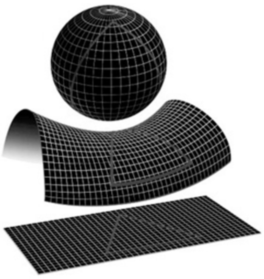
Cosmology 1FA209, 2017
Lecture 2: The cosmological principle and cosmic expansion



Outline

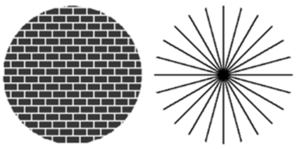
- The cosmological principle:
  - Isotropy
  - Homogeneity
- Big Bang vs. Steady State cosmology
- Redshift and Hubble's law
- Scale factor, Hubble time, Horizon distance
- Olbers' paradox: Why is the sky dark at night?
- Particles and forces
- Theories of gravity: Einstein vs. Newton
- Cosmic curvature

Covers chapter 2 + half of chapter 3 in Ryden

The Cosmological Principle I

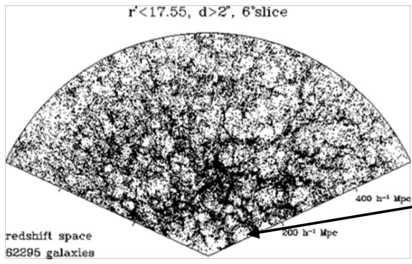
- Modern cosmology is based on the assumption that the Universe is:
  - Homogeneous
  - Isotropic

} The cosmological principle

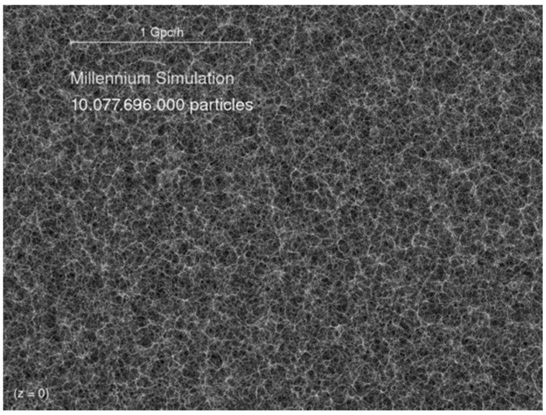


The Cosmological Principle II

- These tenets *seem* to hold on large scales (>100 Mpc), but definitely not on small



Voids typically 70 Mpc across



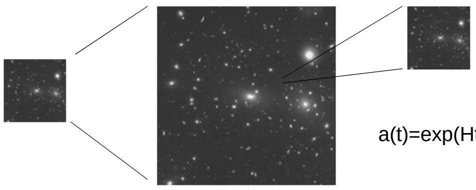
The Perfect Cosmological Principle

- In this case, one assumes that the Universe on large scales is:
  - Homogeneous
  - Isotropic
  - *Non-evolving*

This is incompatible with the Big Bang scenario, but the *Steady State model* (popular in the 1940-1960s) was based on this idea

Steady State Cosmology

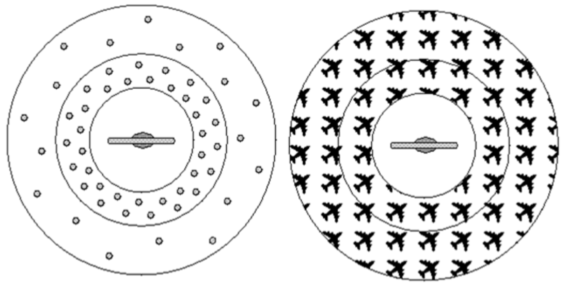
Universe continuously expands, but due to continuous creation of matter, no dilution occurs → Steady State, no hot initial Big Bang and no initial singularity…



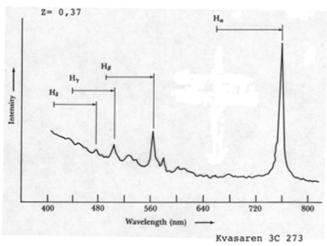
$a(t)=\exp(Ht)$

… But this cosmology fails to explain the CMBR, $T_{CMBR}(z)$, the production of light elements and the redshift evolution of galaxies & AGN

Intermission: Do these Universes obey the cosmological principle?



