
Magnetic monopoles: zero-dimensional objects which act as isolated north or south poles of a magnet

Many GUT models predict huge numbers of these! While subdominant at creation, they would soon come to dominate the energy density of the Universe

Problem: No such objects have ever been observed! Where are the magnetic monopoles?

# Inflation

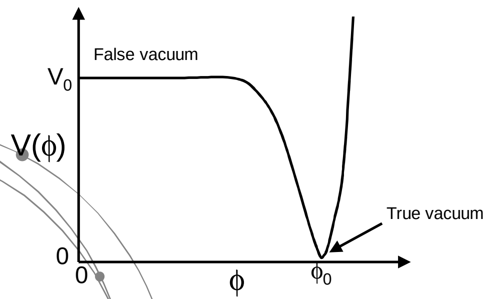
What is inflation?
A short period of fast expansion, happening very early in the history of the Universe



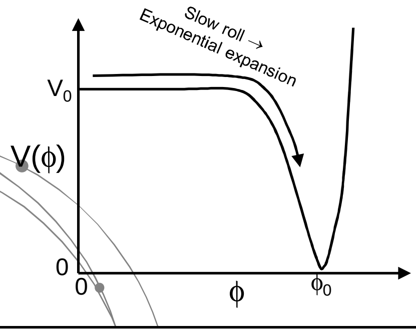

One possible model:
$t_{start}$ ~ $10^{-36}$ s after the Big Bang
$t_{stop}$ ~ $10^{-34}$ s after the Big Bang
$a(t_{stop})/a(t_{start})$ ~ $e^{100}$ ~$10^{43}$

# The inflaton field I

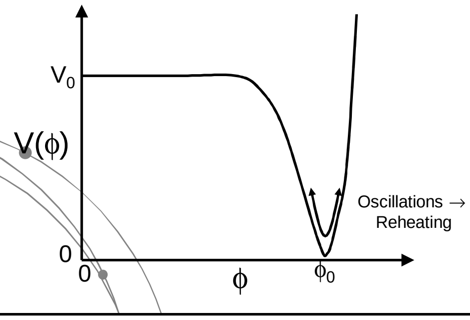
Consider a scalar field $\phi(r,t)$ with potential energy $V(\phi)$



## The inflaton field II




## The inflaton field III




## Slow-roll

$$\varepsilon_\phi = \frac{1}{2}\frac{1}{hc^3}\dot{\phi}^2 + V(\phi)$$

$$P_\phi = \frac{1}{2}\frac{1}{hc^3}\dot{\phi}^2 - V(\phi)$$

Slow roll:

$$\dot{\phi}^2 << hc^3 V(\phi) \Rightarrow$$

$$\varepsilon_\phi \approx -P_\phi \approx V(\phi)$$

Negative pressure!
Λ-like expansion!
de Sitter phase!

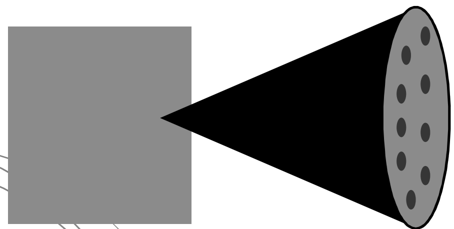
## Reheating

If the Universe expands by a factor of $\sim e^{100}$ →
Temperature drops by $e^{-100}$
and the radiation energy denstiy gets extremely small

*How come it's not small after inflation then?*

Oscillations of ϕ around $\phi_0$ →
Some of the energy of the inflaton field are
being carried away by radiation
These photons *reheat* the Universe
Hence, no shortage of photons after inflation!

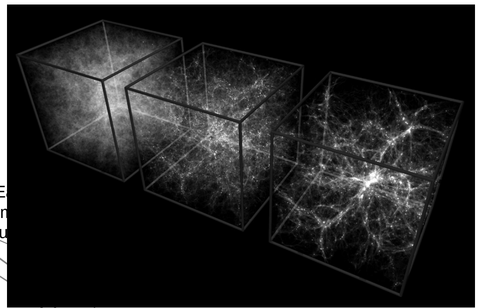
## Seeds for structure formation



Prior to inflation:
Microscopic quantum fluctuations
Virtual particles created and annihilated

