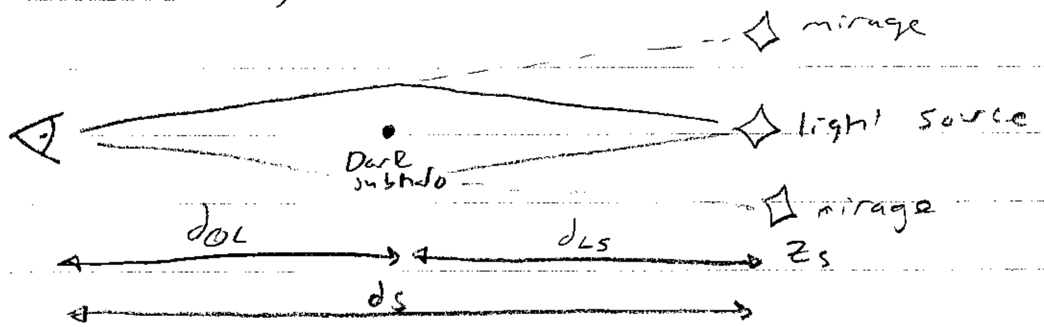
Problem set 3

[13] Gravitational lensing



Note: d_L, d_{LS}, d_S are angular size distances

& $d_{OL} + d_{LS} \neq d_S$ at cosmological distances.

Kayser et al. ($\eta = 1 \Rightarrow$ homogeneous (non-clumped) Universe) Flat Universe

$$D_{xy} = \frac{c}{H_0(1+z_y)} \int_{z_x}^{z_y} \frac{dz}{\sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}}$$

$$\theta_E = \left(\frac{4 G M_{SUB} D_{LS}}{c^2 D_{OL} D_{OS}} \right)^{1/2}$$

Numerically:

$z_{s} = 2.0$ (source redshift)

$M_{SUB} = 10^7 M_\odot = 1.99 \cdot 10^{37}$ kg

$H_0 = 72$ km/s/Mpc $\quad \frac{c}{H_0} = 4.15$ Gpc

a) $z_L = 0.5 \Rightarrow$

$D_{OS} = 1.67$ Gpc

$D_{OL} = 1.22$ Gpc

$D_{LS} = 1.06$ Gpc

$\Rightarrow \theta_E = 3.2263 \cdot 10^{-8}$ radians

arcsec $= \frac{1}{3600}°$ $\Rightarrow \theta_E = 6.7 \cdot 10^{-3}$ arcsec (millilensing)

b) $z_L = 1.0$

$D_{OS} = 1.67$ Gpc $\quad \theta_E = 4.4 \cdot 10^{-3}$ arcsec

$D_{OL} = 1.60$ Gpc

$D_{LS} = 0.610$ Gpc

c, $D_{OL} = 10^5$ pc (not angular size)

$D_{OS} = 1.67$ Gpc

$D_{LS} = D_{OS} - D_{OL}$

$\theta_E = 0.9''$

__But__ In case c, the point mass approximation for the lens breaks down!

Based on Ryden

$\boxed{14}$ Age, redshift & Temperature

Radiation temperature:

$$T(t) \propto \frac{1}{a(t)} = (1+z) \Rightarrow$$

$$T(t) = (1+z)\, T(t_0) \qquad (1, \sim p. 24 \text{ in Ryden})$$

(1) $\Rightarrow$

$$z_{recomb} = \frac{T_{recomb}}{T_{now}} - 1 \qquad (2)$$

To get age of the universe at z_{recomb} in EdS:

$$t_0 = \frac{2}{3H_0} \qquad (3, \ 5.57)$$

$$a(t) = \frac{1}{1+z} = \left(\frac{t_z}{t_0}\right)^{2/3} \qquad (4, \ 5.59)$$

(3) in (4) after rearranging:

$$t_z = \frac{2}{3H_0}(1+z)^{-3/2} \qquad (5)$$

<u>Numerically:</u>

$$\left.\begin{array}{l} T_{recomb} = 4000\,K \\ T_{now} = 2.73\,K \end{array}\right\} \Rightarrow (2) \Rightarrow z_{recomb} \approx 1460$$

$$H_0 = 70 \text{ km s}^{-1}\text{ Mpc}^{-1} \Rightarrow (5) \Rightarrow t_{z,recomb} = 1.7 \cdot 10^5 \text{ yr}$$

