Galaxies MN1

Exercises 2004

1. *Integrated photometry.* Two stars in a close binary system have $m_{\rm B} = 18.2$ and $m_{\rm B} = 19.6$, respectively. The first star has a colour $B - V = -0.2$ and the second $B - V = 0.5$. If this system is observed in a telescope which cannot resolve the two components, what would the integrated $m_{\rm B}$ and $(B - V)$ of this object be?

2. *SFR and IMF.* The star formation rate (SFR) describes the rate per unit volume at which gas mass is converted into stars. The present value for the SFR within the Galactic disk is believed to be $5.0 \pm 0.5\ M_{\odot}\ pc^{-2}\ Gyr^{-1}$. Estimate the number of stars currently being formed each year in the Milky Way under the assumption of a Salpeter initial mass function (IMF) in the interval $0.1 - 120\ M_{\odot}$.

3. *Population synthesis.* Do a simple population synthesis model using the table of stellar parameters below. Assume that the stellar population of your synthetic galaxy only consists of three types of stars (O5, A0 and M0) and that the relative number of stars of each type is given by the Salpeter IMF.

a) Assuming that all stars are still on the main sequence, what is the (B-V) colour of this population? What is the M/L_V ratio?

b) Assuming that the population has aged sufficiently for all the O stars to die (and no longer contribute to the light emitted), what is the (B-V) colour and the M/L_V ratio (where M is defined as $M = M_{\rm stars} + M_{\rm gas} + M_{\rm remnants}$)?

Table 1: Stellar parameters

Stellar type	Mass ($M_{\odot}$)	Luminosity in V ($L_{\odot,V}$)	(B-V)	Main sequence lifetime (yr)
O5	40	$2.5 \cdot 10^5$	-0.35	$1.6 \cdot 10^6$
A0	4	80	0.00	$5.0 \cdot 10^8$
M0	0.5	0.06	1.45	$7.9 \cdot 10^{10}$

4. *Surface brightness I.* A flat, two-dimensional disk galaxy has an exponentially declining surface brightness distribution given by $I(r) = I_0 e^{-\frac{r}{a}}$, where a represents the scale length. Derive an approximate expression for the total luminosity of the disk.

5. *Surface brightness II.* A galaxy with the same surface brightness distribution as in exercise 4 has a scale length of 5 kpc and a rest-frame B-band luminosity of $L_B = 10^{10} L_{\odot,\rm B}$.

a) Derive an expression for r as a function of a, $I(r)$ (in units of $L_{\odot}/{\rm pc}^2$) and L.

b) Derive an expression which converts surface brightness in units of (mag/arcsec^2) to ($L_{\odot}/{\rm pc}^2$) and show that surface brightness is independent of distance (as long as redshift dimming is neglected).

c) Calculate the Holmberg radius (in kpc) of this galaxy.

6. *Rotation curves.* The spiral galaxy NGC 157 has a radial velocity of 1759 km/s. Use the rotation curve in Figure 1 to estimate a lower limit to the mass of this system.

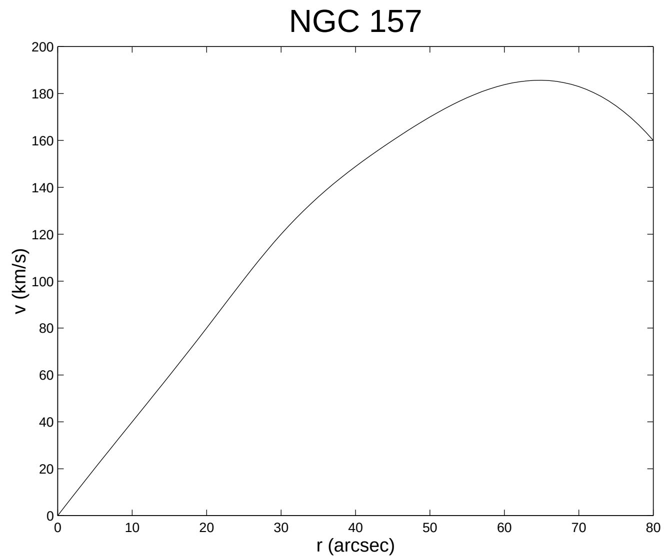

Figure 1: The rotation curve of NGC 157.

