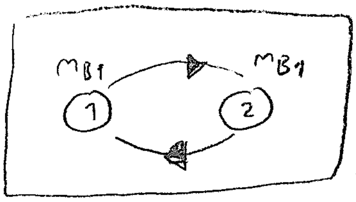
Integrated photometry

1 see opposite page



Could be much more complicated system - like a galaxy with 10^{10} stars...

Integrate flux inside rectangle

What is the integrated magnitude and colour?

Will look like single object at large distance

$$L_{B_{tot}} = L_{B_1} + L_{B_2} \qquad (1)$$

(Note! You can not add magnitudes!)

Have to derive L_{B_1} and L_{B_2}!

$$L_B = 10^{\left(\frac{M_B - M_{B\odot}}{-2.5}\right)} L_{B\odot} \qquad (2)$$

$$M_B = m_B - 5\log d \qquad (3)$$ 1.14 in S&6

in units of 10pc

(3) in (2) ⇒

$$L_B = 10^{\left(\frac{m_B - 5\log d - M_{B\odot}}{-2.5}\right)} L_{B\odot} \qquad (4)$$

(4) in (1) ⇒

$$L_{B_{tot}} = \left(10^{\left(\frac{m_{B1}}{-2.5}\right)} + 10^{\left(\frac{m_{B2}}{-2.5}\right)}\right) 10^{\frac{-5\log d - M_{B\odot}}{-2.5}} L_{B\odot} \qquad (5)$$

Calculation of total apparent magnitude

Inverse of (4) ⇒

$$m_{B_{tot}} = -2.5\log\left(\frac{L_{B_{tot}}}{L_{B\odot}}\right) + M_{B\odot} + 5\log d \qquad (6)$$

(5) in (6) ⇒

$$\boxed{m_{B_{tot}} = -2.5\log\left(10^{\frac{m_{B1}}{-2.5}} + 10^{\frac{m_{B2}}{-2.5}}\right)} \qquad (7)$$

· Same for $m_{V_{tot}}$ ...

$$(B-V) = m_B - m_V \Rightarrow \qquad (8)$$
$$m_V = m_B - (B-V)$$

Numerically:

$m_{B_1} = 18.2 \qquad m_{B_2} = 19.6$

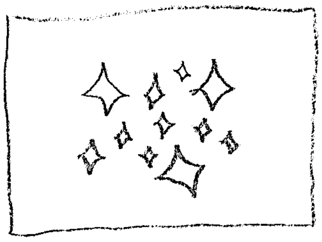
$(B-V)_1 = -0.2 \qquad (B-V)_2 = 0.5$

(8) ⇒ $m_{V_1} = 18.4 \qquad m_{V_2} = 19.1$

(7) ⇒ $m_{B_{tot}} = 17.9359$, $m_{V_{tot}} = 17.9420$ } $(B-V)_{tot} = -0.0061 \approx 0.0$

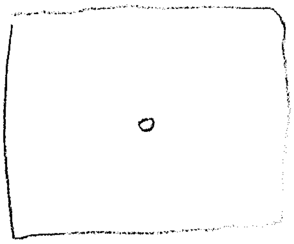
The brightest star (1) dominates the colour!

Nearby system



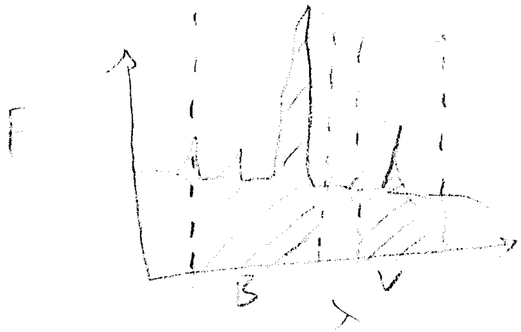
Individual stars resolved

Distant system



Stars unresolved

Only integrated properties observable



2 SFR and IMF

Total SFR in the MW:

$$\dot{M}_{area} \cdot A = \dot{M}_{MW} \qquad (1)$$

SFR/area ↗ $\dot{M}_{area}$ SFR ↗ $\dot{M}_{MW}$

$$A = \pi r^2 \qquad (2)$$

(1) in (2) $\Rightarrow$

$$\dot{M}_{MW} = \pi r^2 \dot{M}_{area} \qquad (3)$$

Number of stars formed each year in the MW:

