Early Universe Particle Physics

Joakim Edsjö
edsjo@physto.se

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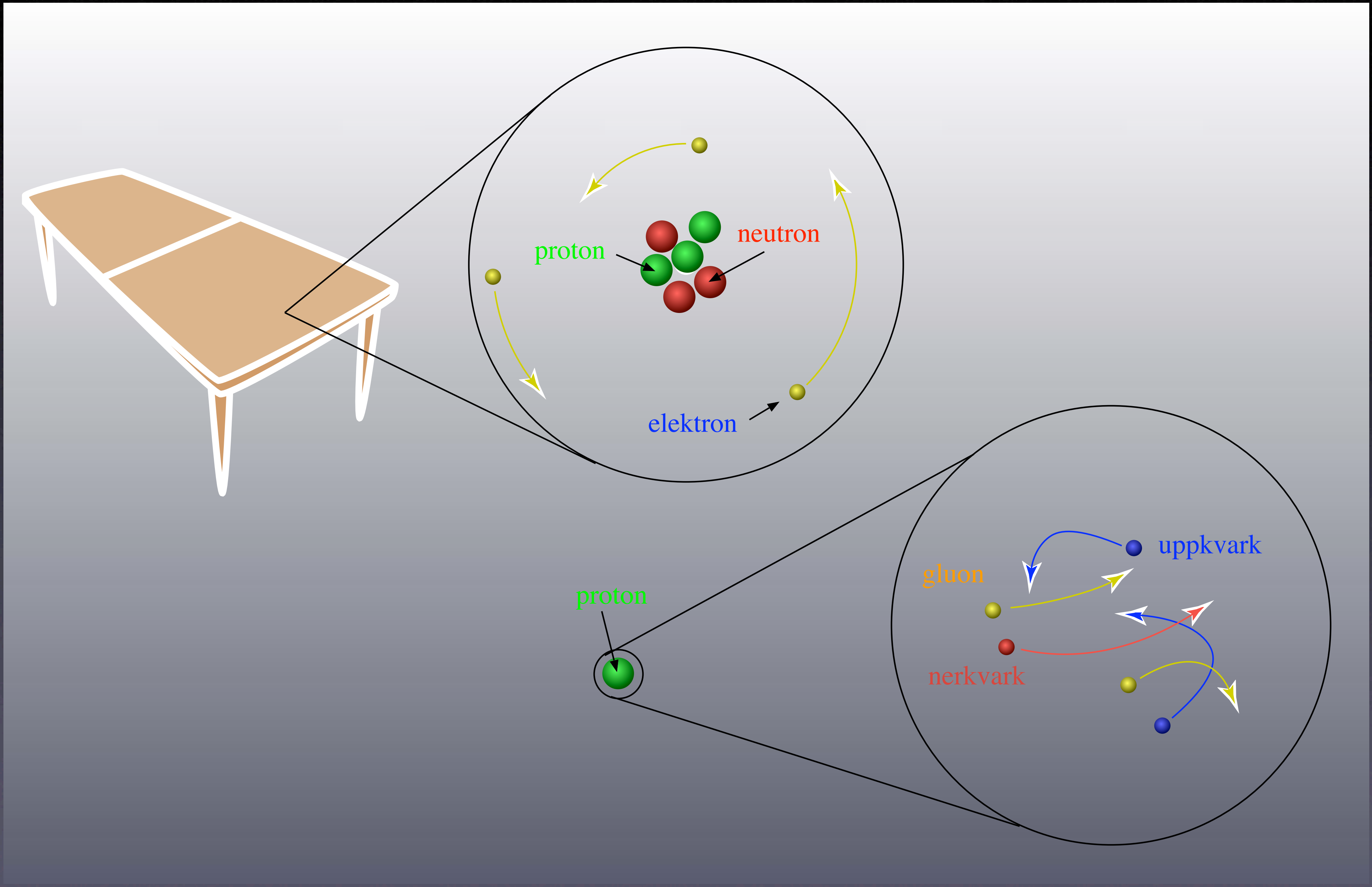
Outline

- Introduction to the *Standard Model* for particle physics
- Introduction to supersymmetry *and the Minimal Supersymmetric Standard Model (MSSM)*
- *Grand Unified Theories (GUTs).*
- Thermal relics: freeze-out

What is matter?



The Standard Model

Quarks

First family	Q=+2/3 u s=1/2	Q=-1/3 d s=1/2
Second family	Q=+2/3 c s=1/2	Q=-1/3 s s=1/2
Third family	Q=+2/3 t s=1/2	Q=-1/3 b s=1/2

The Standard Model

Leptons

First family	Q=0 ν_e s=1/2	Q=-1 e s=1/2
Second family	Q=0 ν_μ s=1/2	Q=-1 μ s=1/2
Third family	Q=0 ν_τ s=1/2	Q=-1 τ s=1/2

Quantum numbers

- In addition to conserved classical quantities like momentum and angular momentum, quantum mechanics allows for more conserved quantities, quantum numbers
- Examples are
  - spin (arising from rotational invariance)
  - electric charge (arising from gauge invariance)
  - lepton number
  - ...