7. *Dark matter halos I.* Dark matter halos are often assumed to be cored isothermal spheres, with density profiles given by:

$$\rho(R) = \frac{\rho_{\text{core}}}{1 + (R/R_{\text{core}})^2}, \tag{1}$$

where ρ_{core} is the core density and R_{core} the core radius. Derive an expression for the rotation curve of a disk (of negligible mass) situated in this dark halo. What velocity does the relation approach at large radii?

8. *Dark matter halos II.* The density profile of the dark halo of the Milky Way may be described by:

$$\rho(R) = \rho_0 \frac{R_c^2 + R_0^2}{R_c^2 + r^2}. \tag{2}$$

Here $\rho_0 = 0.0079\ M_\odot \text{pc}^{-3}$ is the local dark matter density, $R_c \approx 5$ kpc and $R_0 \approx 8.5$ kpc. Assuming the mass of the dark halo to be $2 \times 10^{12}\ M_\odot$, how large is the radius of the dark halo compared to that of the stellar disk of the Milky Way?

9. *The Tully-Fisher relation.* An edge-on disk galaxy with an HI profile given by Figure 2 is observed to have an H-band flux of $m_{\text{H}} = 15.1$. Use the Tully-Fisher relation (5.5) in Sparke & Gallagher to calculate the distance to this object, under the assumption of negligible k-correction and interstellar extinction. What is the corresponding distance derived from Hubble's law? You may find it useful to know that $M_{H\odot} = 3.48$.

10. *Galaxy classification.* Fig. 3 displays the optical images of four galaxies with absolute magnitudes $M_B = -20.8$, -12.2, -20.1, -18.5 (from left to right). What Hubble types would you assign to these objects?

11. *Metallicities.* The galaxy ESO 114-G07 is a low surface brightness galaxy. Observations of one of its HII regions give the emission line fluxes listed in Table 2. Theory predicts a ratio $F(\text{H}\alpha)/F(\text{H}\beta) \approx 2.85$ at densities and temperatures typical of HII regions. The extinction due to dust at wavelength λ is given by $I_\lambda = I_{\lambda 0} e^{-\tau_\lambda}$, where τ_λ represents the optical depth and may

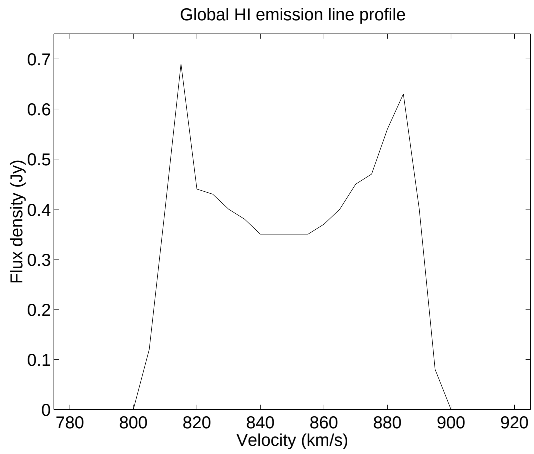

Figure 2: The global HI profile of an edge-on disk galaxy.

Figure 3: The optical images of four nearby galaxies.

be approximated by $\tau_\lambda = Cf(\lambda)$. Values of $f(\lambda)$ for a standard extinction curve can be found in Table 3.

a) Calculate the constant C using the data and the predicted $F(\mathrm{H}\alpha)/F(\mathrm{H}\beta)$ ratio.

b) Correct the observed emission line ratios for extinction.

c) Use the empirical $\log(\mathrm{R}_{23})$ versus (O/H) abundance relation (Figure 4) to get an estimate (with error bars) of the oxygen abundance in this HII region, assuming that the object is located on the so-called lower branch $(12 + log(O/H) \leq 8.3)$, as may be inferred from e.g. an analysis of the OII/NII line ratio.

Table 2: Observed emission line fluxes

λ (Å)	element	Flux ($\mathrm{F(H_\beta)}$=100)
3727	[OII]	127
4959	[OIII]	178
5007	[OIII]	531
4861	Hβ	100
6563	Hα	307

12. *Supermassive black holes.* A research group claims to have detected a star orbiting a Supermassive black hole (SMBH) at a distance of only 10 light minutes. Show that this is inconsistent with the previously estimated mass of this SMBH: $M_{\mathrm{SMBH}} = 2 \pm 1 \times 10^8\ M_\odot$.

