Seminar II: Dark Energy and the Age of the Universe

1. General instructions

This document provides preparation instructions for the second of the two seminars forming part of the examination for the 2005 version of the graduate-level course *Modern Cosmology, 5 points*. The topic of this seminar is dark energy, and how it relates to the age of the Universe at different epochs.

In preparing for the seminar, you should try to:

- Develop some insight into the field of dark energy by studying the relevant literature. In doing so, the seminar questions listed below may serve as guidance as to what you should focus on.
- Prepare to explain and discuss various concepts and recent results relevant to this field in front of the class. The use of the blackboard or overhead projector is highly encouraged.
- Solve the exercises listed below and prepare to present your findings to the class.

You are perfectly welcome to collaborate with your classmates when preparing for the seminar, but once there – everyone is on their own. This means that you are not supposed to rely on the calculation, notes, viewgraphs etc. of others.

2. Seminar questions

Here are a few examples (i.e. not a complete list) of questions that may come up during the seminar:

- What exactly is dark energy?
- What is the connection between dark energy and Einstein's cosmological constant?
- Why did the interest in dark energy receive such momentum in the mid-1990s?
- Consider a matter-dominated Friedmann model for the Universe ($\Omega_{\mathrm{M}} = X$, $\Omega_\Lambda = 0$) – why does the age of the Universe increase if a cosmological constant is introduced ($\Omega_{\mathrm{M}} = X$, $\Omega_\Lambda = Y$)?
- What constraints are there on the dark energy equation of state?
- What upcoming measurements are believed to be able to distinguish between a cosmological constant and dynamical dark energy?

3. The age of the Universe in dark energy cosmologies

The point of this exercise is to:

- Learn how to derive observable quantities from the Friedmann equations
- Get a rough idea of how to derive cosmological parameters from observational constraints
- Learn not to be afraid of dark energy. Thinking about dark energy will (probably) not make you crazy :)

Starting from the Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3c^2}\epsilon(t) - \frac{H_0^2}{a(t)^2}(\Omega_{\mathrm{tot}} - 1), \tag{1}$$

it is possible to derive an expression for the age of the Universe at an epoch corresponding to the scale factor a (with $a = 1/(1+z)$):

$$t_{\mathrm{Univ}}(a) = \frac{1}{H_0}\int_0^a \frac{da}{\sqrt{\Omega_{\mathrm{rad},0}/a^2 + \Omega_{\mathrm{M},0}/a + \Omega_{\Lambda,0}a^2 + (1 - \Omega_{\mathrm{tot},0})}} \tag{2}$$

under the assumption that the energy density $\epsilon(t)$ is dominated by the three components radiation, matter and a cosmological constant, regulated by $\Omega_{\mathrm{tot},0} = \Omega_{\mathrm{rad},0} + \Omega_{\mathrm{M},0} + \Omega_{\Lambda,0}$. In the general case, (2) doesn't have any simple analytical solution, and therefore needs to be evaluated numerically.

Here, only two observational constraints on the age of the Universe will be considered. These are:

- I. The age of the Universe at the present epoch ($a = 1$, $z = 0$) is $t_{\mathrm{Univ}} > 12.5$ Gyr (based on the ages of globular clusters, which set a lower limit to the age of the Universe, e.g. Krauss & Chaboyer 2003, Science 299, 65);
- II. The age of the Universe at an epoch corresponding to $z = 4$ is $t_{\mathrm{Univ}} > 2$ Gyr (based on the chemical abundances of the quasar APM 08279+5255, e.g. Friaca et al. 2005, astro-ph/0504031).

Although these constraints are subject to substantial errorbars, you may initially neglect such complications.

3.1 Simple dark energy models

- Write a program[1] which numerically evaluates (2) and search the parameter space $\Omega_M = 0.0 - 1.0$, $\Omega_\Lambda = 0.0 - 1.0$ for combinations which can meet:

  a) Age constraint I

  b) Age constraints I & II

  One way of doing would be to create a diagram of Ω_M versus Ω_Λ and to plot a marker for each set of parameter values that satisfies the required constraints. You may assume $H_0 = 72$ km s^{-1} Mpc^{-1} and $\Omega_{\mathrm{rad}} = 8.4 \times 10^{-5}$. If done correctly, you will find that the concordance model ($\Omega_M \approx 0.3$, $\Omega_\Lambda \approx 0.7$) meets constraint I, but has severe problems in meeting constraint II.

- Exchange the cosmological constant Λ implicitly present in $\epsilon(t)$ in (1) for a dark energy (DE) model with constant equation of state w_{DE} ($p = w_{\mathrm{DE}}\rho c^2$, where $w_{\mathrm{DE}} = -1$ corresponds to the cosmological constant Λ) and modify (2) accordingly. Explore to what extent the $\Omega_M \approx 0.3$, $\Omega_{\mathrm{DE}} \approx 0.7$ model can simultaneously meet constraints I and II once $w_{\mathrm{DE}} \neq -1$. What value of w_{DE} is required? Is slightly changing Ω_M, Ω_{DE} or H_0 a better option? Why/why not?

- Implement the option of having dark energy with a non-constant equation of state $w_{\mathrm{DE}}(a)$ and test a few different a-dependences (no theoretical justification necessary for the functional form – could be $w_{\mathrm{DE}}(a) \propto \log(a)$, $\cos(a)$ or whatever you feel like). Can the age criteria I & II be met this way? Be prepared to present your models of $w_{\mathrm{DE}}(a)$ in class using plots like e.g. z versus $t_{\mathrm{Univ}}(a)$, z versus $w_{\mathrm{DE}}(a)$ and z versus ρ_{DE}. Coming up with funny names for your dark energy models is not compulsory, but highly encouraged :)

[1] Personally, I would use Matlab for this. An example of how numerical integration is done in this language can be downloaded from: www.astro.uu.se/∼ez/kurs/gradU/matlab_tutorial/

3.2 Going deeper

Choose *one* of the following tasks, and prepare to present your work and results (with transparencies etc.) in class:

- Explore to what extent a cosmological model based on Cardassian expansion (see e.g. Freese 2005, New Astron.Rev. 49, 103; Koivisto et al. 2005, Phys.Rev. D71,064027) can meet the age constraints I & II. In this phenomenological scenario, the Friedmann equation (1) is altered to allow a second density term on the right hand side. Through this modification, it becomes possible to explain e.g. type Ia supernova and CMBR data in a Universe containing only matter and radiation. Are there some additional constraints (not related to ages) which can be used to constrain the additional parameters that are introduced in this approach?

- Same for a cosmological model based on Chaplygin gas (see e.g. Alcaniz et al. 2003, Phys.Rev. D67 , 043514; Alam et al. 2003, MNRAS 344, 1057 and references therein).

- Same for some braneworld scenario (see e.g. Sahni, astro-ph/0502032).

- Look deeper into the age constraints I & II – how robust are they? Are the errorbars random or systematic? Are there additional uncertainties not taken into account? How can the errorbars be reduced?

- Are there even stronger age constraints than I & II?

- Can age determinations be used in some more clever way to constrain dark energy models? (Hint: See Jimenez & Loeb 2002, ApJ 573, 37) What astrophysical objects represent the optimal targets? Can these be studied using current telescopes?

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