After inflation:
Quantum fluctuations blow up
to macroscopic scales
act as seeds for structure formation

## Seeds for structure formation II



Current Universe
Primordial seeds have
generated very
complicated structure

## Inflation as a solution to the flatness problem I

The acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$$

During inflation, the Universe is temporarily dominated by a component with P < -ε/3 (i.e. w<-1/3), giving positive acceleration.
One often assumes a *cosmological constant* $\Lambda_{\text{inflation}}$ to be responsible.
Note: This is a constant very different from the $\Lambda$ driving the cosmic acceleration today.
$\Lambda_{\text{inflation}} \sim 10^{107}\,\Lambda\ldots$

## Inflation as a solution to the flatness problem II

Hubble parameter and scale factor during inflation:

$$H_{\text{inflation}} = \left(\frac{\Lambda_{\text{inflation}}}{3}\right)^{1/2}$$

$$a(t) \propto \mathrm{e}^{H_{\text{inflation}} t}$$

Number of e-foldings during inflation:

$$N = H_{\text{inflation}}(t_{\text{stop}} - t_{\text{start}})$$

$$N \sim 100$$

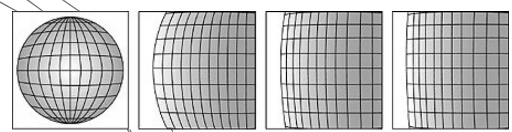
## Inflation as a solution to the flatness problem III

$$|1-\Omega(t_{\text{stop}})| = \mathrm{e}^{-2N}|1-\Omega(t_{\text{start}})|$$

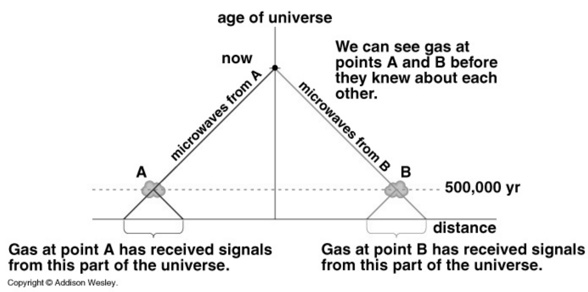
Example:

$$|1-\Omega(t_{\text{start}})| \approx 1 \Rightarrow |1-\Omega(t_{\text{stop}})| \approx 0$$

Inflation makes a curved Universe flat!

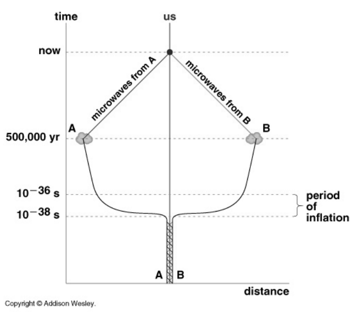


## Inflation as a solution to the horizon problem I




The horizon problem

## Inflation as a solution to the horizon problem II




The solution

## Inflation as a solution to the horizon problem III

Horizon before and after inflation:

$$d_{\text{hor}}(t_2) = c\int_{t_1}^{t_2}\frac{\mathrm{d}t}{a(t)}$$

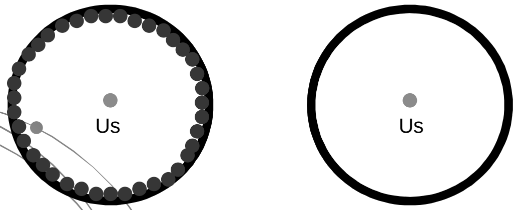
Before inflation:

$$d_{\text{hor}} = 2ct_{\text{start}} \sim 6\times10^{-28}\,\text{m}$$

After inflation:

$$d_{\text{hor}} \approx \mathrm{e}^{N}\,3ct_{\text{start}} \sim 2\times10^{16}\,\text{m}$$

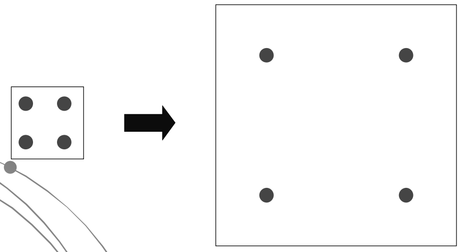
## Inflation as a solution to the horizon problem IV




Regions in causal contact
~20000 patches in the CMBR sky

Just one patch in the sky!