Table 3: Standard interstellar extinction curve

λ (Å)	f(λ)-f(Hβ)
20000	-1.02
12500	-0.86
10000	-0.72
8333	-0.56
7143	-0.43
6563	-0.35
6250	-0.29
5556	-0.16
5000	-0.04
4861	-0.00
4545	0.09
4167	0.20
4000	0.25
3846	0.29
3727	0.33
3571	0.39

13. *The luminosity function of galaxies I.* Estimate $\Omega_{\rm galaxies}$, the contribution from galaxies to the cosmological matter density, assuming all galaxies to have a mass-to-luminosity ratio $M/L_{\rm R} = 20$ and to follow the follow a Schechter function with $L_{\rm R\star} = 8 \times 10^9 h^{-2}\ L_{\rm R\odot}$, $n_\star = 0.019h^3$ Mpc^{-3} and $\alpha = -0.7$.

14. *The luminosity function of galaxies II.* You aim your telescope at a galaxy cluster at a luminosity distance of 350 Mpc and discover 328 galaxies brighter than an apparent R magnitude of $m_R = 20$ (the detection threshold). Assuming the luminosity function given in exercise 13 to be valid in the range 0.01–10 $L_{\rm R\star}$ for the cluster galaxies, how many do you expect to lie faintward of the detection threshold in this system? What fraction of the total mass of galaxies in this cluster do the undetected galaxies represent, assuming constant M/L for all galaxies? You may neglect the effects of interstellar reddening and cosmological k-correction. Numerical integration (e.g. in Matlab) may be necessary.

15. *Starbursts.* A local irregular galaxy with a gas mass of $3.8 \times 10^9\ M_\odot$ exhibits very strong Hα-emission, $L_{\rm H\alpha} = 6.0 \times 10^{42}$ erg/s, indicating ferocious star formation. Assuming a standard Salpeter IMF, the current star formation rate may be derived using:

$$\mathrm{SFR}(M_\odot\ \mathrm{yr}^{-1}) = 7.9 \times 10^{-42} L_{\rm H\alpha}(\mathrm{erg\ s}^{-1}). \quad (3)$$

For how long can this starburst be sustained, assuming:
a) the SFR to be constant over time?
b) the SFR to decrease with time, t, according to SFR(t) $\propto \exp(-t/\tau)$, where the e-folding decay rate is $\tau = 100$ Myr?

16. *Taxonomy of active galactic nuclei (AGN).* In Fig. 5, the optical images of four active galaxies (A, B, C and D) are displayed, and in Fig. 6 their optical spectra. The absolute V-magnitudes of these objects are $M_{\rm V} = -21.8$ (A), -24.6 (B), -22.1 (C) and -21.2 (D) respectively. How would you classify these AGN?

17. *Quasar lifetimes.* A supermassive black hole of mass $4 \times 10^8\ M_\odot$ is situated in a host galaxy with a total gas mass of $1 \times 10^9\ M_\odot$ and is radiating at the Eddington luminosity. What is the estimated lifetime of this quasar?

18. *The spectral energy distribution of quasars I.* The electromagnetic spectrum of a quasar may in the UV–optical region be approximated by a power-law $L_\nu \propto \nu^{-\alpha}$ where L_ν is given in units of erg/s/Hz. Derive an expression for the luminosity emitted between frequencies ν_2 and ν_3 in terms of α, ν_2, ν_3 and L_{ν_1} at some frequency ν_1.

19. *The spectral energy distribution of quasars II.* A quasar at $z = 2$ is observed to have a luminosity of $L_\lambda = 3 \times 10^{43}$ erg/s/nm at $\lambda = 450$ nm in the observer's frame. What total luminosity emitted inside the quasar *rest frame* wavelength range 360–520 nm (the B-filter) does that correspond to (this would be the luminosity in the k-corrected B-filter) if $\alpha = 0.5$ is assumed?

20. *Superluminal motion.* In certain active galaxies, changes in the position of substructures may be detected over intervals as short as a year. For the quasar 3C 273 at a redshift of z=0.158, one component is seen to move away from the nucleus with an apparent angular velocity of 2.2×10^{-3} arcseconds/yr.

a) What is the corresponding apparent, projected linear velocity of this component?

b) Assuming that the phenomenon of superluminal motion is due to matter moving with a relativistic velocity v close to the line of sight, and that Θ is the angle between the line of sight and the direction of ejected mater, what value of Θ would produce the largest apparent velocity of the ejected component?

c) What is the minimum matter ejection velocity required in order to produce the apparent superluminal speed observed in a) ?