Based on Ryden

$\boxed{15}$ CMBR I

Energy density
$$\rho_{CMBR} \propto T^4 \Rightarrow \rho_{CMBR}(T) \doteq \rho_{CMBR,0} T^4 \quad (1,\ 9.1 \text{ in Ryden})$$

$$T(t)_{CMBR} \propto \frac{1}{a(t)} = 1+z \quad (2,\ p.24)$$

Energy per CMBR photon:
$$E_\gamma = h\nu \quad (3)$$

$$n_{CMBR} = \frac{\rho_{CMBR} \cdot c^2}{E_\gamma} \quad (4)$$

(3) in (4) $\Rightarrow$

$$n_{CMBR} = \frac{\rho_{CMBR} \cdot c^2}{h\nu_{CMBR}} \quad (5)$$

Expansion of space $\rightarrow$

$$\nu \propto \frac{1}{a(t)} = 1+z \Rightarrow \nu = \nu_0 (1+z) \quad (6)$$

(6) in (5) $\Rightarrow$

$$n_{CMBR}(z) = \frac{\rho_{CMBR}(z) \cdot c^2}{h\nu_{CMBR,0}(1+z)} \quad (7)$$

(1) and (2) in (7) $\Rightarrow$

$$\boxed{n_{CMBR}(z) = \frac{\rho_{CMBR,0}(1+z)^3 c^2}{h\nu_{CMBR,0}} = n_{CMBR,0}(1+z)^3} \quad (8)$$

Numerically:

$$n_{CMBR,0} = 4.11 \cdot 10^8 \, m^{-3} \quad (9.2 \text{ in Ryden})$$

$$z = 5 \Rightarrow n_{CMBR} = 8.88 \cdot 10^{10} \, m^{-3}$$

Ryden / Roos

16 Big Bang Nucleosynthesis

~All neutrons end up in ^4He

$$Y(^4He) \equiv \frac{4n_{He}}{n_{tot}} \approx \frac{4\left(\frac{n_n}{2}\right)}{n_n + n_p} = \frac{2n_n/n_p}{1 + \frac{n_n}{n_p}} \quad (1, p. 178 \text{ below } 10.21)$$

if all neutrons are assumed to end up in ^4He.

What is $\frac{n_n}{n_p}$ at the time of ^4He formation?

$$n_n = n_{n_0} e^{-t/\tau} \quad (2) \quad (\text{Ph. M. } 8.1)$$

(τ: mean lifetime of neutron)

$$n_p = n_{p_0} + \left(1 - e^{-t/\tau}\right) n_{n_0} \quad (3)$$

(2)/(3) after some rearranging

$$\frac{n_n}{n_p} = \frac{1}{\frac{n_{p_0}}{n_{n_0}} e^{t/\tau} + e^{t/\tau} - 1} \quad (4)$$

<u>Numerically</u>

$$\left.\begin{array}{l} \frac{n_{p_0}}{n_{n_0}} = 1/0.18 = 7.0178 \\ t = 180\,s - 1\,s = 179\,s \\ \tau = 889\,s \quad (\text{mean lifetime of neutrons}) \end{array}\right\} \frac{n_n}{n_p} = 0.1425$$

$$(1) \Rightarrow Y(^4He) \approx 0.25$$

Based on Ryden

Flatness problem

If $\Omega_{tot}-1$ small now, it must have been really small in the early universe. How small? Need to know how it evolves with time!