## Inflation as a solution to the magnetic monopole problem I



Expansion dilutes the number densities of objects, and inflation did this extremely efficiently

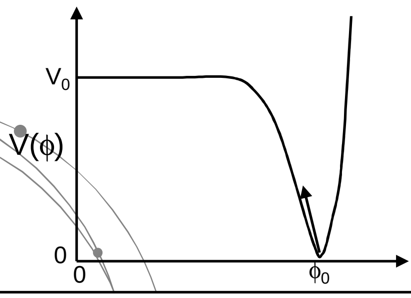
## Inflation as a solution to the magnetic monopole problem II

$$\text{At the end of inflation}:$$
$$n_{\text{monopoles}}(t_{\text{stop}}) \sim e^{-300} n_{\text{monopoles}}(t_{\text{GUT}})$$

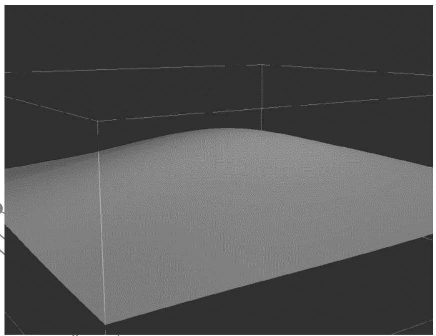
A realistic number density of monopoles at the GUT epoch would correspond to less than one monopole within the volume spanned by the last scattering surface

## Eternal inflation I

Once the inflaton field has come to rest at $\phi_0$, inflation ends. But in some regions of space equantum fluctuations can make the inflation field move up the potential again $\rightarrow$ Some regions keep inflating

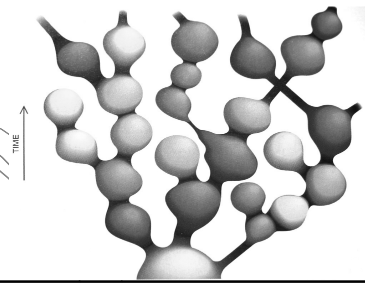



## Eternal inflation II



## Eternal inflation III

Differently inflated regions may end up with very different properties ('mutations') $\rightarrow$ Multiverse
Good 'genes' may promote further expansion (self-reproducing Universes).



## Eternal inflation IV

- Quantum fluctuations in $\phi \rightarrow$ *Future-eternal inflation*

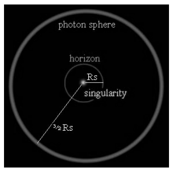
  Inflation will always continue (somewhere)
- Past-eternal inflation models also exist:

  *Revives the perfect cosmological principle!* The interior of each inflating bubble may be described by the Big Bang theory, but the multiverse as a whole has been around forever

## Stellar Black Holes

| Initial mass: | Last stages: | Remnant mass: |
|---|---|---|
| $30<M/M_{solar}$ | SN $\rightarrow$ BH | ~ 3—20 $M_{solar}$ |

→




Supernova

Stellar Black Hole

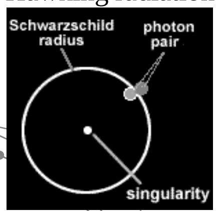
Since the progenitors of stellar black holes are baryonic, these black holes cannot contribute much to the matter density of the Universe

## Primordial Black Holes

- High-density regions in the early Universe (t « 1 s) may collapse into primordial black holes
- PBHs could in principle form with masses from $M_{Planck}$ - $10^{15}$ $M_{solar}$
- Remains a viable candidate for the cold dark matter: $\Omega_{PBH}$ could be ~0.3!
- Example:
  - $M_{PBH}$~$10^{-8}$ $M_{solar}$ (mass of the Moon) would have a size (event horizon) of R~0.1 mm

## Primordial Black Holes II

Hawking radiation:




$$\tau_{evap} \sim 9 \cdot 10^{-18} M^3 s$$

$$T_{rad} = \frac{hc^3}{16\pi^2 kGM}$$

Objects with M > $5\times10^{11}$ kg would still be around!

·Observational constraints:
- BBNS abundances
- Gamma-ray background
- CMBR

Unclear what happens at $M_{Planck}$. Relics may form!