21. *Masses of galaxy clusters I.* The Coma cluster has a radial velocity of 6750 km/s, an effective radius of 50' and a 3-dimensional velocity dispersion of 900 km/s. Use the virial theorem $M = \frac{<v^2>R_{\rm grav}}{G}$ to estimate the total mass of the galaxy cluster, assuming the gravitational radius to be 2.5 times larger than the effective radius.

22. *Masses of galaxy clusters II.* Estimate the mass of the ionized gas in the Coma cluster described in exercise 21, using the information that the central gas number density (atomic nuclei per volume) is 2×10^{-3} cm^{-3} and assuming M/L as a function of radius to be constant. The density distribution of ionized gas may be approximated by the relation: $\rho = \rho_0[1 + (\frac{r}{a})^2]^{-\frac{3}{2}}$, where $a = 0.5$ Mpc.

23. *Gravitational lensing I.* A faint galaxy is observed at 1' from the centre of a galaxy cluster. The redshift of this object indicates that it is a distant background object, gravitationally lensed by the mass of the foreground cluster. The cluster is located at an angular size distance of 200 Mpc and the galaxy at 500 Mpc. Assuming the cluster mass to be spherically symmetrically distributed, and that the mass inside 1' from the centre amounts to $10^{14}\ M_\odot$, at what angle from the centre of the cluster would the galaxy be observed in the absence of the gravitational bending of light?

24. *Gravitational lensing II.* The faint galaxy in exercise 23 is observed to have a redshift of $z = 0.165$ and an apparent B-band magnitude of $m_B = 20$. Estimate its absolute B-magnitude, correcting for the gravitational magnification of the foreground galaxy cluster, but neglecting the effects of dust extinction and k-correction. The flux magnification factor A may be calculated using:

$$A = \frac{1 + x^2/2}{x\sqrt{1 + x^2/4}} \tag{4}$$

where $x = \beta/\theta$ and the angles are defined as in Fig. 2.22 in Sparke & Gallagher.

25. *Cosmic star formation.* The Madau diagram (Figure 7), which describes the comoving cosmic UV luminosity density as a function of redshift, may also be converted into a diagram of cosmic SFR as a function of time. By assuming a Salpeter IMF in the mass range $0.1 - 125\ M_\odot$, models of galaxy evolution predict the relation between 0.28 μm UV luminosity and SFR to be:

$$L_{UV} = 7.9 \cdot 10^{27} \times \frac{SFR}{M_\odot yr^{-1}}\ \mathrm{erg\ s^{-1}Hz^{-1}}. \tag{5}$$

If this relation is assumed to be valid over the entire redshift range plotted, then the cosmic SFR appears to drop at $z > 1.2$. Assuming that the luminosity in the UV filter is linearly proportional to the mass fraction in $10 - 125\ M_\odot$ stars, what would happen to the shape of the cosmic SFR curve if the IMF exponent α ($dN/dM \propto M^{-\alpha}$, where $\alpha = 2.35$ represents the Salpeter IMF) gradually started to increase with redshift at $z > 1.2$ and reached $\alpha = 2.75$ at $z = 4$?

26. *Galaxy formation I.* Assume that protogalaxies can form once

$$\bar{\rho} \geq 200\rho_{\mathrm{M}}(z_{\mathrm{f}}), \tag{6}$$

where $\bar{\rho}$ is the density averaged over a spherical volume delimited by radius r and $\rho_{\mathrm{M}}(z_{\mathrm{f}})$ is the matter density of the universe at the formation redshift z_{f}. Determine the highest redshifts at which the protogalaxies with radii of $r = 10$, 30 and 100 kpc may have assembled, assuming their circular velocities to be 200 km/s.

27. *Galaxy formation II.* Assume that we live in an Einstein-de Sitter universe and that the most distant quasars observed (z$\sim$5) represent young galaxies at the end of the collapse phase. Estimate the redshift, the approximate age of the universe and the mean density of the clouds when the contraction started, assuming the collapse time to be roughly equal to the time from the Big Bang to the beginning of contraction.

Preliminary schedule for exercise classes

Remember that it will be much easier to understand the solutions presented during the exercise sessions if you have already attempted to solve the problems yourself.

Exercise session 1 - Problems 1, 2, 4, 6, 9, 10
Exercise session 2 - Problems 11, 12, 13, 16, 18
Exercise session 3 - Problems 15, 17, 21, 23, 25

Hand-in exercises

Your final grade on this course will partly be based partly on your solutions to these.

