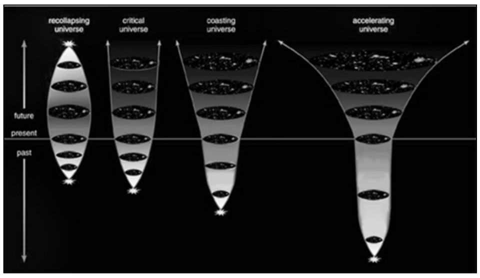
Cosmology 1FA209, 2017
Lecture 4: Cosmological models and the fate of the Universe



Seminar I request

October 2, 13-15
Emma Fransson
Lifi Huang
Thomas Mathisen
Elise Sänger
Charles Taylor
Eva Laura Winter

Group B

October 3, 10-12
Brivael Laloux
Vlad Rastau
Josefine Nittler
Larisa Castellanos
Amanda Bernmalm
Lawen Ahmedi
Luis Tabera

Group C

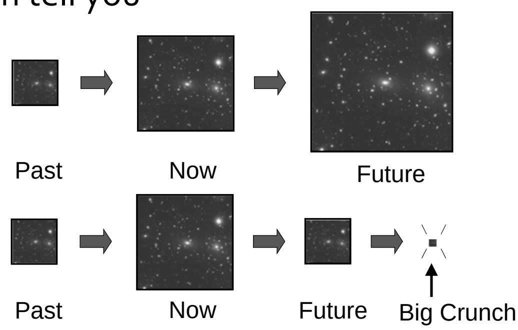
Is anybody from group C willing to move to group B?
If so, please contact me!

Outline

- Properties and fate of single-component Universes
  - Empty Universe
  - Flat Universe
  - Matter/Radiation/Lambda-dominated Universe
  - Einstein-de Sitter Universe
- Properties and fate of multiple-component Universes
  - Matter + curvature
  - Benchmark model
  - Other dark energy scenarios

Covers chapter 5 in Ryden

What the dynamics of the Universe can tell you



The fate of the Universe!

Density evolution (general)

- Friedmann equation + fluid equation + equation of state → a(t), ε(t) or t(a), ε(a)
- Problem: There are many components in the Universe →
  - Evolution complicated
  - Different components dominate evolution at different times

Multiple energy components

$$\varepsilon_{\rm tot} = \sum_w \varepsilon_w$$

$$\Omega_{\rm tot} = \sum_w \frac{\varepsilon_w}{\varepsilon_c}$$

Often, one just writes ε and Ω

Remember:

$w = 0$ (non – relativistic matter)

$w = 1/3$ (radiation and relativistic matter)

$w = -1$ (cosmological constant)

$w < -1/3$ (dark energy) → Positive acceleration

Density evolution

Fluid equation + eq. of state (see exercise session) $\rightarrow$

$$\varepsilon_w(a) = \varepsilon_{w,0} a^{-3(1+w)}$$

$$\begin{cases} w_m = 0 \;\Rightarrow\; \varepsilon_m(a) = \varepsilon_{m,0} a^{-3} \\ w_r = 1/3 \;\Rightarrow\; \varepsilon_r(a) = \varepsilon_{r,0} a^{-4} \\ w_\Lambda = -1 \;\Rightarrow\; \varepsilon_\Lambda(a) = \varepsilon_{\Lambda,0} \end{cases}$$

The benchmark model

The currently favoured cosmological model:

$\Omega_M \approx 0.31$

- $\Omega_M = \Omega_{\text{Non-baryons (CDM)}} + \Omega_{\text{Baryons}}$
- $\Omega_{\text{Non-baryons (CDM)}} \approx 0.26$
- $\Omega_{\text{Baryons}} \approx 0.05$

} More on this in the dark matter lecture

$\Omega_\Lambda \approx 0.69$

$\Omega_R \approx 8.4 \times 10^{-5}$

- $\Omega_R = \Omega_{CMB} + \Omega_\nu + \Omega_{\text{starlight}}$
- $\Omega_{CMB} \approx 5.4 \times 10^{-5}$
- $\Omega_\nu \approx 3.6 \times 10^{-5}$
- $\Omega_{\text{starlight}} \approx 1.5 \times 10^{-6}$

$\Omega_{tot} = \Omega_M + \Omega_\Lambda + \Omega_R \approx 1.0 \rightarrow$
Flat Universe ($\kappa = 0$)