Definition of IMF:

$$\xi = \int_{M_L}^{M_u} \xi_0 M^{-\alpha} \, dM \qquad (\alpha = 2.35 \Rightarrow \text{Salpeter}) \qquad (4)$$

ξ: Xi, number of stars

ξ_0: Normalization constant

Normalize to $\dot{M}_{MW}$ in order to find ξ_0:

$$\dot{M}_{MW} = \int_{\text{all stars}} M \cdot d\xi \ \text{ per year} = \xi_0 \int_{M_L}^{M_u} M^{-\alpha+1} \, dM \qquad (5)$$

(3) in (5) $\Rightarrow$

$$\xi_0 = \frac{\pi r^2 \dot{M}_{area}}{\int_{M_L}^{M_u} M^{-\alpha+1} \, dM} \qquad (6)$$

Finally, using (4) and (6):

$$\boxed{\xi \text{ per year} = \frac{\pi r^2 \dot{M}_{area}}{\int_{M_L}^{M_u} M^{-\alpha+1} \, dM} \cdot \int_{M_L}^{M_u} M^{-\alpha} \, dM}$$

Numerically:

$$\xi \text{ per year} = \frac{\pi (15 \cdot 10^3 \, pc)^2 \cdot 5.0 \cdot 10^{-9} \, M_\odot pc^{-2} yr^{-1}}{\left[\frac{M^{-0.35}}{-0.35}\right]_{0.1 M_\odot}^{120 M_\odot}} \left[\frac{M^{-1.35}}{-1.35}\right]_{0.1 M_\odot}^{120 M_\odot} \approx 10 \text{ stars/yr}$$

$\dot{M}_{MW} = 3.53 \, M_\odot/yr$

$\xi_0 \approx 0.6$

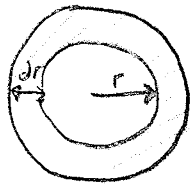
$$\left(\begin{array}{l} \dot{M}_{area} = 4.5 \, M_\odot pc^{-2} \, Gyr^{-1} \Rightarrow 9 \text{ stars/yr} \\ \phantom{\dot{M}_{area} = } 5.5 \quad \text{— '' —} \Rightarrow 11 \text{ stars/yr} \end{array} \right)$$

Answer: 10 ± 1 stars/yr

$\boxed{4}$ Surface brightness I

Surface brightness = Luminosity per surface area (e.g. $L_\odot/pc^2$)

$$I(r) = I_0 e^{-r/a} \qquad (1)$$



Luminosity from within the shaded area

$$L = \int \underbrace{\pi 2 r}_{\text{circumference}} I(r)\, dr \qquad (2) \qquad *$$

Integrate over whole disc to get total luminosity!

(1) in (2):

$$L = 2\pi I_0 \int_0^\infty r e^{-r/a} = \left[\begin{matrix} \text{Math. Handbook} \\ \text{p. 171, F 320} \end{matrix} \right] =$$

$$= 2\pi I_0 \left[a^2 e^{-r/a} \left(-\frac{r}{a} - 1 \right) \right]_0^\infty = 2\pi I_0 (0 + a^2) =$$

$$= 2\pi I_0 a^2 \qquad \square$$

Why integrate to $r = \infty$?

$e^{-r/a}$ drops so quickly that integrating to some large r or ∞ doesn't make that much of a difference.

However, if the scale length is large, this expression could give an unreasonably high luminosity.

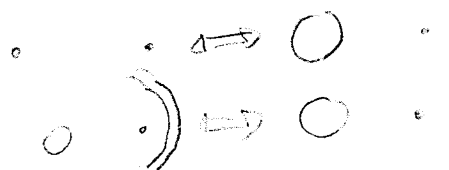
$*$ Alternatively:

$r\,d\phi$ since $\frac{d\phi}{2\pi} \cdot \pi \cdot 2r = r\,d\phi$

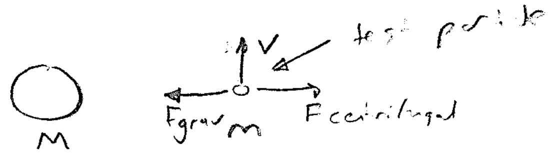
$$\Rightarrow L = \int_0^\infty I\, ds = I_0 \int_0^{2\pi} d\phi \int_0^\infty r e^{-r/a} dr =$$

$$= 2\pi I_0 \int_0^\infty r e^{-r/a} dr$$

6 Rotation curves



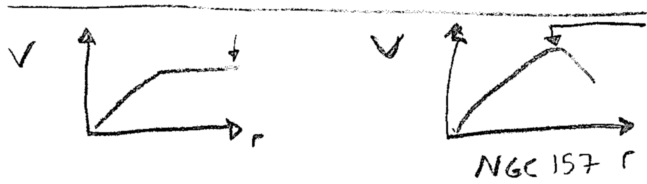
Show theorems, p 44 in S&G



$F_{grav} = F_{centrifugal} \Rightarrow$ * (Note! This is a simplistic derivation. See S&G for a more rigorous one)

$$\frac{GmM}{r^2} = \frac{mv^2}{r} \Rightarrow \boxed{M(<r) = \frac{V^2(r)\, r}{G} \quad (1)}$$ (218, 320 in S&G)

What values of V(r) and r should be used?



good bet, because $M \propto v^2$ (v more important than r)

(Should really try to find (r, V) that maximizes $M(<r)$, but difficult without a diagram is available)

Note! Depends on what cause of drop is - Newtonian or due to measurement uncertainty / non-circular motion etc

Conversion from r_{arcsec} to r_{lin}:

$$V_{(r)} = H_0 d \Rightarrow d = \frac{V_{rad}}{H_0} \quad (2)$$

$$r_{lin} = \frac{r_{arcsec}\, 2\pi \cdot d}{3600 \cdot 360} \quad (3)$$

$\alpha = \frac{D}{d}$ - diff in velocity, d = distance

(2) in (3) $\Rightarrow$

$$\boxed{r_{lin} = \frac{r_{arcsec} \cdot 2\pi \cdot V_{rad}}{3600 \cdot 360 \cdot H_0}} \quad (4)$$

Numerically:

$H_0 = 65$ km/s/Mpc

$r_{V_{max}} = 65''$, $V_{max} = 190$ km/s

$V_{rad} = 1759$ km/s

$$(4) \Rightarrow r_{lin} = \frac{65 \cdot 2\pi \cdot 1759}{3600 \cdot 360 \cdot 65} \cdot 10^6 \text{ pc} = 8.528 \text{ kpc}$$

$$(1) \Rightarrow M(<r_{V_{max}}) = \frac{(190 \cdot 10^3)^2 \cdot 8.528 \cdot 10^3 \cdot 3.0857 \cdot 10^{16} \text{ m/pc}}{6.6726 \cdot 10^{-11}} = 1.424 \cdot 10^{41} \text{ kg}$$

$$= 7.155 \cdot 10^{10}\, M_\odot$$

Answer: $7.2 \cdot 10^{10}\, M_\odot$

* Alt: Kepler III $p^2 = \frac{4\pi r^3}{GM}$, $p = \frac{2\pi r}{V_{rot}} \Rightarrow M = \frac{r V_{rot}^2}{G}$

Numerically

$w = 65\ km/s$

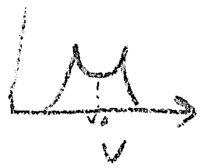
$m_H = 15.1$

$M_{H\odot} = 3.48$

$i = 90°$ (angle from face-on)

$\Rightarrow\ d_{TF} = 1.2019 \cdot 10^7\ pc \approx 17.0\ Mpc$

$v_0 = 850 \cdot 10^3\ m/s$



$H_0 = 72\ km/s/Mpc$

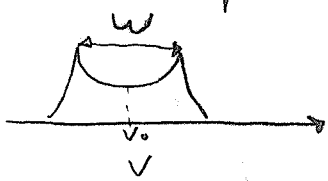
$\Rightarrow\ d_{Hubble} \approx 3.64 \cdot 10^{23}\ m \approx 11.8\ Mpc$

9 The Tully Fisher relation

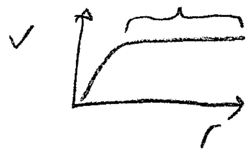
(Line width - lum relation (or mass))

Emission Line profile:

Fig 7:



Peaks come from:



$\Rightarrow$ w measure of mass of galaxy

$$\frac{L_H}{3\cdot 10^{10} L_{H\odot}} \approx \left(\frac{V_{max}}{196\ \text{km/s}}\right)^{3.8} \qquad (1,\ 5.5 \text{ in S\&G})$$

$$V_{max} \approx \frac{w}{2\sin i} \qquad (2,\ \text{p. 201 in S\&G})$$

So, we know the luminosity and the magnitude - how do we get the distance?

$$M = m - 5\log d_{pc} + 5 \qquad (3)$$

$$L_H = 10^{\left(\frac{M_H - M_{H\odot}}{-2.5}\right)} L_{H\odot} \qquad (4)$$

(3) in (4) $\Rightarrow$

$$L_H = 10^{\left(\frac{m_H - 5\log d - M_{H\odot} + 5}{-2.5}\right)} L_{H\odot} \qquad (5)$$

(2) in (1) $\Rightarrow$

$$L_H \approx 3\cdot 10^{10} L_{H\odot} \left(\frac{w}{2\cdot 196\cdot \sin i}\right)^{3.8} \qquad (6)$$

(5) = (6) after rearranging:

$$d_{TF} = 10^{\left(\frac{-2.5\log\left[3\cdot 10^{10}\left(\frac{w}{2\cdot 196\cdot \sin i}\right)^{3.8}\right] - m_H + M_{H\odot} - 5}{-5}\right)} \qquad (7)$$

Distance according to Hubbles law:

$$d_{Hubble} \approx \frac{v_0}{H_0} \qquad (8)$$

10 Galaxy classification

	a	b	c	d
	$M_B = -20.8$	$M_B = -17.2$	$M_B = -20.1$	$M_B = -18.5$
Rough classification:	E	E	S	SB
Hubble type:	see below	see below	Sc	SBc
Luminosity (Show fig 4.18, p. 166 in S&G)	Normal	Dwarf	Normal	Normal (Almost dwarf)
Conclusion:	E	dE	Sc	SBc

Hubble type, a:

a = 4 mm
b = 2.5 mm

$$10\left(1-\frac{b}{a}\right) = 3.8$$

→ E4

(literature says E3–E5 on M59)

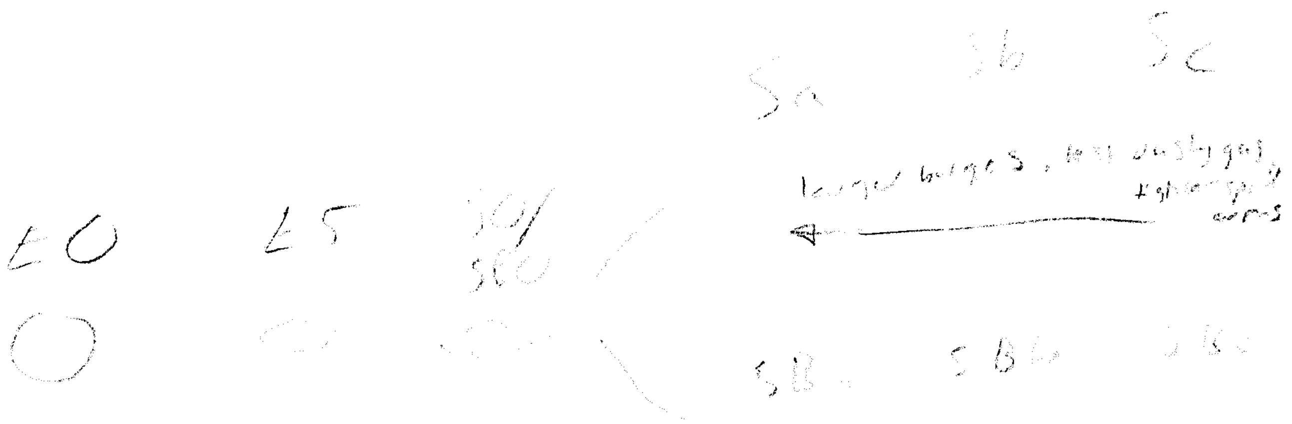
Hubble type, b:

a = 10 mm
b = 7 mm

$$10\left(1-\frac{b}{a}\right) = 3$$

→ E3

(literature says E2 on M32)


E0
E5
S0/
SBO
Sa
Sb
Sc
larger bulges, less dust/gas,
tightly wound
arms
SBa
SBb
SBc