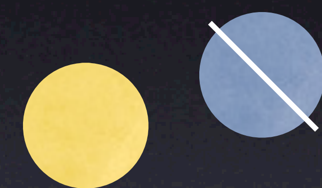
Baryons and mesons

Baryons

- three quarks
- baryon number = 1
- Examples:

$$p = uud$$
$$n = udd$$

Mesons



- one quark + one antiquark
- baryon number = 0
- Examples:

$$\pi^0 = (u\bar{u} + d\bar{d})$$
$$\pi^+ = u\bar{d}$$

What else is there?

- Quarks and leptons have spin 1/2 (in units of $h/2\pi$), and are called fermions.
- They are the building blocks of matter
- What makes them talk to each other?

Interactions

Interaction	Charge	Force carrier
Electromagnetic	Electric charge	Photon, γ
Weak	Weak isospin	$W^{\pm}$, Z^0
Strong	Colour	Gluon, g

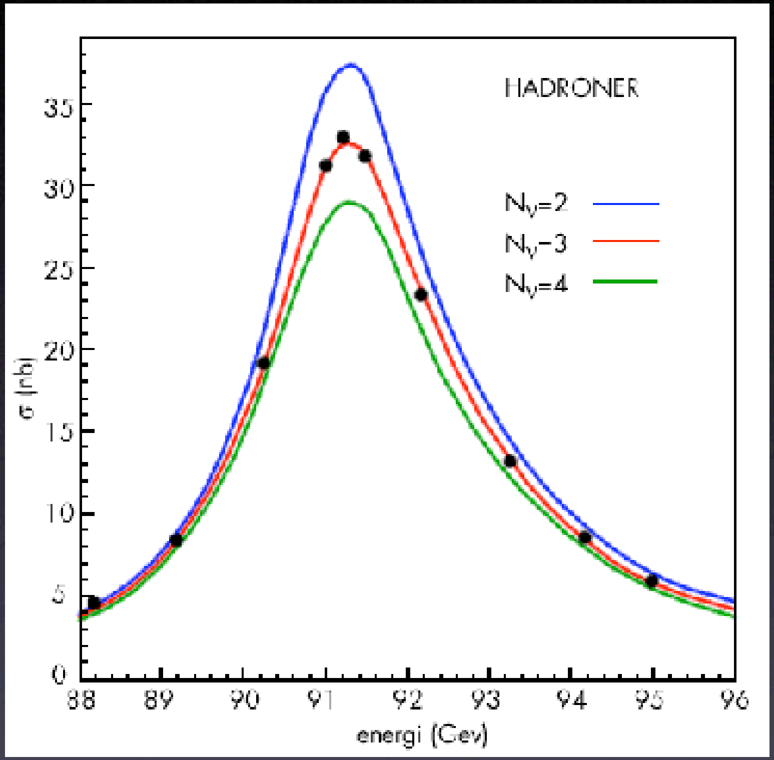
Force carriers

Force carrier	Charge	Spin	Mass
Photon, γ	0	1	0
Z^0	0	1	91 GeV
$W^\pm$	± 1	1	80 GeV
g	0	1	0

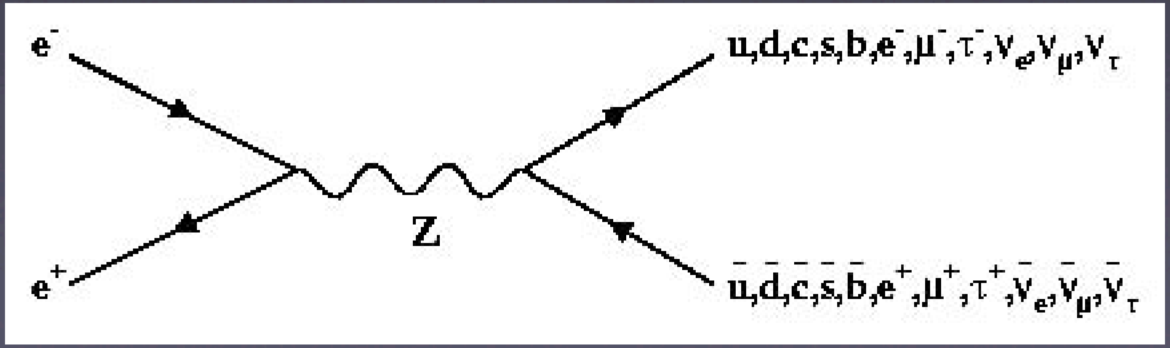
Interaction strengths

Interaction	Coupling constant
Electromagnetic	$\alpha_{\rm em} = \dfrac{e^2}{4\pi} \sim \dfrac{1}{137}$
Weak	$\alpha_{\rm weak} = \dfrac{3}{5}\dfrac{\cos^2\theta_W}{\sin\theta_W}\alpha_{\rm em} \sim 2.47\alpha_{\rm em}$
Strong	$\alpha_{\rm s} = \dfrac{g_s^2}{4\pi} \sim 0.3$

Why three families?