Why are single-component Universes relevant then?

$$\begin{aligned} w_m = 0 \;&\Rightarrow\; \varepsilon_m(a) = \varepsilon_{m,0} a^{-3} \\ w_r = 1/3 \;&\Rightarrow\; \varepsilon_r(a) = \varepsilon_{r,0} a^{-4} \\ w_\Lambda = -1 \;&\Rightarrow\; \varepsilon_\Lambda(a) = \varepsilon_{\Lambda,0} \end{aligned}$$

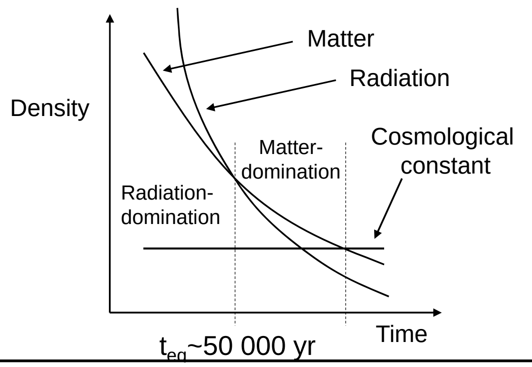
Values of Ω_M, Ω_Λ, Ω_R today + these relations $\rightarrow$

$\varepsilon_M \geq \varepsilon_\Lambda$ at some point in the past (matter-dominated Universe)

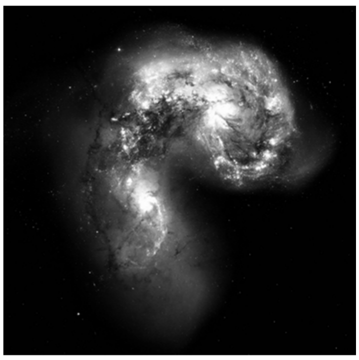
$\varepsilon_R \geq \varepsilon_M$ even further back (radiation-dominated Universe)

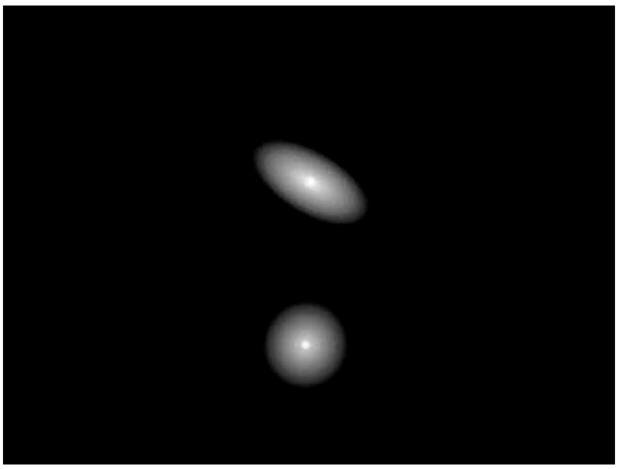
The different epochs of the Universe can be *approximated* by single-component evolution

Different components dominate the sum at different times



Intermission: What are you looking at?





The Milne Universe (empty)

Of pretty limited relevance for our Universe, but provides simple demonstration of how to derive current age of Universe, a(t) and t(z) from the Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

$$\varepsilon(t) = 0 \quad \Rightarrow$$

$$H(t)^2 = -\frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2} \Rightarrow$$

$$\dot{a}(t)^2 = -\frac{\kappa c^2}{R_0^2}$$

The Milne Universe (empty) II

$$\dot{a}(t)^2 = -\frac{\kappa c^2}{R_0^2}$$

Empty, static Universe: $\kappa = 0 \quad \Rightarrow \quad \dot{a}(t) = 0$

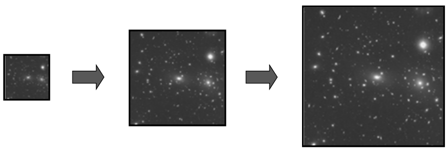
Empty, negatively curved: $\kappa = -1 \ \Rightarrow$

$$\dot{a}(t) = \frac{\mathrm{d}a}{\mathrm{d}t} = \pm\frac{c}{R_0}$$

If expanding (not contracting) $\Rightarrow$

Current age of Universe: $t_0 = \frac{R_0}{c}, \quad a(t) = \frac{t}{t_0}, \quad t(z) = \frac{t_0}{1+z}$