Deadline May 5 - Problems 3, 5, 7
Deadline May 19 - Problems 14, 19, 20
Deadline June 4 - Problems 22, 24, 26

Erik Zackrisson, March 2004

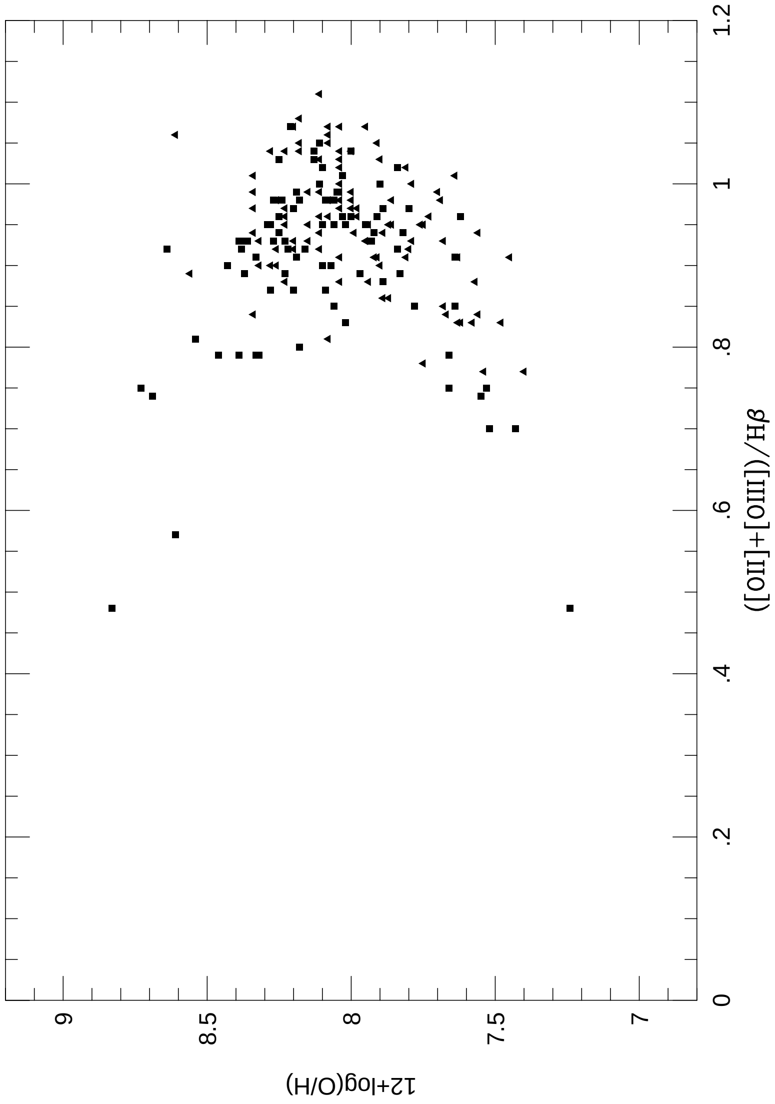

Figure 4: The empirical $\log R_{23} = \log(([OII]\lambda3727 + [OIII]\lambda\lambda4959,5007)/H\beta)$ versus oxygen abundance relation, adopted from Olofsson 1997, Astronomy & Astrophysics 321, 29. The different markers represent the location of model star forming regions of different ages and chemical composition. In this notation, 8.93 represents the solar 12+log(O/H) value.

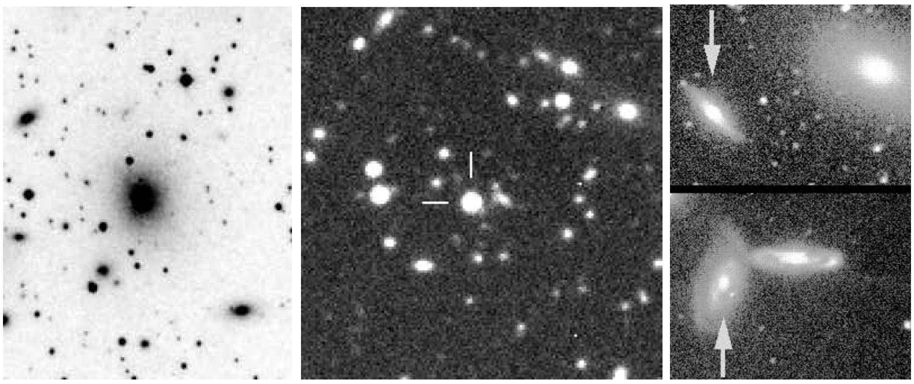

Figure 5: The optical images of four active galaxies: A (left), B (centre), C (upper right) and D (lower right)