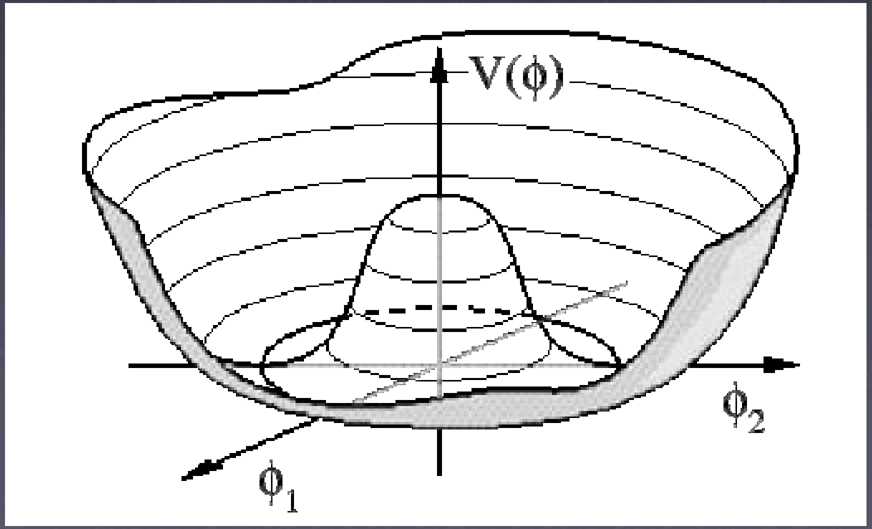


Measurement of the Z width at LEP shows that there are three neutrino families



Why are particles massive?

- Current belief is that a scalar field, the Higgs field, is required to give mass the Z^0, $W^{\pm}$ and the quarks and leptons.
- When particles move in this background field, the "drag" from the field will appear as a mass.



Tests of the Standard Model

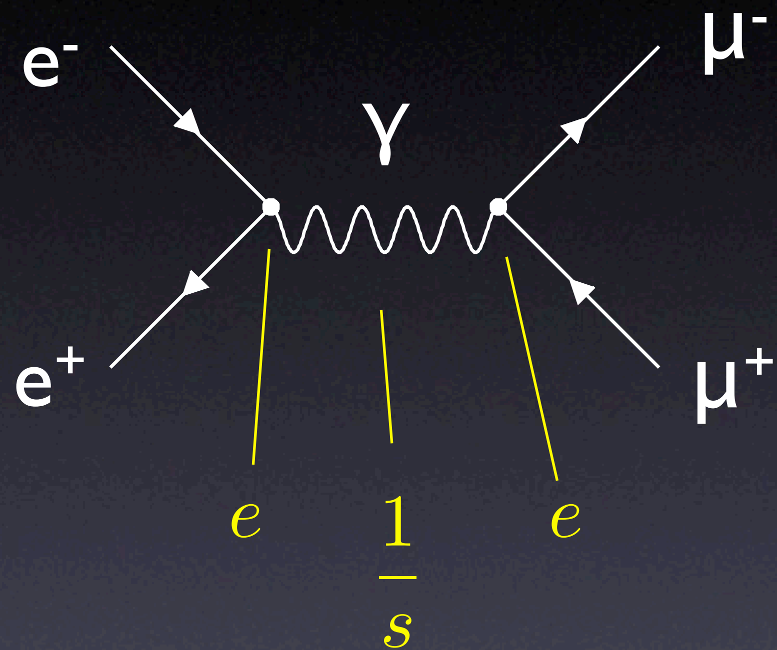
At LEP @ CERN, extensive tests of the standard model has been made in $e^+ e^-$ collissions.

Delphi, one of the detectors at LEP.

LHC coming up in 2007!

Feynman diagrams

Example of EM cross section estimate



The particle physics model can be formulated as a set of *Feynman rules*, which makes cross section calculations relatively easy

