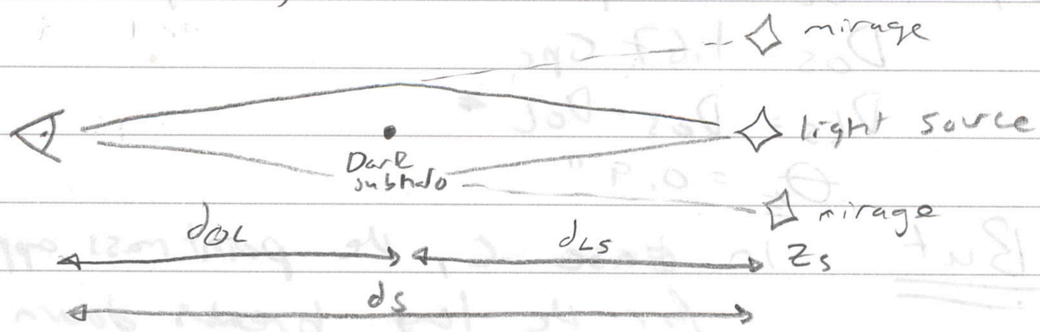
Problem set 3

13 Gravitational lensing



Note: d_L, d_{LS}, d_S are angular size distances

& $d_{OL} + d_{LS} \neq d_S$ at cosmological distances.

Kayser et al. ($\eta = 1 \Rightarrow$ homogeneous (non-clumpy) universe) Flat Universe

$$D_{xy} = \frac{c}{H_0(1+z_y)} \int_{z_x}^{z_y} \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}$$

$$\theta_E = \left(\frac{4 G M_{sub} D_{LS}}{c^2 D_{OL} D_{OS}}\right)^{1/2}$$

<u>Numerically:</u>

$z_{s} = 2.0$ (source redshift)

$M_{sub} = 10^7 M_\odot = 1.99 \cdot 10^{37}$ kg

$H_0 = 72$ km/s/Mpc $\quad \frac{c}{H_0} = 4.15$ Gpc

a) $z_L = 0.5 \Rightarrow$

$D_{OS} = 1.67$ Gpc

$D_{OL} = 1.22$ Gpc

$D_{LS} = 1.06$ Gpc

$\Rightarrow \theta_E = 3.2263 \cdot 10^{-8}$ radians

arcsec $= \frac{1}{3600}°$ $\Rightarrow \theta_E = 6.7 \cdot 10^{-3}$ arcsec (millilensing)

b) $z_L = 1.0$

$D_{OS} = 1.67$ Gpc $\quad \theta_E = 4.4 \cdot 10^{-3}$ arcsec

$D_{OL} = 1.60$ Gpc

$D_{LS} = 0.610$ Gpc

c, $D_{OL} = 10^5\ pc$ (not angular size)

$D_{OS} = 1.67\ Gpc$

$D_{LS} = D_{OS} - D_{OL}$

$\theta_E = 0.9''$

<u>**But**</u> In case c, the point mass approximation for the lens breaks down!

New version 2011

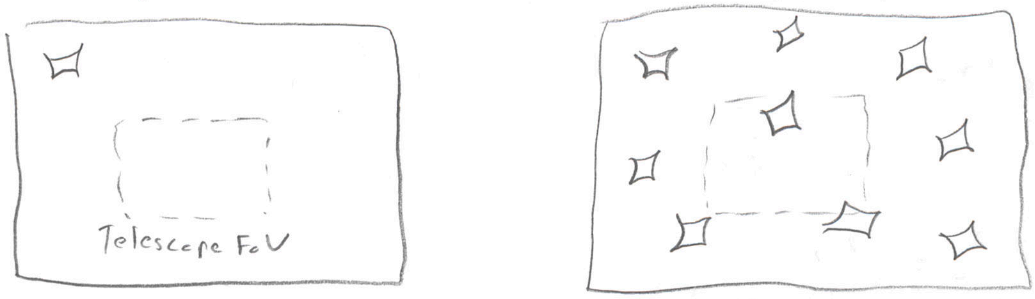
14. Hunting for high redshift objects

In general, the prospects of detecting a specific class of point-like, high redshift objects depends on:

- m_{object} – their apparent magnitudes (how bright are they?)
- n_{object} – their surface number densities (how many are there in a given patch of the sky)

Both aspects are crucial, but here we will only deal with the second one.

Example



Bleak detection prospects! Good detection prospects!

