Modern Cosmology 2005

Exercise set 2

To be solved in exercise session 2

1. *Freeze-out I.*
Particles in the early Universe will remain in thermal equilibrium as long as their interaction rate Γ dominates over the cosmic expansion rate, i.e. $\Gamma > H$. When $\Gamma < H$, the particles will decouple (freeze out). To estimate the freeze-out temperature T, it is useful to have a relation for H as a function of T. Derive such a relation for the radiation-dominated era of the Universe.

2. *Freeze-out II.*
A hypothetical fermion with two internal degrees of freedom freezes out while it is still relativistic. Make a rough estimate of the freeze-out temperature, assuming $< \sigma|v| > \approx 4 \times 10^{-42} (\frac{T}{\mathrm{Gev}})^2$ m$^3 s^{-1}$.

3. *Cosmic microwave background radiation (CMBR) I.*
The CMBR can be very well represented by a black-body spectrum with a current temperature of 2.728 K. Use this information to estimate its contribution Ω_{CMBR} to the cosmological density.

4. *Standard candles.*
Estimate the expected apparent magnitude m_{B} of a supernova type Ia (absolute magnitude $M_{\mathrm{B}} \approx -19.6$) at a redshift of $z = 0.8$ in a Universe described by $\Omega_{\mathrm{M}} = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 72$ km s^{-1} Mpc^{-1}. Cosmological k-corrections and dust effects may be neglected.

5. *The era of dark-energy domination.*
Estimate the redshift at which the Universe became dark-energy dominated, assuming $\Omega_{\mathrm{M}} = 0.3$ and $\Omega_{\mathrm{DE}} = 0.7$ today, and that the dark energy has an equation of state ($p = wc^2\rho$):
a) $w = -1.0$ (i.e. a cosmological constant)
b) $w = -1.5$

6. *The Big Rip.*
If the dark energy has an equation of state $w < -1$, the Universe may be ripped apart as the scale factor $a \to \infty$ when $t \to t_{\mathrm{Rip}}$. Derive an analytical expression for t_{Rip}, under the assumption that the Universe has a flat geometry and is currently dominated by dark energy with constant w. Predict the time remaining before the Big Rip, in scenarios where:
a) $w = -1.1$
b) $w = -1.5$
c) $w = -2.0$

Hand-in exercises (deadline October 14)

7. *The radiation-dominated epoch*
Starting from the Friedmann equations, derive an expression which relates time to temperature in the radiation-dominated Universe (assuming a flat geometry), and derive the time corresponding to a temperature of 3 MeV.

8. *CMBR II*
The flux of the sky-averaged CMBR is peaking at about 3.8×10^{-15} ergs cm^{-2} sr^{-1} s^{-1} Hz^{-1}. Let's assume, that due to a region of unusually low density, the temperature of the CMBR is

reduced by one part in 10^5 over a certain patch in the sky, compared to the average. Calculate the corresponding flux from that region at

a) The frequency at which the flux in the patch is peaking, assuming it to follow the Planck distribution

b) A wavelength of 0.7 mm, under the same assumption.

9. *Dark energy and supernovae type Ia.*

The redshifts and apparent magnitudes of a small sample of supernovae type Ia are listed in Table 1. Use this data to determine which of the following three cosmological models is the most likely:

a) $\Omega_\mathrm{M} = 1.0$, $\Omega_\Lambda = 0.0$

b) $\Omega_\mathrm{M} = 0.3$, $\Omega_\Lambda = 0.7$

c) $\Omega_\mathrm{M} = 0.5$, $\Omega_\Lambda = 1.5$

You can assume $H_0 = 72$ km s^{-1} Mpc^{-1} in all cases. Both dust extinction corrections and cosmological k-corrections may be ignored. The estimated photometric 1σ errors of the measurements are $\sigma_m = 0.3$ magnitudes. Errors in the redshift determinations can be assumed negligible. Note: As this is just a toy example with artificial data, you should not expect the concordance model to win by default :)

Table 1: Supernova Type Ia data

z	m_B
0.022	15.3
0.041	17.0
0.057	17.4
0.083	18.3
0.11	18.7
0.35	22.0
0.42	22.1
0.57	23.5
0.83	24.9
0.90	24.5
0.98	25.2
1.10	25.6
1.30	25.2
1.50	25.7
1.9	26.9

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