Fate of the Universe in a Milne Universe



Past Now Future

Eternal, constant expansion ('Big Chill')

Proper distance in a Milne Universe

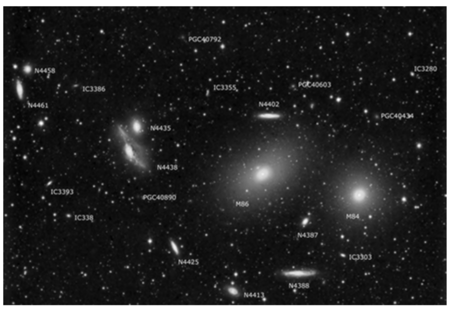
Proper distance from us to object which emitted light at t_e:

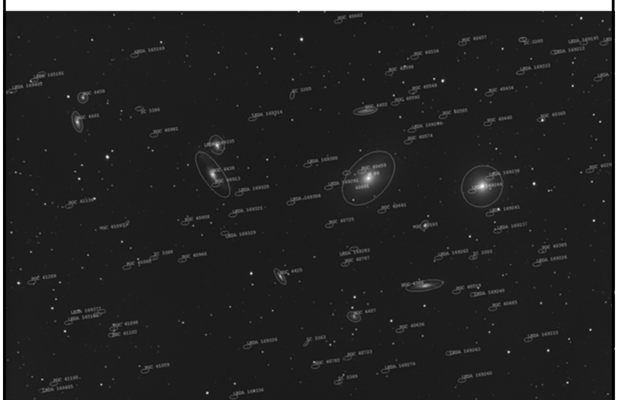
$$d_p(t_0) = c\int_{t_e}^{t_0}\frac{\mathrm{d}t}{a(t)}$$

$$a(t) = \frac{t}{t_0} \quad \Rightarrow d_p(t_0) = ct_0\int_{t_e}^{t_0}\frac{\mathrm{d}t}{t} = ct_0\ln\left(\frac{t_0}{t_e}\right)$$

$$t_e = \frac{t_0}{1+z} \Rightarrow d_p(t_0) = ct_0\ln(1+z) = \frac{c}{H_0}\ln(1+z)$$

Intermission: What are you looking at?





Flat, single-component Universes

For $w \neq -1$:

$$a(t) = \left(\frac{t}{t_0}\right)^{2/(3+3w)}$$

$$\varepsilon(t) = \varepsilon_0 \left(\frac{t}{t_0}\right)^{-2}$$

Important types of flat, single-component Universes

Radiation-only Universe (radiation-dominated epoch):

$$t_0 = \frac{1}{2H_0}, \quad a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

Matter-only Universe (matter-dominated epoch):

$$t_0 = \frac{2}{3H_0}, \quad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Λ-only Universe I

Λ has $w = -1 \Rightarrow$

$$\dot{a}^2 = \frac{8\pi G \varepsilon_\Lambda}{3c^2} a^2$$

Rearrange:

$$\dot{a}^2 = H_0^{\,2} a^2 \quad \text{if}$$

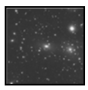
$$H_0 = \left(\frac{8\pi G \varepsilon_\Lambda}{3c^2}\right)^{1/2}$$

Λ-only Universe II

Solution :

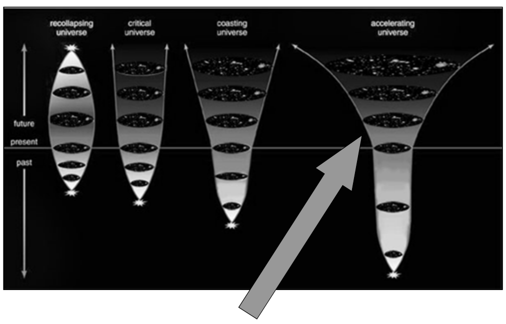
$$a(t) = e^{H_0 (t - t_0)}$$

de Sitter Universe (de Sitter phase)
Same growth as in Steady state cosmology

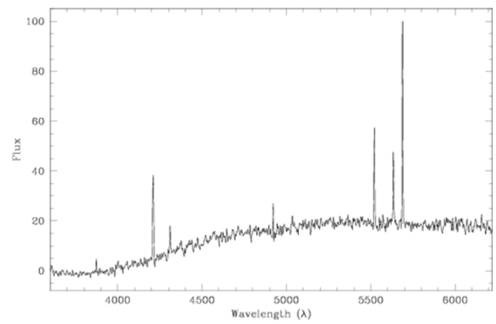
(...)

