Modern Cosmology 2005

Exercise set 3

To be solved in exercise session 3

1. *Radioactive dating*
The abundance ratio of unstable Thorium (half-life 14.5 Gyr) to stable Europium in a star is measured to be $n(\mathrm{Th})/n(\mathrm{Eu}) = 0.27$, whereas stellar nucleosynthesis predictions suggests that the star should have formed with a ratio of $n(\mathrm{Th})/n(\mathrm{Eu}) = 0.50$. The Thorium hence appears to have depleted due to radioactive decay. Use the measured and predicted abundance ratios to infer a lower limit to the age of the Universe.

2. *Big Bang Nucleosynthesis (BBNS)*
Estimate the ^{4}He abundance (mass fraction of baryons in the form of ^{4}He) expected from BBNS, assuming that after proton-neutron freezeout, every neutron was used up in the production ^{4}He nuclei. You may for simplicity assume that the neutron-to-proton ratio stayed constant at a value of $N_\mathrm{n}/N_\mathrm{p} \approx 0.13$ throughout the BBNS epoch.

3. *Dark matter in disk galaxies I*
A disk galaxy at a redshift of $z = 0.0020$ displays an apparent magnitude of $m_\mathrm{B} = 11.1$ and the inclination-corrected rotation curve shown in Fig. 1. Use this data to set a lower limit to the mass and mass-to-light ratio M/L_B (in units of $\frac{M_\odot}{L_{\mathrm{B},\odot}}$) of this system, and a lower limit to the contribution from dark matter to the total mass, given that models indicate that the mass-to-light ratio of the visible component should be $M/L_\mathrm{B} \approx 2$. You may find it useful to know that $M_{B,\odot} = 5.48$.

4. *Gravitational lensing*
Dark halo substructure can in principle be detected by its gravitational lensing effects (so-called meso- or millilensing) on background light sources. If the substructures are approximated to be point-masses of mass M_sub, the image-splitting angle can be estimated from the angular Einstein radius

$$\theta_\mathrm{E} = \left(\frac{4GM_\mathrm{sub}D_\mathrm{ls}}{c^2 D_\mathrm{ol} D_\mathrm{os}}\right)^{1/2}, \qquad (1)$$

where D_ls, D_ol and D_os are the angular size distances between lens and light source, observer and lens, observer and light source, respectively. Calculate the image separation angle (in arcseconds) for a light source at $z = 2$, produced by a dark subhalo of mass $M = 10^7\ M_\odot$ located at

a) a redshift of $z = 0.5$
b) a redshift of $z = 1.0$
c) a distance of 100 kpc (Note: This is inside the dark halo of the Milky Way!)

5. *High-redshift light sources I*
Compare the hydrogen-ionizing luminosity of a 1000 $M_\odot$ population III star to that of a 100 $M_\odot$ population I star, assuming that both exhibit black body spectra. You may assume that the population III star has a bolometric luminosity of $\log(L/L_\odot) = 7.45$ and an effective temperature (K) of $\log T_\mathrm{eff} = 5.05$, whereas the population I star has $\log(L/L_\odot) = 6.10$ and $\log T_\mathrm{eff} = 4.74$.

6. *High-redshift light sources II*
For a light source at redshift z, the apparent magnitude m in a given filter is related to the absolute magnitude M in the same filter through:

$$m = M + 5\log_{10}\left(\frac{d_\mathrm{L}}{1\ \mathrm{Mpc}}\right) + 25 + k \qquad (2)$$

where d_{L} is the luminosity distance to the object and k is the cosmological k-correction, which takes into account that the light observed at a certain wavelength (i.e. in a certain filter) on Earth was emitted at a much shorter wavelength, because of the redshift. In the case of a light source with a power-law spectrum ($f_\nu \propto \nu^{-\alpha}$), k is given by:

$$k = (\alpha - 1) \times 2.5 \log_{10}(1 + z). \tag{3}$$

Use this information to estimate the K-band (mean $\lambda = 2.2 \times 10^{-6}$ m) apparent magnitude for a $z = 5.5$ quasar with an absolute K-band magnitude of $M_K = -25$ and a power-law spectrum with $\alpha = 0.5$.

