Cosmology 1FA209
Hand-in exercises 2017

Instructions: Hand-written solutions are quite acceptable, but submission via email is highly encouraged (and will allow for a swifter evaluation), so please consider scanning your solutions (or taking photos of them) and submitting them in electronic format. The deadline for handing in solutions to these problems is **October 29, 2017**.

1. *Hubble's law and luminosity distance*
A galaxy is observed at a redshift of $z = 0.25$. How distant is this object according to Hubble's law? How accurate is Hubble's law for estimating the luminosity distance at this redshift, under the assumption of a cosmological model with $\Omega_{\rm M} = 0.3$ and $\Omega_\Lambda = 0.7$?

2. *Fate of the Universe*
Starting from the Friedmann equation

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{\kappa c^2}{R_0^2 a(t)^2}, \tag{1}$$

demonstrate that a currently expanding, matter-only universe will continue to expand forever if $\Omega_{\rm M} \leq 1$, but not if $\Omega_{\rm M} > 1$.

3. *The era of dark-energy domination*
Estimate the redshift at which the Universe became dark-energy dominated, assuming $\Omega_{\rm M} = 0.3$ and $\Omega_{\rm DE} = 0.7$ today, and that the dark energy has an equation of state ($p = wc^2\rho$):
a) $w = -1.0$ (i.e. a cosmological constant)
b) $w = -1.5$

4. *Dark energy and supernovae type Ia*
The redshifts and apparent magnitudes of a small sample of supernovae type Ia are listed in Table 1. Use this data to determine which of the following three cosmological models is the most likely:
a) $\Omega_{\rm M} = 1.0$, $\Omega_\Lambda = 0.0$
b) $\Omega_{\rm M} = 0.3$, $\Omega_\Lambda = 0.7$
c) $\Omega_{\rm M} = 0.5$, $\Omega_\Lambda = 0.5$
You can assume $H_0 = 72$ km s^{-1} Mpc^{-1} and an absolute magnitude for type Ia supernovae of $M_B = -19.3$ in all cases. Both dust extinction corrections and cosmological k-corrections may be ignored. The estimated photometric 1σ errors of the measurements are $\sigma_m = 0.1$ magnitudes. Errors in the redshift determinations can be assumed negligible. Note: As this is just a toy example with artificial data, you should not expect the concordance model to win by default.

5. *Fermi problem: Dark matter*
Let's assume that 10% of the dark matter consists of Earth-mass primordial black holes. If you were to set a Death star-sized space station on autopilot along a straight path at near-light speed (and turning off all automatic evasive-maneuver capabilities) in intergalactic space for $\sim 10^8$ yrs, how many such black holes do you expect to collide with? Make an order-of-magnitude estimate of this, quantify the uncertainty and make a top-3 list of the most important shortcomings/simplications that are likely to affect your estimate (and clearly explain why this is so).

6. *The flatness problem II*
If $|\Omega_{\text{tot}} - 1| < 0.1$ now, what value of $|\Omega_{\text{tot}} - 1|$ does that imply at
a) the epoch of matter-radiation equality?
b) the Planck time, assuming no inflation?
In both-cases, late-time domination by dark energy may be neglected.

Table 1: Supernova Type Ia data

z	m_B
0.022	15.51
0.041	16.93
0.057	17.68
0.083	18.64
0.11	19.11
0.35	21.94
0.42	22.41
0.57	23.19
0.83	23.94
0.90	24.27
0.98	24.49
1.10	24.62
1.30	25.12
1.50	25.67
1.90	26.15

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