Now Future



de Sitter Universe/phase
(after 'present')

Intermission: What are you looking at?



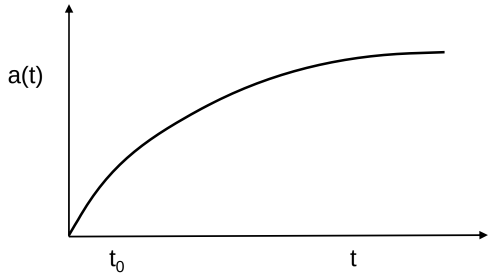
Einstein-de Sitter Universe I

This served as the benchmark model up until the mid-1990s

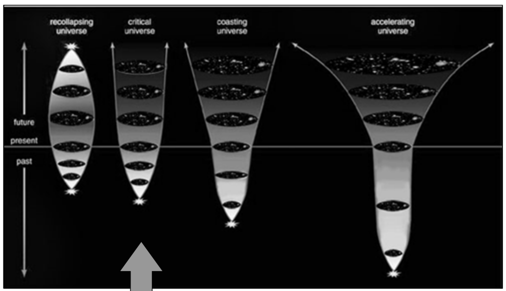
- Flat (i.e. critical-density), matter-dominated Universe
- $\Omega_M = 1.0$, $\Omega_{tot} = 1.0$, $\kappa = 0$

$$t_0 = \frac{2}{3H_0}, \quad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

Einstein-de Sitter Universe II

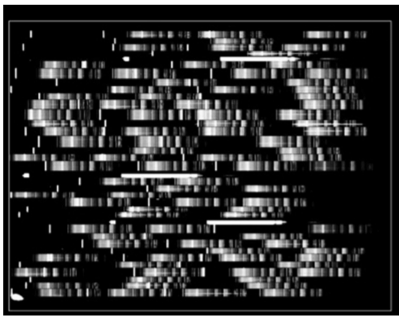


Asymptotically approaches zero expansion, but no recollapse

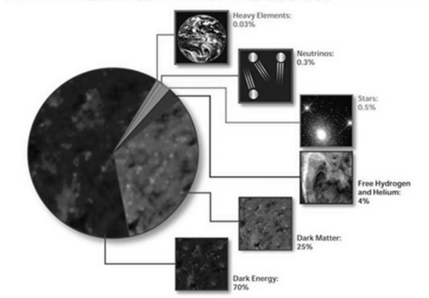


Einstein-de Sitter Universe

Intermission: What are you looking at?



Multiple-component Universes I



Note: Valid for current time only!

Multiple-component Universes II

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a(t)^2}$$

Recall:

$$\varepsilon(t) = \varepsilon_M(t) + \varepsilon_R(t) + \varepsilon_\Lambda(t) + \ldots$$

Multiple-component Universes III

For 3 components with arbitrary curvature, the FE may be rewritten:

$$\frac{H(t)^2}{H_0^2} = \underbrace{\frac{\Omega_{R,0}}{a(t)^4} + \frac{\Omega_{M,0}}{a(t)^3} + \Omega_{\Lambda,0}}_{\text{The 3 components}} + \underbrace{\frac{1-\Omega_{\text{tot},0}}{a(t)^2}}_{\text{Curvature}}$$

More components can easily be added as additional terms, as long as you know their a(t)-dependence

Multiple-component Universes IV

Time (from Big Bang):

$$t(a) = \frac{1}{H_0}\int_0^a \frac{da}{\left[\Omega_{R,0}a^{-2} + \Omega_{M,0}a^{-1} + \Omega_{\Lambda,0}a + (1-\Omega_{\text{tot},0})\right]^{1/2}}$$

Lookback-time = t_0-t(a)

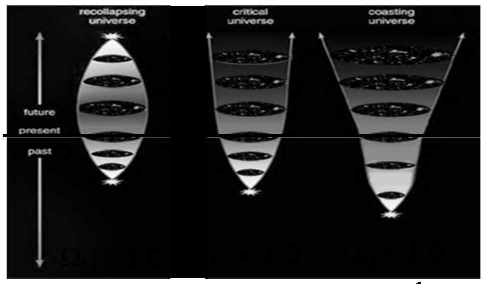
Nasty integral! No simple analytical solution in general case → Use numerical integration!

