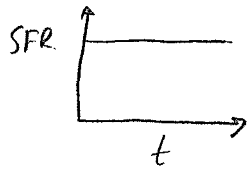
15 Starbursts

Now
$$\downarrow$$
$$SFR(0) = 7.9 \cdot 10^{-42} L_{H\alpha} \qquad (1)$$

$$\int_0^{t_{stop}} SFR(t)\, dt = M_{gas} \qquad (2)$$

a) 

$SFR = SFR(0) \Rightarrow$ (2) becomes

$$SFR(0) \cdot t_{stop} = M_{gas} \Rightarrow$$

$$t_{stop} = \frac{M_{gas}}{SFR(0)} \qquad (3)$$

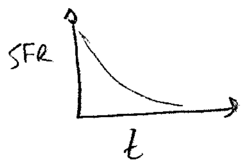
(1) in (3) $\Rightarrow$

$$t_{stop} = \frac{M_{gas}}{7.9 \cdot 10^{-42} L_{H\alpha}} \qquad (4)$$

Numerically:

$$t_{stop} = \frac{3.8 \cdot 10^{9}\, M_\odot}{7.9 \cdot 10^{-42} \cdot 6 \cdot 10^{42}\ M_\odot/yr} \approx 8.0 \cdot 10^{7}\, yr$$

__Answer:__ 80 Myr (Means: Starburst phenomenon must be transient!)

b) 

$$SFR(t) \propto \exp\left(-\frac{t}{\tau}\right) \Rightarrow$$

(2) becomes

$$\int_0^{t_{stop}} SFR(0) \cdot \exp\left(-\frac{t}{\tau}\right) = M_{gas} \Rightarrow \quad SFR(0)\left[-\tau \exp\left(-\frac{t}{\tau}\right)\right]_0^{t_{stop}} = M_{gas} \Rightarrow$$

$$SFR(0)\, \tau \left(1 - \exp\left(-\frac{t_{stop}}{\tau}\right)\right) = M_{gas} \Rightarrow$$

$$t_{stop} = -\tau \ln\left(1 - \frac{M_{gas}}{SFR(0)\,\tau}\right) \qquad (5)$$

(1) in (5) $\Rightarrow$

$$t_{stop} = -\tau \ln\left(1 - \frac{M_{gas}}{7.9 \cdot 10^{-42} L_{H\alpha}\,\tau}\right) \qquad (6)$$

Numerically:

$t_{stop} = 1.6179 \cdot 10^{8}\,\mathrm{yr} \approx 160\,\mathrm{Myr}$

(Twice as long, but still a transient phenomenon)

17 Quasar lifetimes

Eddington luminosity:

$$L_E \approx 1.3 \cdot 10^{31} \left(\frac{M_{SMBH}}{M_\odot}\right) W \qquad (1,\ 8.4 \text{ in } S\&G)$$

$$L_E t = \eta m_{gas} c^2 \qquad (2)$$

$$t = \frac{\eta m_{gas} c^2}{L_E} \qquad (3)$$

(1) in (2) $\Rightarrow$

$$t = \frac{\eta m_{gas} c^2}{1.3 \cdot 10^{31} \left(\frac{M_{SMBH}}{M_\odot}\right)} \qquad (4)$$

! will M_{SMBH} ... increase with time if ε ?

<u>Numerically:</u>

Typical $\eta = 0.1$

$m_{gas} = 1 \cdot 10^9\, M_\odot = 1.99 \cdot 10^{39}$ kg

$M_{SMBH} = 4 \cdot 10^8\, M_\odot$

$\Rightarrow t = 3.44 \cdot 10^{15}$ s ≈ 109 Myr (typical lifetimes ≈ 100 Myr)

Problem

21 Masses of galaxy clusters

The virial theorem describes the balance between kinetic and potential energy (may be applied if the system is approximately stable)

Two-body relaxation → Maxwellian velocity distribution → arbitrarily high velocities → objects escape from the system on long time scale → final collapse

Birney & Tremaine, Galactic dynamics, p.213:

$$M_{vir} = \frac{\langle v^2 \rangle R_{grav}}{G} \quad (1) \qquad \left(R_{grav} = \frac{GM^2}{|W|} \right)$$