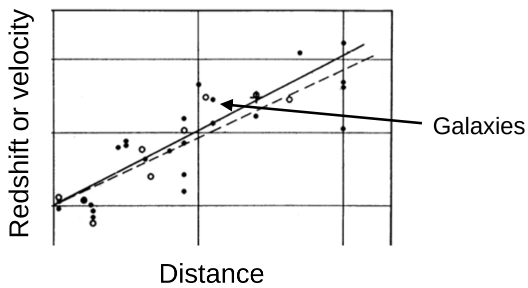
Redshift



Definition of redshift:

$$z=\frac{\lambda_{obs}-\lambda_{em}}{\lambda_{em}}$$

Hubble's law I



Hubble's law II

Hubble's law:

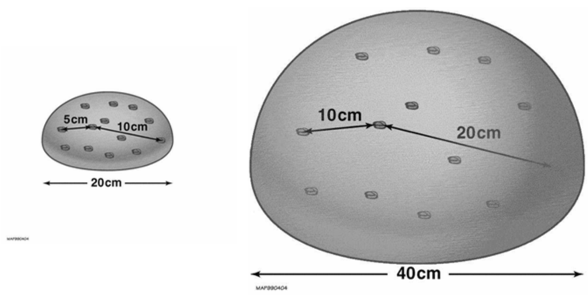
The Hubble "constant" → H_0; Luminosity distance → d

$$z=\frac{H_0 d}{c}$$

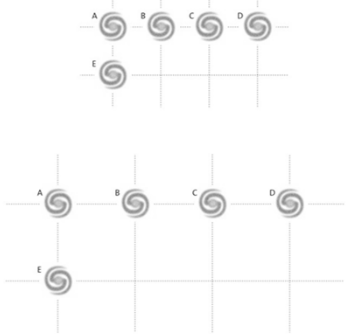
In observational astronomy, the term recession velocity, v, occurs frequently:

At low z: $z\approx\frac{v}{c}$ → $v=H_0 d$

Expansion of the Universe I



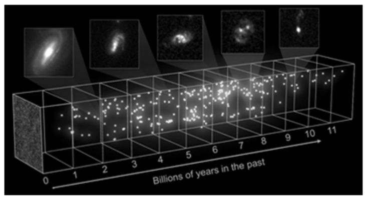
Expansion of the Universe II



Redshift and distance I

- Low redshift (z≈0) corresponds to:
  - Small distance (local Universe)
  - Present epoch in the history of the Universe
- High redshift corresponds to:
  - Large distance
  - Earlier epoch in the history of the Universe

Low redshift



High redshift

Redshift and distance II

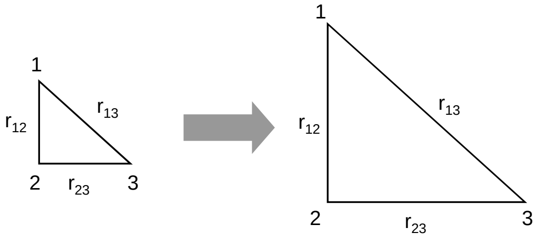
- But beware:
  - At low redshift, Doppler components coming from peculiar motions may be substantial – must be corrected for before distance *d* is derived from *z* or *v*
  - The redshift coming from cosmic expansion is *not* a Doppler shift – don't treat it like one!
  - The linear version of Hubble's law is only appropriate at z<0.15 (at 10% accuracy)

Intermission

$z \approx -0.001$

What does it mean?

Scale factor



Time t (earlier)
Scale factor a(t)

Time t_0 (now)
Scale factor $a(t_0)=1$

$$r_{12}(t) = a(t)/a(t_0)\, r_{12}(t_0) = a(t)\, r_{12}(t_0)$$
$$v_{12}(t) = a'/a\, r_{12}(t)$$

Scale factor and redshift

$$1+z = \frac{a_0}{a} = \frac{1}{a}$$

Cosmic scale factor today (at t_0) — can be set to $a_0=1$

Cosmic scale factor when the Light was emitted (the epoch corresponding to the redshift z)

The Hubble "constant"

$H_0 \approx 70 \pm 5$ km s^{-1} Mpc^{-1} [s^{-1}]

Today

Errorbars possibly underestimated…

Note: Sloppy astronomers often write km/s/Mpc…

In general:

$$H \equiv \frac{\dot{a}}{a}$$

Not a constant in our Universe!

Hubble time

In the case of constant expansion rate, the Hubble time gives the age of the Universe:

$$t_{\mathrm{H}} = \frac{1}{H_0} \approx 14 \text{ Gyr}$$

In more realistic scenarios, the expansion rate changes over time, but the currently favoured age of the Universe is still pretty close – around 13—14 Gyr.