Criterion:

$$N_{object} = A_{FoV} \cdot n_{object} \gtrsim 1 \Rightarrow \text{Good detection prospects!}$$

N_{object}: objects in FoV

A_{FoV}: Area in arcmin^2

n_{object}: objects per arcmin^2

(Targets with $N_{object} << 1$ can also be detected, but this takes a wide-area survey (mosaic) which is not practical for targets that are also very faint, due to the excessive observing times required

Pop III stars in FoV at $z \approx 10 (\pm 0.5)$

$$N_{object} \sim A_{FoV} \cdot \frac{\overset{M_\odot \, Mpc^{-3} \, yr^{-1}}{SFR}}{M_{popIII}} \cdot \Delta d \cdot \Delta t \left(\frac{\tau}{\Delta t} \right) \cdot \frac{1}{\theta^2} \qquad (1)$$

Δd: proper distance between $z = 9.5$ and 10.5

Δt: Cosmic age difference — " —

$\left(\frac{\tau}{\Delta t}\right)$: Only living stars should be counted

θ: arcmin per Mpc

Δd

Proper distance at current epoch:

$$d_p(0) = \frac{d_L(z)}{(1+z)} \qquad (2, \text{ 7.29 in Ryden})$$

d_L-formula from exercise 7 in (2) →

$$d_p(0) = \frac{c}{H_0} \int_0^z \frac{dz}{\sqrt{\Omega_m (1+z)^3 - (\Omega_m + \Omega_\Lambda - 1)(1+z)^2 + \Omega_\Lambda}} \qquad (5)$$

Convert to proper distance at z:

$$\Delta d_z \approx \frac{d_{p_{z1}} - d_{p_{z2}}}{1+z} \qquad (6)$$

Very dirty approximation

Δt

Equation (13) from exercise [14]:

$$t_z = \frac{1}{H_0} \int_z^\infty \frac{dz}{(1+z)\sqrt{\Omega_m (1+z)^3 - (\Omega_m + \Omega_\Lambda - 1)(1+z)^2 + \Omega_\Lambda}} \qquad (7)$$

$$\Delta t = t_{z_2} - t_{z_1} \qquad (8)$$

14 forts.

~~θ~~

$$\theta_{arcmin} = \frac{L}{d_A} \cdot \left(\frac{360 \cdot 60}{2\pi}\right) \qquad (9)$$

$$d_A = \frac{d_L}{(1+z)^2} \qquad (10, 7.37 \text{ in Ryden})$$

(10) in (12) $\Rightarrow$

$$\theta_{arcmin} \; \frac{L\,(1+z)}{d_p(0)} \left(\frac{360 \cdot 60}{2\pi}\right) \qquad (11)$$

Numerically

$H_0 = 70$ km s^{-1} Mpc^{-1}

$\Omega_M = 0{,}3$

$\Omega_\Lambda = 0{,}7$

$$\left.\begin{matrix} z_1 = 10{,}5 \\ z_2 = 9.5 \end{matrix}\right\} \Rightarrow \begin{matrix} \Delta d_z \approx 19{,}5 \text{ Mpc} \\ \Delta t = 63 \cdot 10^6 \text{ yr} \end{matrix}$$

$$\left.\begin{matrix} L = 1 \text{ Mpc} \\ z = 10 \end{matrix}\right\} \overset{(11)}{\Rightarrow} \theta_{arcmin} \approx 4.0$$

N_{object}:

(~~4~~) $\Rightarrow$ $A_{FoV} = 2.2^2 \text{ arcmin}^2 = 4.84 \text{ arcmin}^2$

$SFR = 10^{-5}\, M_\odot \text{ Mpc}^{-3} \text{ yr}^{-1}$

$M = 500\, M_\odot \Rightarrow \tau = 2 \cdot 10^6 \text{ yr} \Rightarrow N_{object} \approx 0.2$

$50\ M_\odot \Rightarrow \tau = 4 \cdot 10^6 \Rightarrow N_{object} \approx 5$

$5\ M_\odot \Rightarrow \tau \approx 6 \cdot 10^7 \Rightarrow N_{object} \approx 700$

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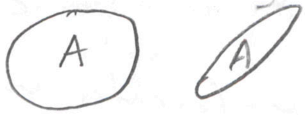
15 The Tolman test

In astronomy, surface brightness is measured using ~~the~~ quantities I [$L_\odot pc^{-2}$] or μ [mag arcsec^{-2}]



$$I = \frac{L}{A}$$

- At small distances (very low z), I and μ are independent of distance*
- At large distances (non-negligible z), I and μ are subject to surface brightness dimming due to the expansion of the Universe:

$$I \propto (1+z)^{-4} \qquad (1)$$

Why is this?