Hand-in exercises (deadline November 11)

7. *Dark matter in disk galaxies II*
Modified Newtonian Dynamics (MOND) has been suggested to explain the rotation curves of disk galaxies without the need for dark matter. MOND postulates that:

$$\mu\left(\frac{a}{a_0}\right) a = \frac{MG}{R^2}, \tag{4}$$

where a is the gravitational acceleration, a_0 is a constant introduced by MOND, and μ is an interpolation function with the property that $\mu(x) \approx 1$ for $x >> 1$ and $\mu(x) \approx x$ for $x << 1$. In the limit of small accelerations ($x << 1$), the MONDian acceleration is given by:

$$a = \sqrt{a_{\mathrm{N}} a_0}, \tag{5}$$

where a_{N} is the usual Newtonian acceleration. One possible way to test MOND is to check to what extent kinematic data from different systems really can be fitted with the same value of a_0. Use the disk-galaxy data presented in exercise 3 to test to what extent this object can be reconciled with $a_0 \approx 1.2 \times 10^{-10}$ ms^{-2}, which is the value for a_0 suggested by most previous attempts to fit galaxy data with MOND. Treating the galaxy as a point-mass is OK as long as you use the data in a way that minimizes the errors introduced by this approximation.

8. *High-redshift light sources III.*
With the James Webb Space Telescope (JWST; to be launched ∼2011), it should be possible to detect objects as faint as $m_K = 27.4$ in the K-band (mean $\lambda = 2.2 \times 10^{-6}$ m). At low redshifts, most quasars have intrinsic, absolute B-band (mean $\lambda = 4400$ Å) magnitudes of $M_B \approx -23$ and colours of $M_B - M_K \approx 2.5$.

a) Make a rough estimate of the highest redshift at which such objects can be seen with the JWST in the K-band, assuming the quasars to have power-law continua with $\alpha = 0.5$.

b) Is it really realistic to expect to detect quasars of this luminosity up to the derived redshift limit with JWST in the K-band? Hints: What are the implicit and explicit assumptions going into the calculation? Will they break down at some redshift? Why?

Erik Zackrisson, October 2005

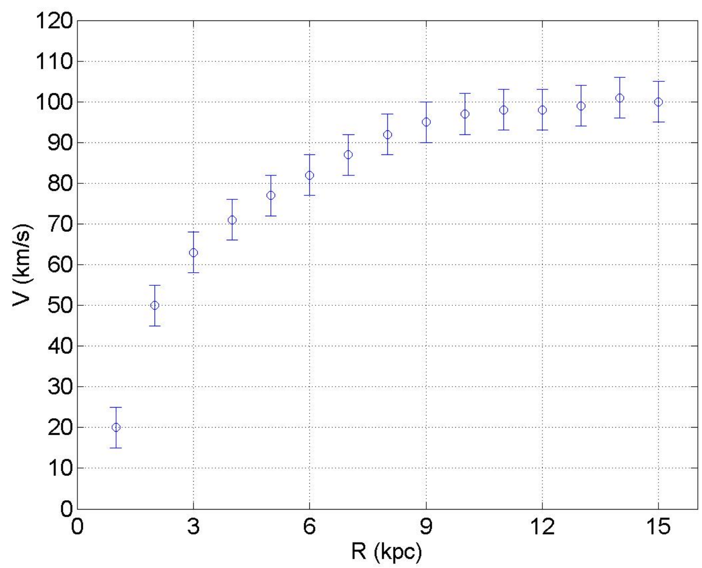

Figure 1: The rotation curve of a disk galaxy, projected to edge-on orientation.