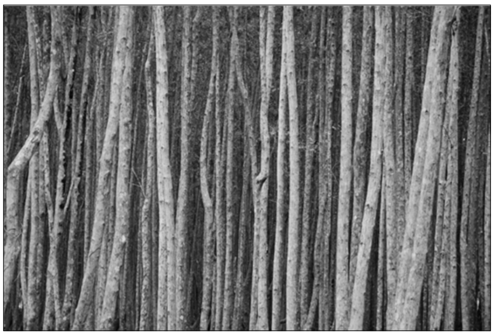
Olbers' paradox I

"Why is the sky dark at night?"
(Heinrich Olbers 1926)

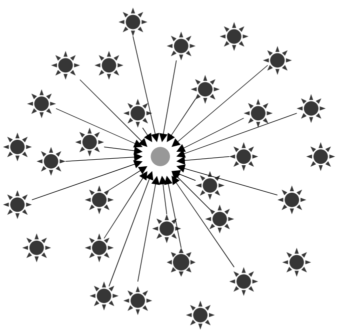
If the Universe is:
- Spatially infinite (i.e. infinite volume)
- Infinitely old and unevolving

- then the night sky should be bright!

Intermission: What does this have to do with Olbers' paradox?

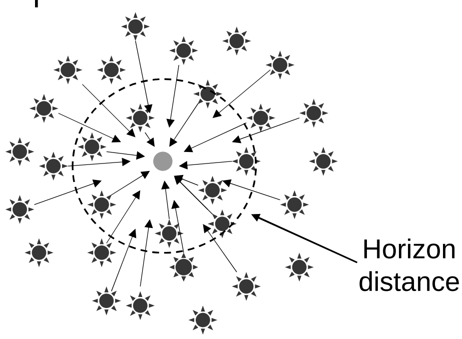


Olbers' paradox III



Planet Earth surrounded by stars in an infinte, unevolving Universe

Olbers' paradox IV



Main solution: The Universe has finite age
The light from most stars have not had time to reach us!

Horizon distance

- Horizon distance = Current distance to the most faraway region from which light has had time to reach us
- This delimits the causally connected part of the Universe an observer can see at any given time
- Horizon distance at time t_1:

$$d_{\text{hor}}(t_1) = c\int_{t=0}^{t_1} \frac{\mathrm{d}t}{a(t)}$$

- Most realistic scenarios give:
  $d_{hor}(t_0)$~c/H_0 (the so-called Hubble radius)

Particles and forces I

- The particles that make up the matter we encounter in everyday life:
  - Protons, p
    938.3 MeV
  - Neutrons, n
    939.5 MeV
  - Electrons, e^-
    0.511 MeV

Protons and neutrons: Baryons (made of 3 quarks)
Electrons: Lepton

Since most of the mass of 'ordinary matter' is contributed by protons and neutrons, such matter is often referred to as *baryonic*. Examples of mostly baryonic objects: Planets, stars, gas clouds (but not galaxies or galaxy clusters)

Particles and forces II

- Other important particles (for this course):
  - Photon, γ
    Massless, velocity: c
  - Neutrinos, $\nu_e\ \nu_\mu\ \nu_\tau$
    ~eV (?), velocity close to c
    Interacts via weak nuclear force only

Neutrinos: Leptons

The four forces of Nature

- Strong force
  - Very strong, but has short range (~10^{-15} m)
  - Holds atomic nuclei together
- Weak force
  - Weak and has short range
  - Responsible for radioactice decay and neutrino interactions
- Electromagnetic force
  - Weak but long-range
  - Acts on matter carrying electric charge
- Gravity
  - Weak, very long-range and always attractive

On the large scales involved in cosmology, gravity is by far the dominant one

Newtonian gravity

- Space is Euclidian (i.e. flat)
- Planet are kept in their orbits because of the gravitational force:

$$F = -\frac{GM_g m_g}{r^2}$$

Gravitational mass (M_g, m_g)

- The acceleration resulting from the gravitational force:

$$F = m_i a$$

Inertial mass (m_i)