- Every photon loses energy due to redshift $\Rightarrow$ factor $(1+z)^{-1}$
- Cosmological time dilation gives fewer photons per unit time from high-z objects

$\}$ Affects L

- Apparent area increased by factor $(1+z)^2$ since the photons were emitted at a time when the object was closer (sometimes referred to as "abberation")

$\}$ Affects A

* as derived in the Galaxies course, at least when I teach it.

Tolman test (1930)

Redshift due to expanding Universe:

$I \propto (1+z)^{-4} \Rightarrow \mu = A\,[2.5\log_{10}(1+z)] + C$ with $A=4$

Redshift due to "tired light" in a static Universe:

$I \propto (1+z)^{-1} \Rightarrow$ same μ-formula with $A=1$

photons lose energy due to redshift

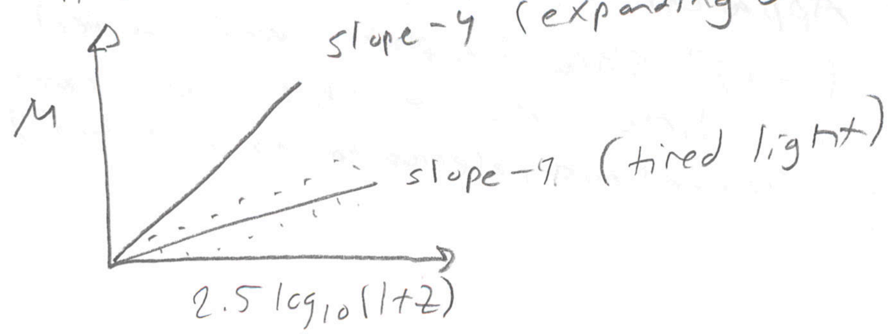
These relations are independent of other cosmological parameters (cosmic curvature, H_0, dark energy etc.)

Basic idea:

Target galaxies over a range in z, measure surface brightness across these objects, plot I against $(1+z)$ or μ against $2.5\log_{10}(1+z)$ and read off the slope.

Disturbing: To this day, a minority of researchers argue that the test proves that the Universe is static!

Show Andrews (2009) plot!



WTF?

No need to solve this, but what are the likely reasons why this test is difficult to apply in practice?

Let the students think ...

15 Parts.

Complications

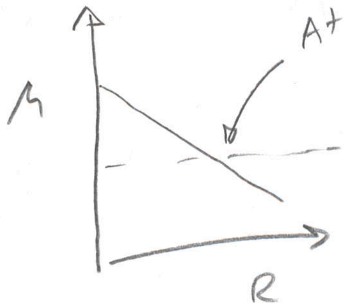
Evolution

In the standard scenario, hierarchical build-up ⇒ galaxies evolve in L and size with z (no large galaxies at high z)

Higher intrinsic μ at high z → slower drop off than $(1+z)^{-4}$

- Galaxies do not have constant surface brightness

Typical μ-profile:



At some point, the outskirts drown in the sky noise → whole profile never probed

What area do you sum up the light over?

- Objects at high z subtend a smaller angle in the sky → more difficult to resolve. Limited resolution smears object

- Observations at high and low z do not probe the same rest-frame wavelength – a k-correction needs to be applied

Kyder / Koos

16 Big Bang Nucleosynthesis

~All neutrons end up in ^4He

$$Y(^4He) \equiv \frac{4n_{He}}{n_{tot}} \approx \frac{4\left(\frac{n_n}{2}\right)}{n_n+n_p} = \frac{2n_n/n_p}{1+\frac{n_n}{n_p}} \quad (1, p.178 \text{ below } 10.21)$$

if all neutrons are assumed to end up in ^4He.