$$e = \sqrt{4\pi\alpha}$$

$$s = (p_1 + p_2)^2 = E_{\mathrm{CM}}^2$$

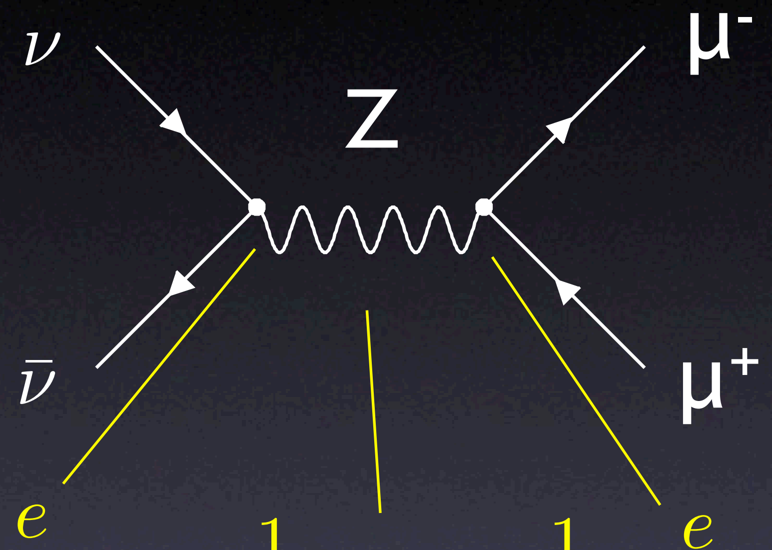
$$A \sim \frac{e^2}{s} \sim \frac{\alpha}{s}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) \propto \phi\frac{\alpha^2}{s^2}$$

$$\phi \sim s \Rightarrow \sigma(e^+e^- \to \mu^+\mu^-) \propto \frac{\alpha^2}{s}$$

Feynman diagrams

Example of weak cross section estimate



$$\frac{1}{s - m_Z^2} \sim \frac{1}{m_Z^2}$$

$$A \sim \frac{e^2}{m_Z^2} \sim \frac{\alpha}{m_W^2}$$

$$\sigma(\nu\bar{\nu} \to \mu^+\mu^-) \propto \phi\frac{\alpha^2}{m_W^4}$$

$$\phi \sim s \Rightarrow \sigma(\nu\bar{\nu} \to \mu^+\mu^-) \propto \frac{\alpha^2 s}{m_W^4}$$

The particle physics model can be formulated as a set of *Feynman rules*, which makes cross section calculations relatively easy

$$e = \sqrt{4\pi\alpha}$$

$$s = (p_1 + p_2)^2 = E_{\mathrm{CM}}^2$$

Spin quantization

- Due to quantization of angular momentum, a particle of spin s can only have 2s+1 different spin components (along an arbitrary axis)
- E.g., a spin 1/2-particle can have two different spin directions:

$\uparrow$ Spin up $\downarrow$ Spin down

- A (massive) spin 1-particle can have three different spin directions:

$\uparrow$ Spin up $\longrightarrow$ Spin sideways $\downarrow$ Spin down

Degrees of freedom in the Standard Model

- Each spin state, colour and antiparticle state can be viewed as a separate particle and will contribute to the available degrees of freedom.

Particle	Degrees of freddom (d.o.f.)	Sum of d.o.f.
Charged lepton	2 x 2 = 4	4 x 3 = 12
Neutrinos	2 (only left-handed)	2 x 3 = 6
Quarks	2 x 3 x 2 = 12	12 x 6 = 72
Total number of d.o.f.		**12 + 6 + 72 = 90**

Why supersymmetry?

- The Standard Model is highly successful in describing observations.
- However, we can make though experiments at higher energies, where the Standard Model breaks down, cross sections going to infinity could occur e.g.
- Adding a symmetry between fermions and bosons restores these problems.
- String theory also seems to need supersymmetry to be consistent.

Short introduction to supersymmetry

- In the simplest models, add one superpartner for every standard model particle (N=1)
- The superpartners have the same properties as their standard model counterpart, except for the spin that differs by 1/2
- At low energies, supersymmetry must be broken (otherwise, the masses would be the same as the standard model particles). There are many different scenarios for how supersymmetry breaking could have occurred.
- Examples:
  a) The Minimal Supersymmetric Standard Model, MSSM.
  b) Minimal SUGRA, mSUGRA

The supersymmetric mass spectrum

Normal particles/fields		Supersymmetric particles/fields			
		Interaction eigenstates		Mass eigenstates	
Symbol	Name	Symbol	Name	Symbol	Name
$= d, c, b, u, s,$	quark	$\tilde{q}_L, \tilde{q}_R$	squark	$\tilde{q}_1, \tilde{q}_2$	squark
$l = e, \mu, \tau$	lepton	l_L, l_R	slepton	l_1, l_2	slepton
$\nu = \nu_e, \nu_\mu, \nu_\tau$	neutrino	$\tilde{\nu}$	sneutrino	$\tilde{\nu}$	sneutrino
g	gluon	$\tilde{g}$	gluino	$\tilde{g}$	gluino
$W^{\pm}$	W-boson	$\tilde{W}^{\pm}$	wino	$\tilde{\chi}^{\pm}_{1,2}$	chargino
$H^{\pm}$	Higgs boson	$\tilde{H}^{\pm}$	Higgsino		
B	B-field	$\tilde{B}$	Bino	$\tilde{\chi}^0_{1,2,3,4}$	neutralino
W^3	W^3-field	$\tilde{W}^3$	Wino		
H^0_1	Higgs boson	$\tilde{H}^0_1$	Higgsino		
H^0_2	Higgs boson				
H^0_3	Higgs boson	$\tilde{H}^0_2$	Higgsino		

The lightest neutralino is a good dark matter candidate!