TL7 The flatness problem I

The Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_{tot}}{3} - \frac{\kappa c^2}{R_0 a^2} \qquad (1)$$

$$\Omega_{tot} = \frac{\rho_{tot}}{\rho_c} \qquad (2)$$

$$\rho_c = \frac{3H^2}{8\pi G} \qquad (3)$$

(2) and (3) $\Rightarrow$

$$\rho_{tot} = \frac{3H^2\Omega_{tot}}{8\pi G} \qquad (4)$$

(4) in (1) $\Rightarrow$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2\Omega_{tot} - \frac{\kappa c^2}{R_0 a^2} \qquad (5)$$

$\frac{\dot{a}}{a} = H \Rightarrow$ (5) becomes, after rearranging:

$$\Omega_{tot} - 1 = \frac{\kappa c^2}{R_0 a^2 H^2} \Rightarrow \boxed{\Omega_{tot} - 1 \propto \frac{1}{a^2 H^2}} \qquad (6)$$

will be usefull in hand-in 8 ¡!

$\frac{\kappa c^2}{R_0}$ is a constant

Roos/Ryden

$\boxed{18}$ Size-redshift relation

$$\theta_{rad} = \frac{l}{D_A} \qquad (1) \qquad (\sim 7.33 \text{ in Ryden})$$

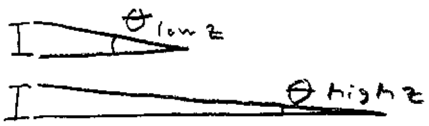
$$D_A = \frac{c}{H_0(1+z)} \int_1^{1+z} \frac{dx}{(\Omega_M x^3 + (1-\Omega_M-\Omega_\Lambda)x^2 + \Omega_\Lambda)^{1/2}} \qquad (2)$$

$$\theta_{arcsec} = \underbrace{\frac{\theta_{rad}}{2\pi}}_{\text{fraction of circle}} \cdot \underbrace{360}_{\text{degrees in circle}} \cdot \underbrace{3600}_{'' \text{ per degrees}} \qquad (3)$$

a) Visa plot + program på OH

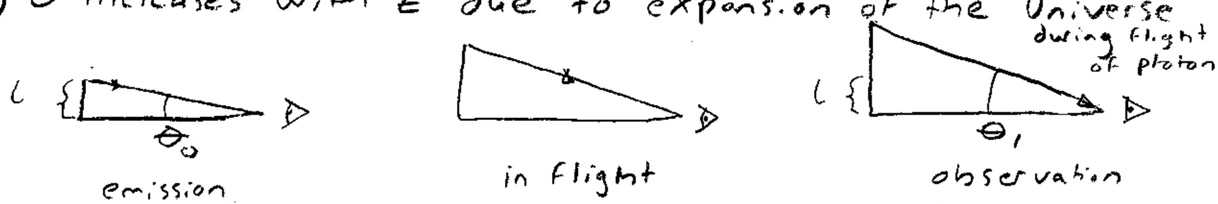
b) $\theta = 3.93''$ vid $z=1$ $(1.9077 \cdot 10^{-5}$ rad$)$
$\theta = 7.57''$ vid $z=10$ $(3.672 \cdot 10^{-5}$ rad$)$

Why does θ increase at high z?
Two competing effects:
1) θ decreases with z due to increased distance

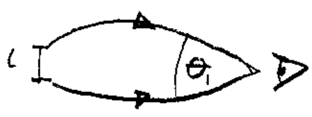


$\theta_{low\,z} > \theta_{high\,z}$

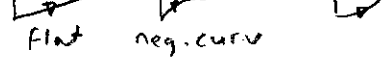
2) θ increases with z due to expansion of the Universe during flight of photon



$\theta_1 > \theta_0$



2 starts to become dominant at high z!

(The detailed θ-z relation also depends on the geometry of the universe:


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Hints for hand-in exercises

Set 3

7: Pretty simple - more or less solved on page 147 (first page of chapter 9) in the textbook

8: Tricky ... You need to work out the time dependence of $|\Omega_{tot}-1|$ and remember that the dependence is different in the radiation-dominated and matter-dominated eras

9: Not that hard - express H as a function of scale factor and study its behaviour as a increases.
Why does it behave like that? It has something to do with the definition of H...