What is $\frac{n_n}{n_p}$ at the time of ^4He formation?

mean lifetime of neutron

$$n_n = n_{n_0} e^{-t/\tau} \quad (2) \quad (\text{Ph.M. } 8.1)$$

$$n_p = n_{p_0} + \left(1 - e^{-t/\tau}\right) n_{n_0} \quad (3)$$

(2)/(3) after some rearranging

$$\frac{n_n}{n_p} = \frac{1}{\frac{n_{p_0}}{n_{n_0}} e^{t/\tau} + e^{t/\tau} - 1} \quad (4)$$

Numerically

$$\left.\begin{aligned} \frac{n_{p_0}}{n_{n_0}} &= 1/0.18 = 7.0178 \\ t &= 180\,s - 1\,s = 179\,s \\ \tau &= 889\,s \quad \text{(mean lifetime of neutrons)} \end{aligned}\right\} \frac{n_n}{n_p} = 0.1425$$

$$(1) \Rightarrow Y(^4He) \approx 0.25$$

Based on Ryden

17 The Flatness problem I

The Friedmann equation

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho_{tot}}{3c^2} - \frac{\kappa c^2}{R_0 a^2} \qquad (1)$$

Flatness problem

If $\Omega_{tot}-1$ small now, it must have been really small in the early universe. How small? Need to know how it evolves with time. First step is this exercise, second step in exercise 22 (hand-in)

$$\Omega_{tot} = \frac{\rho_{tot}}{\rho_c} \qquad (2)$$

$$\rho_c = \frac{3H_0^2}{8\pi G} \qquad (3)$$

(2) and (3) $\Rightarrow$

$$\rho_{tot} = \frac{3c^2 H_0^2 \Omega_{tot}}{8\pi G} \qquad (4)$$

(4) in (1) $\Rightarrow$

$$\left(\frac{\dot a}{a}\right)^2 = H^2\Omega_{tot} - \frac{\kappa c^2}{R_0 a^2} \qquad (5)$$

$\frac{\dot a}{a} = H \Rightarrow$ (5) becomes, after rearranging:

$$\Omega_{tot} - 1 = \frac{\kappa c^2}{R_0 a^2 H^2} \Rightarrow \boxed{\Omega_{tot} - 1 \propto \frac{1}{a^2 H^2}} \qquad (6)$$

will be usefull in ~~hand-in~~ 8 22!

$\frac{\kappa c^2}{R_0}$ is a constant

Prob. in Roos/Ryden

$\boxed{18}$ Size-redshift relation

$$\theta_{rad} = \frac{l}{D_A} \qquad (1) \qquad (\sim 7.33 \text{ in Ryden})$$

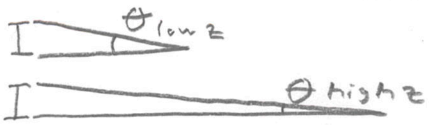
$$D_A = \frac{c}{H_0(1+z)} \int_1^{1+z} \frac{dx}{(\Omega_M x^3 + (1-\Omega_M-\Omega_\Lambda)x^2 + \Omega_\Lambda)^{1/2}} \qquad (2)$$

$$\theta_{arcsec} = \underbrace{\frac{\theta_{rad}}{2\pi}}_{\text{fraction of circle}} \cdot \underbrace{360}_{\text{degrees in circle}} \cdot \underbrace{3600}_{\text{'' per degrees}} \qquad (3)$$

a) Visa plot + program på OH

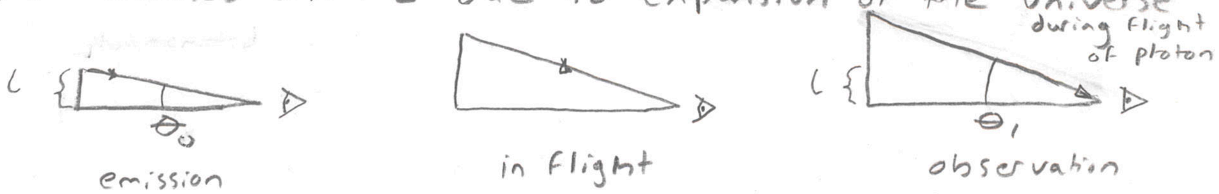
b) $\theta = 3.93''$ vid $z=1$ $(1.9077 \cdot 10^{-5}$ rad$)$
$\theta = 7.57''$ vid $z=10$ $(3.672 \cdot 10^{-5}$ rad$)$

Why does θ increase at high z?
Two competing effects:
1) θ decreases with z due to increased distance

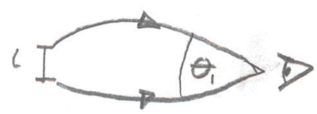


$\theta_{low\,z} > \theta_{high\,z}$

2) θ increases with z due to expansion of the Universe

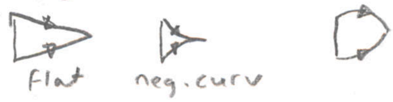


$\theta_1 > \theta_0$



2 starts to become dominant at high z!

(The detailed θ-z relation also depends on the geometry of the universe:



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