Special case: Matter + Curvature I

$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_{M,0}}{a(t)^3} + \frac{1-\Omega_{M,0}}{a(t)^2}$$

H(t)-evolution → Possible fates of Universe
This is one of the hand-in exercises!
(Note: Lots of help on page 86-87 in Ryden)

Special case: Matter + Curvature II



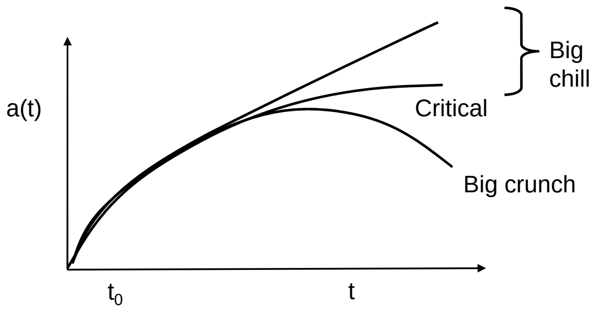
κ =+1	κ = 0	κ =-1
'Big Crunch'	'Big Chill'	'Big Chill'
Finite	Infinite*	Infinite*

* Note: In the case of a simple topology

Intermission: What does "end of the Universe" actually mean?

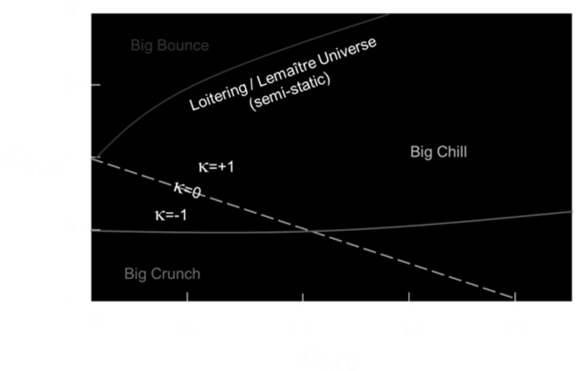
- All life ends?
- All stars fade away?
- All black holes evaporate?
- All matter destroyed?
- Space ends?
- Time ends?

Special case: Matter + Curvature III



Up until the mid-90s, these were the only 3 possible fates of the Universe typically quoted in textbooks

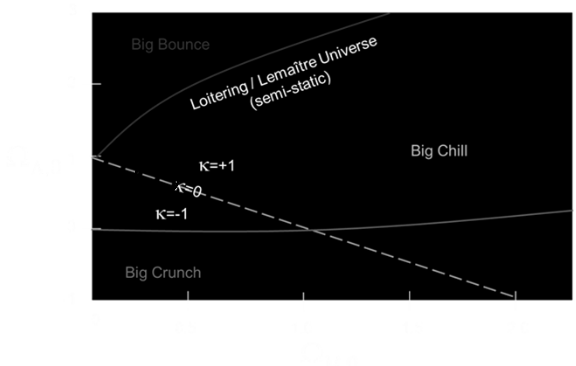
Matter + curvature + Λ



Properties of the benchmark model

- Matter-radiation equality:
  - $a \approx 2.8\times10^{-4}$
  - $t \approx 4.7\times10^{4}$ yr
- Matter-Λ equality:
  - $a \approx 0.75$
  - $t \approx 9.8$ Gyr
- Now:
  - $a = 1$
  - $t \approx 13.7$-13.8 Gyr

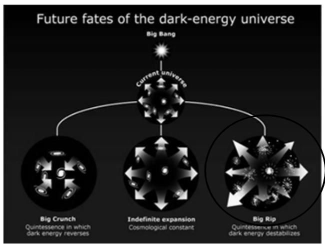
Properties of the benchmark model II



Other Dark Energy Models

What if dark energy is something other than Λ?

- Varying equation of state w(t)?
- Constant $w \neq -1$?



More on this in exercise session

Our ignorance about the nature of dark energy →
Ultimate fate of our Universe unclear