← potential energy

$$\sigma_v^2 = \langle v^2 \rangle - \underbrace{\langle v \rangle^2}_{\approx 0} = \langle v^2 \rangle \quad (2) \; *$$

velocity dispersion

(2) in (1) ⇒ $$M_{vir} = \frac{\sigma_v^2 R_{grav}}{G} \quad (3)$$

$$R_{grav} = 2.5 \cdot R_{eff} \quad (4)$$

$$R_{eff\,lin} = \frac{R_{eff\,arcmin} 2\pi d}{60 \cdot 360} \quad (5)$$

$$d = \frac{v_{rad}}{H_0} \quad (6)$$

(6) in (5) in (4) in (3) ⇒

$$\boxed{M_{vir} = \frac{\sigma_v^2 \cdot 2.5 \cdot R_{eff\,arcmin} \cdot 2\pi \cdot v_{rad}}{G \cdot 60 \cdot 360 \cdot H_0}} \quad (7)$$

Numerically:

$\sigma_v = 900$ km/s

$R_{eff\,arcmin} = 50'$

$v_{rad} = 6750$ km/s

$H_0 = 65$ km/s/Mpc

$$\Rightarrow M_{vir} = \frac{(900 \cdot 10^3)^2 \cdot 2.5 \cdot 50 \cdot 2\pi \cdot 6750 \cdot 10^3}{6.6720 \cdot 10^{-11} \cdot 60 \cdot 360 \cdot 10^3 / 3.0857 \cdot 10^{22}}$$

$$= 1.4145 \cdot 10^{45}\ \text{kg} = 7.1082 \cdot 10^{14} M_\odot$$

Answer: $7.1 \cdot 10^{14} M_\odot$

(* Not obvious whether σ_v or σ_r was specified in the exercise text

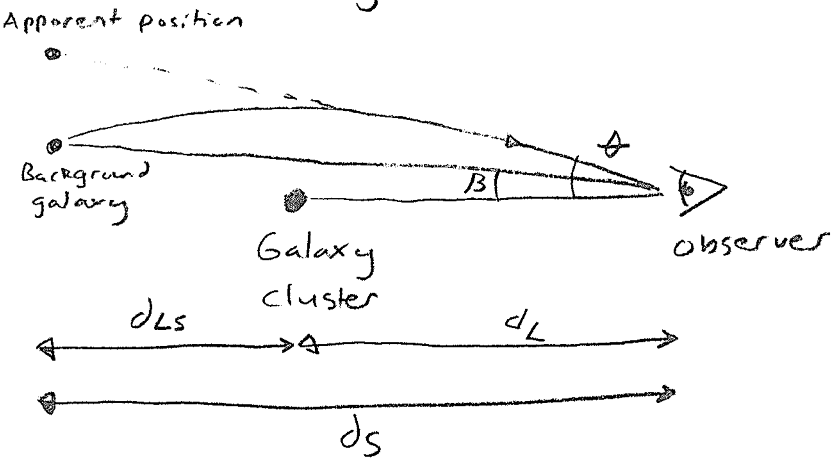
$\sigma_v^2 \approx 3\sigma_r^2$ (e.g. Nilson, 1976, Galaxernas och universum)

$\Rightarrow M_{vir} = 3\sigma_r^2 R_{grav}/G$)

Effective radius: radius inside which half of the light from the cluster originates

Core radius = radius at which the surface mass density drops to half its central value. (if M/L constant with radius)
Annars: surface brightness fallit till halva centralvärdet.

23 Gravitational lensing I



Angular size distances (≠ luminosity distances!)

$d_A = \frac{L}{\theta}$

See. Schneider, Ehlers & Falco p.26

$$\theta - \beta = \frac{4GM\, d_{LS}}{\theta c^2\, d_L\, d_S} \qquad (1,\ 6.49 \text{ in S\&G})$$

Dirty approximation

$$d_{LS} \approx d_S - d_L \qquad (2)$$

(Learn more about this in the cosmology course)

(2) in (1) after rearranging:

$$\beta = \theta - \frac{4GM\,(d_S - d_L)}{\theta c^2\, d_L\, d_S} \qquad (3)$$

Numerically:

$d_L = 200\ \mathrm{Mpc} = 6.1714 \cdot 10^{24}\ \mathrm{m}$

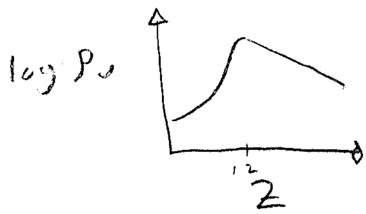
$d_S = 500\ \mathrm{Mpc} = 1.5428 \cdot 10^{25}\ \mathrm{m}$