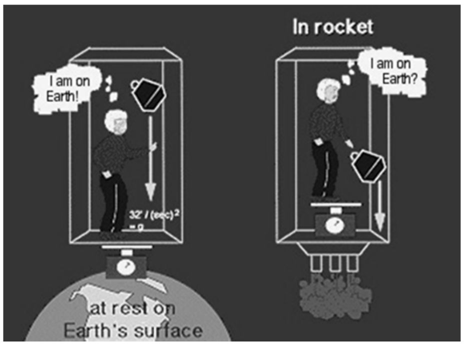
Equivalence Principle

- Gravitational acceleration towards an object with mass M_g is:

$$a = -\frac{GM_g}{r^2}\left(\frac{m_g}{m_i}\right)$$

- Empirically M_g=M_i (to very high precision)
- The equality of gravitational mass and inertial mass is called the equivalence principle
- In Newtonian gravity, M_g=M_i is just a strange coincidence, but in General Relativity, this stems from the idea that masses cause curvature of space

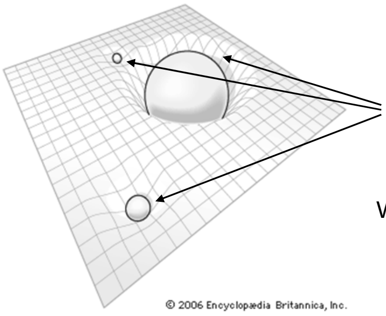
Intermission: What does this have to do with the equivalence principle?



General Relativity

- 4D space-time
- Mass/energy curves space-time
- Gravity = curvature
- Pocket summary:
  - Mass/energy tells space-time how to curve
  - Curved space-time tells mass/energy how to move

Small-scale curvature

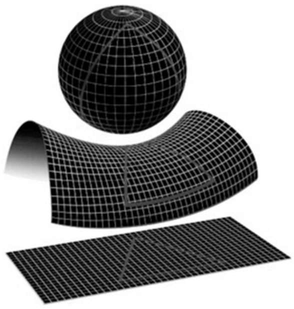


Small-scale distortions caused by astronomical objects

What about the large-scale curvature?

Global Curvature I

In the world models of general relativity, our Universe may have spatial curvature (on global scales)



Positive curvature

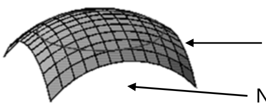
Negative curvature

Zero curvature (flat/Euclidian space)

Global Curvature II

Very tricky stuff…

- This an *intrinsic* curvature in 3D space
- Note: No need for encapsulating our 3D space in 4D space to make this work

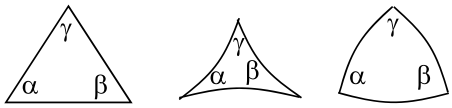


This represents 3D

No need for anything here

Global Curvature III

Angles in curved spaces

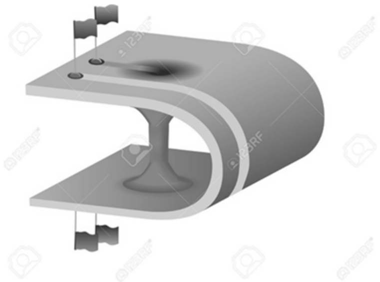


Flat: $\alpha+\beta+\gamma = 180°$

Negative: $\alpha+\beta+\gamma < 180°$

Positive: $\alpha+\beta+\gamma > 180°$

Intermission: What is this figure meant to illustrate?



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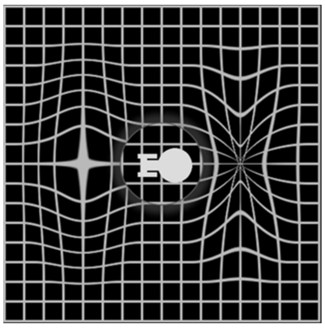

Metrics I

- Metric: A description of the distance between two points
- Metric in 2 dimensional, flat space:

$$ds^2 = dx^2 + dy^2$$ (Pythagoras)

- Metric in 3 dimensional, flat space:

$$ds^2 = dx^2 + dy^2 + dz^2$$

Metrics II

- Metric in 3 dimesions, flat space, polar coordinates:

$$ds^2 = dr^2 + r^2 d\Omega^2$$
$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$$

- Metric in 3 dimensions, arbitrary curvature:

$$ds^2 = \frac{dx^2}{1 - \kappa x^2 / R^2} + x^2 d\Omega^2$$

Flat: $\kappa = 0,\ x = r$

Negative: $\kappa = -1,\ x = R\sinh(r/R)$

Positive: $\kappa = 1,\ x = R\sin(r/R)$

Curvature radius