The MSSM parameters

- The general MSSM includes the most general N=1 supersymmetry Lagrangian that does not break R-parity, B-L, etc. It includes 124 free parameters
- Many ways to break supersymmetry exists. In a phenomenological model, the MSSM, we usually pick 7 parameters.

μ	Higgsino mass parameter
M_2	Gaugino mass parameter
m_A	Mass of CP-odd Higgs boson
$\tan\beta$	Ratio of Higgs vacuum expectation values
m_0	Scalar mass parameter
A_b	Trilinear coupling, bottom sector
A_t	Trilinear coupling, top sector

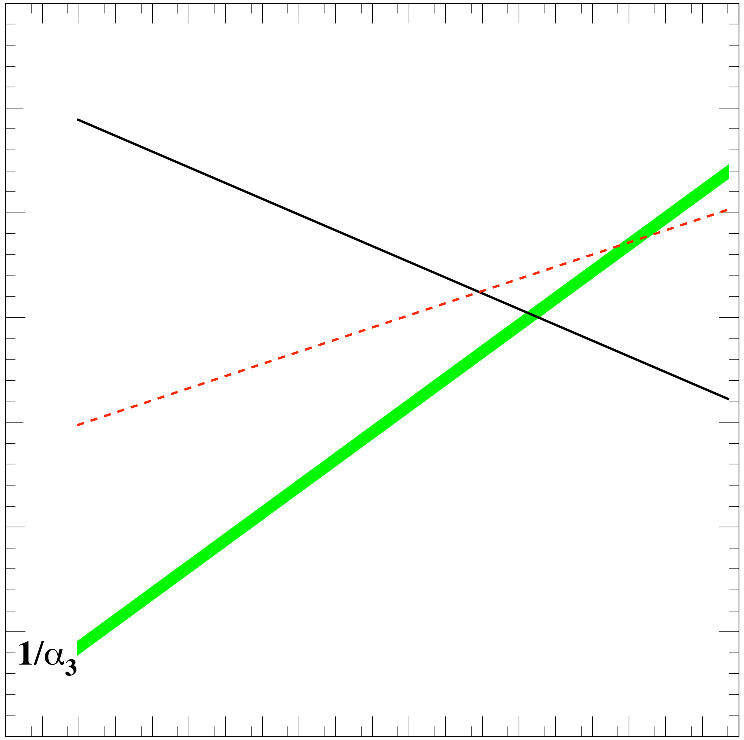
The mSUGRA parameters

- An alternative to MSSM is the GUT-inspired mSUGRA-model.
- Fix mass parameters at GUT scale and run RGEs to low energy scale. Usually we pick 5 parameters in this model.

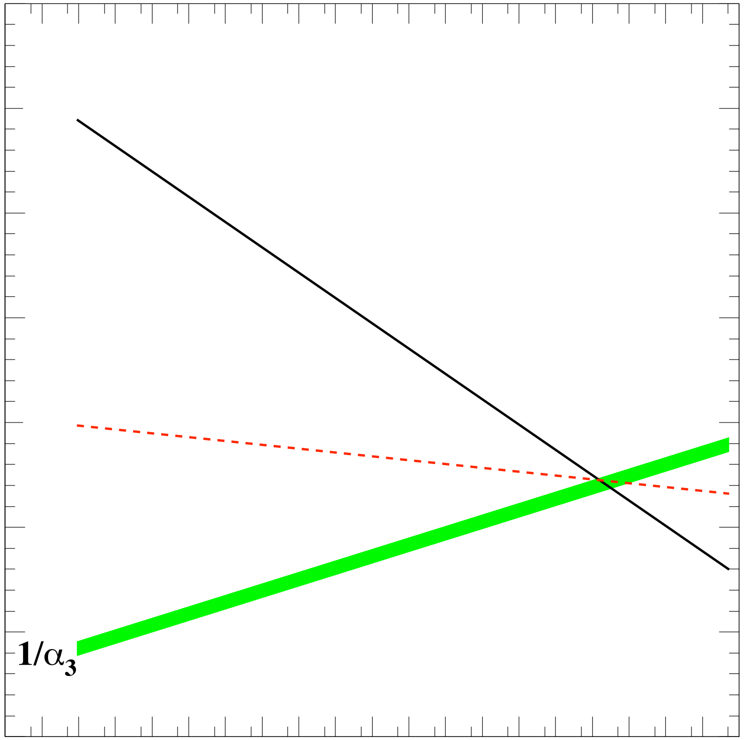
$\mathrm{sgn}(\mu)$	Sign of Higgsino mass parameter
$m_{1/2}$	Gaugino mass parameter (at GUT scale)
$\tan\beta$	Ratio of Higgs vacuum expectation values
m_0	Scalar mass parameter (at GUT scale)
A	Trilinear coupling (at GUT scale)

Grand Unification

Standard Model



MSSM



Thermodynamics in the Early Universe

In the Friedmann Lemaitre Robertsson Walker metric with a flat universe (Ω=1), we have the Friedmann equation

$$H^2(t) = \frac{8\pi G\rho}{3}$$

A very good approximation is that the relativistic particles contribute mostly to ρ. Can we calculate ρ from thermodynamics?