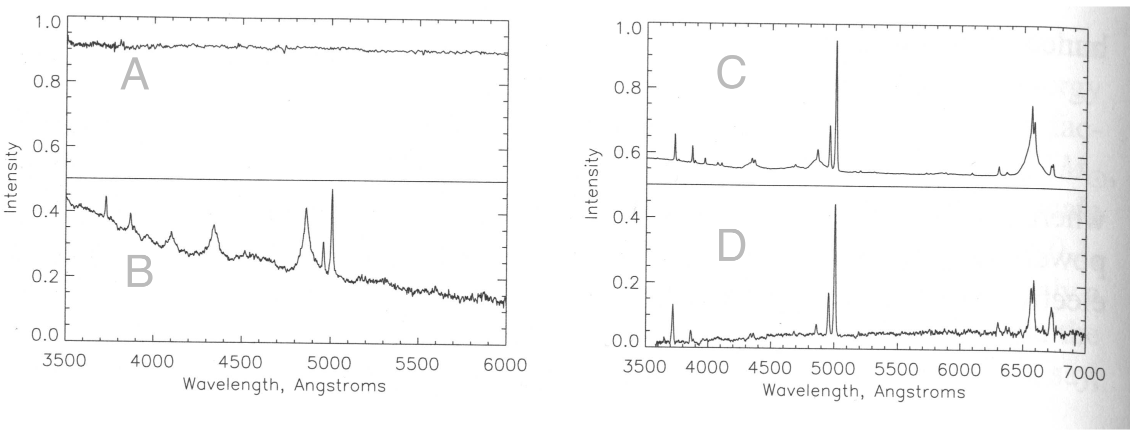

Figure 6: The optical spectra of the four active galaxies depicted in Fig. 5.

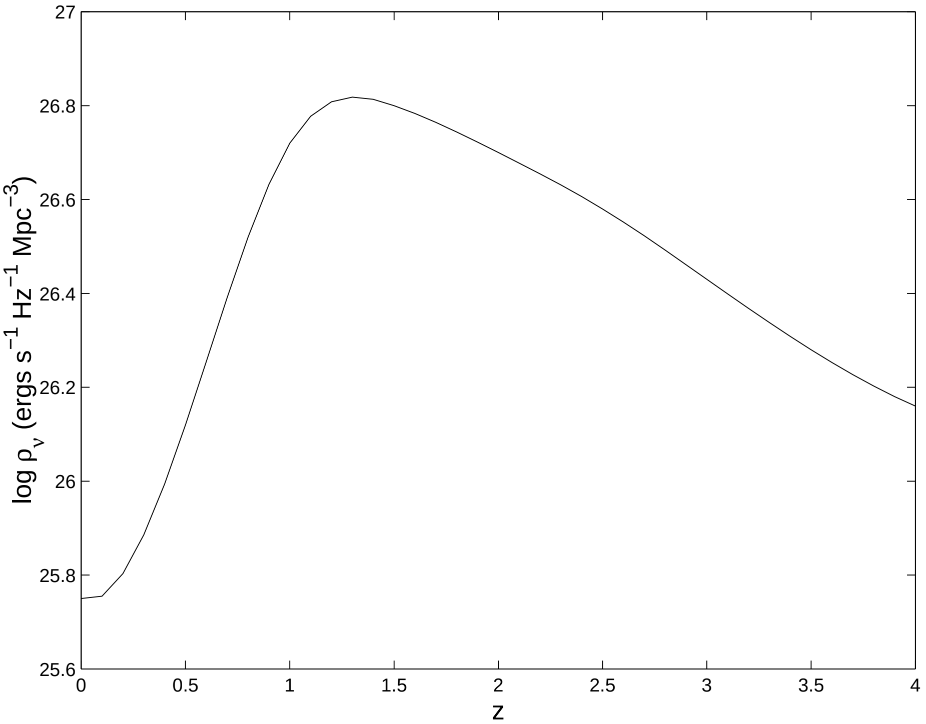

Figure 7: Evolution of the comoving cosmic UV luminosity density at a rest-frame wavelength of 0.28μm.