$\theta = 1' = \frac{2\pi}{60 \cdot 360}\ \mathrm{rad} = 2.9089 \cdot 10^{-4}\ \mathrm{rad}$

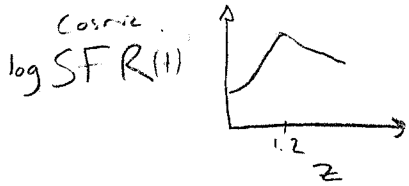
$M = 10^{14} M_\odot = 1.99 \cdot 10^{44}\ \mathrm{kg}$

(3) $\Rightarrow$ $\beta = 9.3660 \cdot 10^{-5}\ \mathrm{rad} = 19.3''$

$\boxed{25}$ Cosmic star formation



$\Longrightarrow$



if $SFR = k\,\rho_\nu$ where k does not change ~~at~~ for $z = 0.0 - 4.0$

$$L_{UV} = 7.9 \cdot 10^{27} \cdot SFR \Rightarrow$$

$$\frac{SFR}{V} = \frac{L_{UV}}{V \cdot 7.9 \cdot 10^{27}} = \frac{\rho_\nu}{7.9 \cdot 10^{27}} \Rightarrow k = \frac{1}{7.9 \cdot 10^{27}} \quad (1)$$

comoving volume

$$L_{UV} \propto f(10-125 M_\odot) = \frac{M_{10-125 M_\odot}}{M_{0.1-125 M_\odot}} \quad (2)$$

Remember:

$$M_{tot} = \int_{M_L}^{M_u} \xi_0 M^{-\alpha+1}\, dM = \xi_0 \left[\frac{M^{-\alpha+2}}{-\alpha+2} \right]_{M_L}^{M_u} \quad (3)$$

(3) in (2), with $M_U = 125 M_\odot$, $M_L = 0.1 M_\odot$

$$L_{UV} \propto \frac{125^{-\alpha+2} - 10^{-\alpha+2}}{125^{-\alpha+2} - 0.1^{-\alpha+2}} \quad (4)$$

$$\left.\begin{array}{ll} z \leq 1.2 & \alpha = 2.35 \\ z = 4 & \alpha = 2.75 \end{array}\right\} \Rightarrow$$

Want to determine

$k_z = \left(\frac{L_{UV}}{SFR}\right)_z$ of equation (1):

$$\frac{k_{z \leq 1.2}}{k_{z=4}} = \frac{\dfrac{125^{-0.35} - 10^{-0.35}}{125^{-0.35} - 0.1^{-0.35}}}{\dfrac{125^{-0.75} - 10^{-0.75}}{125^{-0.75} - 0.1^{-0.75}}} = \frac{0.1276}{0.0270} = 4.76$$

This implies that

$$\left(\frac{SFR}{V}\right)_{z\leq 1.2} = \frac{\rho_\nu}{7.9 \cdot 10^{27}}$$

but

$$\left.\begin{aligned}\left(\frac{SFR}{V}\right)_{z=4} &= \frac{4.76\,\rho_\nu}{7.9 \cdot 10^{27}}\end{aligned}\right\} \quad (5)$$

<u>Numerically:</u>

$$\rho_{\nu_{z=1.2}} = 10^{26.82} \text{ erg/s/Hz/Mpc}^3$$

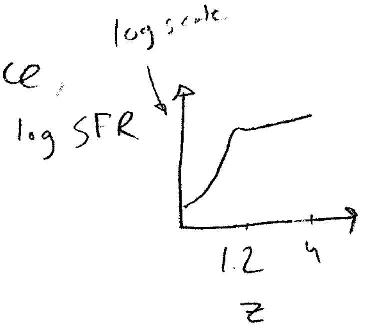
$$\rho_{\nu_{z=4}} = 10^{26.18} \text{ erg/s/Hz/Mpc}^3$$

(5) $\Rightarrow$

$$\left(\frac{SFR}{V}\right)_{z=12} = 0.0836 \ M_\odot/\text{yr}/\text{Mpc}$$

$$\left(\frac{SFR}{V}\right)_{z=4} = 0.0912 \ M_\odot/\text{yr}/\text{Mpc}$$

Hence,



The cosmic star formation rate does no longer drop at $z > 1.2$, but actually increases slightly!

The inferred cosmic SFR(t) is sensitive to assumptions about the high-z stellar IMF!