Equilibrium thermodynamics

The distribution function is given by

$$f_i = \frac{1}{e^{\frac{(E_i - \mu_i)}{T}} \pm 1}$$

- bosons
+ fermions

The number density is given by

$$n_i = \frac{g_i}{(2\pi)^3} \int f_i(\mathbf{p}) d^3p$$

$$g = \text{internal degrees of freedom}$$

and the energy density is

$$\rho_i = \frac{g_i}{(2\pi)^3} \int E_i(\mathbf{p}) f_i(\mathbf{p}) d^3p$$

The energy and number densities

Energy density | Number density

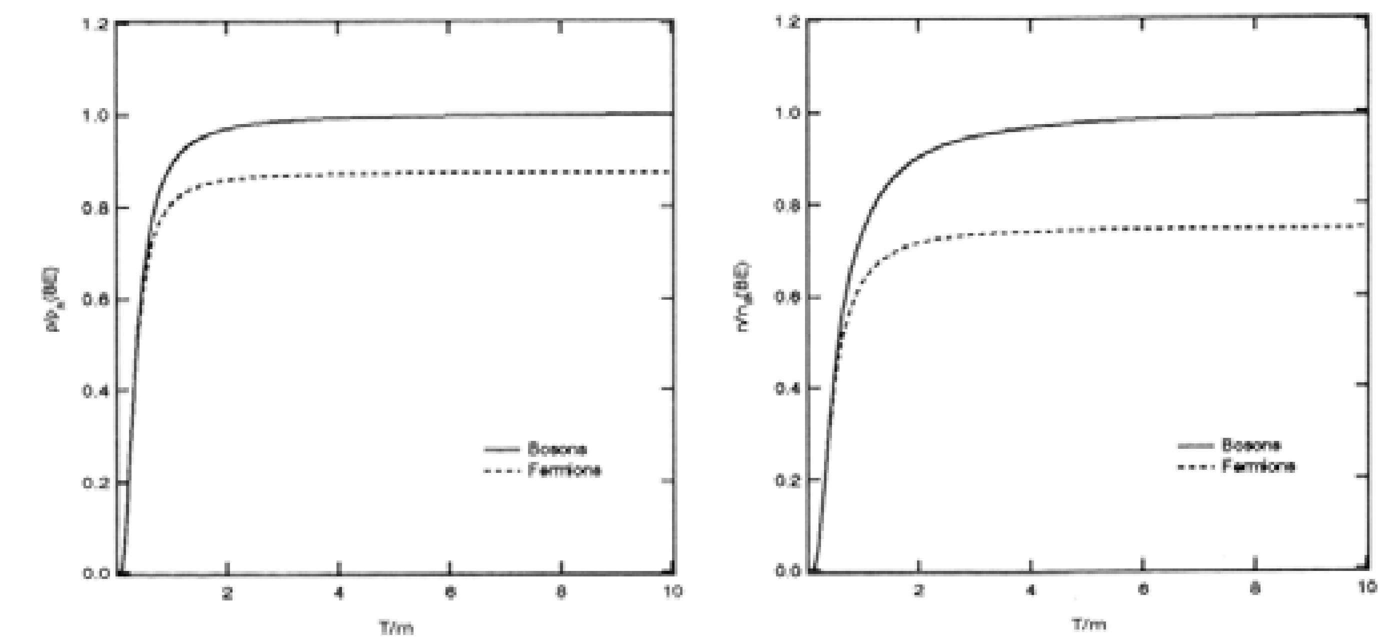


Figure 7.1: The energy (a) and number density (b) for fermions and bosons, as a function of T/m. The quantities have been normalised to the relativistic expressions for bosons, $n_R(BE) = \frac{\zeta(3)}{\pi^2} g_i T^3$ and $\rho_R(BE) = \frac{\pi^2}{30} g_i T^4$, respectively.

For non-relativistic particles, the distributions are the same for fermions and bosons.

Figure from Bergström and Goobar, *Cosmology and Particle Astrophysics*

Equilibrium distributions
non-relativistic limit

In the non-relativistic limit, the integrals can be solved analytically with the result (valid for both bosons and fermions)

$$n = g_i \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}}$$

$$\rho = mn$$

$$p = Tn \ll \rho$$

Equilibrium distributions
ultra-relativistic limit

In the ultra-relativistic limit, the integrals can be solved analytically with the result

$$n_{\mathrm{bosons}} = \frac{\zeta(3)}{\pi^2} g_i T^3 \quad ; \quad \zeta(3) \simeq 1.20$$

$$n_{\mathrm{fermions}} = \frac{3}{4}\left(\frac{\zeta(3)}{\pi^2} g_i T^3\right) \quad ; \quad \zeta(3) \simeq 1.20$$

$$\rho_{\mathrm{bosons}} = \frac{\pi^2}{30} g_i T^4$$

$$\rho_{\mathrm{fermions}} = \frac{7}{8}\left(\frac{\pi^2}{30} g_i T^4\right)$$

$$p = \frac{1}{3}\rho$$

Put into the Friedmann equation

Considering only the relativistic species, we can write

$$\rho = \frac{\pi^2}{30} g_{\text{eff}}(T) T^4 \quad ; \quad p = \frac{1}{3}\rho = \frac{\pi^2}{90} g_{\text{eff}}(T) T^4$$

$$g_{\text{eff}} = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4$$

Inserting this into the Friedmann equations gives

$$H^2(t) = \frac{8\pi G \rho}{3} = \frac{8\pi G}{3} \frac{\pi^2}{30} g_{\text{eff}} T^4 \simeq 2.76 g_{\text{eff}} \frac{T^4}{m_{\text{Pl}}^2}$$

$$\Rightarrow \quad \boxed{H = 1.66 \sqrt{g_{\text{eff}}} \frac{T^2}{m_{\text{Pl}}}}$$

Time and g_{eff}

During radiation domination, we have that

$$a(t) \sim \sqrt{t}$$

Hence, we get

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}$$

Which gives

$$t = 0.30 \frac{m_{\text{Pl}}}{\sqrt{g_{\text{eff}}} T^2}$$

$$\left(\text{Around 1 MeV this becomes } t \sim \left(\frac{1 \text{ MeV}}{T} \right)^2 \text{sec} \right)$$

What is g_{eff}

As an example, calculate g_{eff} at a temperature of 1 TeV, i.e. where all Standard Model particles are relativistic. From earlier we know that g_{tot} = 90 for the fermions. For the remaining particles, we get

Particle	D.o.f.
Z^0	3
$W^{\pm}$	6
γ	2 (no transvere pl.)
g	8 x 2 = 16
H	1
Sum	**28**

→ $$g_{\text{eff}}(T = 1\ \text{TeV}) = 28 + \frac{7}{8} 90 = 106.75$$

Entropy

The entropy in a comoving volume $a^3(t)$ is conserved and one can show that the entropy density is

$$s(T) = \frac{\rho(T) + p(T)}{T}$$

For ultra-relativistic particles, we can then write

$$s = \frac{2\pi^2}{45} g_{\text{eff}}^s T^3$$

$$g_{\text{eff}}^s = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3$$

Example
Neutrino decoupling I

A typical weak interaction cross section (for eg. neutrinos) is

$$\sigma \sim \frac{\alpha^2 T^2}{m_W^4}$$

($s \sim T^2$)

This gives the interaction rate

$$\Gamma = (\sigma v) n \sim \frac{\alpha^2 T^5}{m_W^4}$$

Which should be compared with the expansion rate

$$H \sim \frac{T^2}{m_{\rm Pl}}$$

Example
Neutrino decoupling II

When Γ > H, the neutrinos are kept in chemical equilibrium, but when H > Γ, equilibrium can no longer be sustained and the neutrinos *freeze-out*. This happens when

$$\Gamma \sim H \Rightarrow \frac{\alpha^2 T^5}{m_W^4} \sim \frac{T^2}{m_{\rm Pl}}$$

I.e., when

$$T_{\rm dec} \sim \left(\frac{m_W^4}{\alpha^2 m_{\rm Pl}^2}\right)^{\frac{1}{3}} \sim 4\ {\rm MeV}$$

After this, the neutrinos are no longer in equilibrium.

Example
Neutrino decoupling III

The neutrinos that were present at freeze-out, T_{dec} ~ 4 MeV, are present still today and we can calculate the contribution to the current energy density.

The abundance, $$Y = \frac{n}{s}$$

is conserved (as both s and n goes as T^3) and we get

$$Y(\text{today}) = Y(T = T_{\text{dec}})$$
$$n(\text{today}) = s(\text{today})Y(T = T_{\text{dec}})$$

Putting in numbers for s, ρ etc we can get

$$\Omega_\nu h^2 \sim \left(\frac{\sum m_\nu}{90\ \text{eV}}\right) \quad ; \quad \Omega = \frac{\rho}{\rho_{\text{crit}}}$$

Relic density

simple approach for non-relativistic matter

Decoupling occurs when

$$\Gamma < H$$

We have that

$$\Gamma = \langle \sigma_{\rm ann} v \rangle n_\chi$$

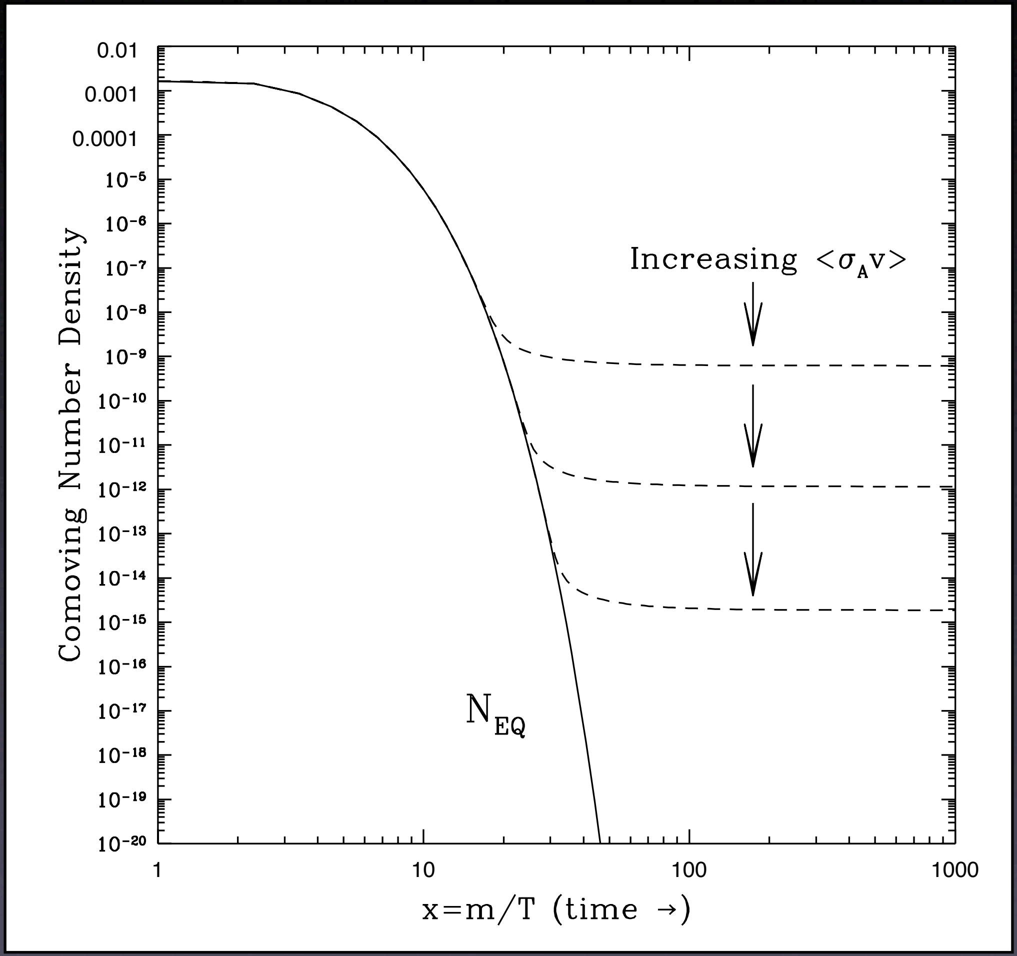
$$n_\chi^{\rm eq} = g_\chi \left(\frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi/T}$$

$$H(T) = 1.66 g_*^{1/2} \frac{T^2}{m_{\rm Planck}}$$

$$\Gamma \simeq H \quad \Rightarrow \quad T_f \simeq \frac{m_\chi}{20}$$

$$\Omega_\chi h^2 \simeq \frac{3 \times 10^{-27} cm^3 s^{-1}}{\langle \sigma_{\rm ann} v \rangle}$$

Figure from Jungman et al, Phys. Rep. 267 (1996) 195



$$\langle \sigma_{\rm ann} v \rangle \simeq \langle \sigma_{\rm ann} v \rangle_{WIMP} \quad \Rightarrow \quad \Omega_\chi h^2 \simeq 1$$

WIMP candidates

- Any weakly interacting massive particle could in principle work
- In supersymmetric extensions of the standard model, the lightest neutralino arises as a natural dark matter candidate. Will focus on this candidate as a concrete example.
- Other examples are e.g. Kaluza-Klein particles.

The neutralino as a WIMP

The neutralino:

$$\tilde{\chi}_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}^3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0$$

The neutralino can be the lightest supersymmetric particle (LSP). If R-partity is conserved, it is stable.

The gaugino fraction

$$Z_g = |N_{11}|^2 + |N_{12}|^2$$

Calculational flowchart

1. Select model parameters
2. Calculate masses etc
3. Check accelerator constraints
4. Calculate the relic density $\Omega_\chi h^2$
5. Check if the relic density is cosmologically OK
6. Calculate fluxes, rates, etc

Calculation done with

DarkSUSY 4.1 available on
www.physto.se/~edsjo/darksusy
astro-ph/0406204
JCAP 07 (2004) 008

The relic density

$$\Omega_\chi h^2 = 0.103^{+0.020}_{-0.022}$$

from WMAP+SDSS
M.Tegmark et al., astro-ph/0310723

References and further reading

- Lars Bergström and Ariel Goobar, *Cosmology and Particle Astrophysics*, chapter 6-8.
- E.W. Kolb and M. S. Turner, *The Early Universe*.