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Stellar Winds: Mechanisms and Dynamics

Lecture Notes

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Most stars have a stellar wind, i.e. and outflow of gas from the stellar surface but the mass loss rates and outflow velocities depend strongly on the stellar parameters. Our sun has a mass loss rate of only $10^{-14} M_\odot$/yr but e.g. cool luminos stars (at a stage that our sun will reach towards the end of its life) can lose matter at much faster rates (probably up to $10^{-4} M_\odot$/yr) so that eventually mass loss may become the process which determines the further evolution, rather than nuclear burning.

Direct observational evidence for stellar winds comes, e.g., from P Cygni profiles of spectral lines (due to Doppler shifts) or an excess of thermal emission at infrared wavelengths originating in cool circumstellar envelopes (compared to the stellar flux at these wavelengths). In recent years direct imaging and interferometric observations have become increasingly important for studying the spatial structures and dynamics of stellar outflows.

1 Wind Mechanisms and Basic Relations

A stellar wind requires a source of momentum, i.e. an outwards directed force which can overcome gravity. The gas has to be accelerated beyond the escape velocity

$$v_{\rm esc} = \left(\frac{2GM}{r} \right)^{1/2} \tag{1}$$

which depends on the mass M of the star and decreases with r (i.e. the distance from the center of the star). According to the nature of the dominating outwards-directed force stellar winds can be divided into several groups.

Radiation-driven winds receive their momentum directly from the radiation field of the star by absorbing the outwards-directed stellar radiation and re-emitting photons into all directions. This creates a net force (radiation pressure) directed away from the star. This is probably the most efficient mass loss mechanism for luminous stars.

Thermal winds are driven by the (gradient of) the gas pressure. This requires a high pressure in the wind formation zone which may be created, e.g., by dissipation of sound waves or by radiative heating of the gas. Models of pressure-driven winds were developed in an early attempt to understand the newly-discovered solar wind.

A further possibility to accelerate gas away from the stellar surface are waves (sound waves, shocks, magneto-hydrodynamic waves, etc.). However, this is a rather inefficient mechanism for directly driving a wind by momentum input. Nevertheless, waves may contribute considerably to create thermal winds (heating, increasing the gas pressure) or even radiation-driven outflows (by enhancing the density of the outer atmospheric layers so that the relevant absorbing species like molecules or dust grains can form more efficiently).

In certain cases – especially if no time-dependent phenomena like waves or a variable stellar luminosity are involved in the acceleration of the outflow – a stationary wind model may be a good description (or at least a useful first approximation). In a stationary wind, by definition, the flow pattern does not explicitly depend on time, i.e. all terms containing partial derivatives with respect to time are zero. The equation of continuity therefore reduces to

$$\nabla\cdot(\rho\, u) = 0\,. \tag{2}$$

In spherical symmetry we find

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho\, u) = 0 \qquad \rightarrow \qquad r^2\rho\, u = \text{const} \qquad \rightarrow \qquad \dot{M} \equiv 4\pi r^2\rho u = \text{const} \tag{3}$$

where $\dot{M}$ is the mass loss rate, i.e. the total mass flowing through a given (spherical) surface per unit time. If we regard the outer wind zone where the wind has reached its constant outflow velocity the equation of continuity therefore implies a density profile $\rho \propto r^{-2}$.

2 A Thermal Wind Model

We consider a simple model for a pressure-driven wind which can be treated analytically and was derived by E.N. Parker (1960).

The model is based on the following assumptions: The outflow is stationary and spherically symmetric. As discussed above in this case the equation of continuity becomes

$$r^2\rho u = r_0^2\rho_0 u_0 = \text{const} \tag{4}$$

where all quantities with subscript '0' correspond to the radius r_0 (e.g. $\rho_0 = \rho(r_0)$). The equation of motion is

$$\rho u\frac{du}{dr} = -\frac{dP}{dr} - \frac{GM\rho}{r^2}, \tag{5}$$

where P denotes the gas pressure, M the stellar mass and G the gravitational constant, i.e. we consider forces due to pressure gradients and gravity, only (no radiation pressure, etc.). As equation of state we assume a ideal gas

$$P = \frac{k}{\mu\, m_u}\,\rho\, T\,. \tag{6}$$

To allow for an analytic treatment of the problem we replace the energy equation by a polytropic law, i.e.

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{7}$$

Using the deal gas law and the equation of continuity to replace P and ρ in the equation of motion, after some manipulations we obtain

$$\left(1 - \frac{T\,k}{u^2\mu\, m_u}\right)\frac{\mu\, m_u}{T_0\, k}\frac{du^2}{dr} = -2\left\{r^2\frac{d}{dr}\left(\frac{T}{T_0\, r^2}\right) + \frac{GM\mu\, m_u}{r^2\, T_0\, k}\right\}. \tag{8}$$

By defining

$$\xi \equiv \frac{r}{r_0} \tag{9}$$

$$\psi(\xi) \equiv \frac{u^2\,\mu\, m_u}{T_0\, k} \tag{10}$$

$$\lambda \equiv \frac{2GM\mu\, m_u}{r_0\, T_0\, k} \tag{11}$$

and substituting these new quantities into the equation we find

$$\left(1-\frac{T}{T_0\psi}\right)\frac{d\psi}{d\xi}=-2\xi^2\frac{d}{d\xi}\left(\frac{T}{T_0\xi^2}\right)-\frac{\lambda}{\xi^2}. \tag{12}$$

This is the general equation for a stationary, spherically symmetric pressure-driven wind for an ideal gas. In principle, the temperature structure $T(r)$ can be obtained by solving the energy transport. However, since we are interested in an analytical solution of the problem we assume a polytropic law as given above. In this case,

$$\frac{T}{T_0}=\left(\frac{\rho}{\rho_0}\right)^{\gamma-1}=\xi^{2(1-\gamma)}\left(\frac{\psi_0}{\psi}\right)^{\frac{\gamma-1}{2}}, \tag{13}$$

where we have used the ideal gas law and the equation of continuity

$$\frac{\rho}{\rho_0}=\xi^{-2}\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}} \tag{14}$$

and we obtain the wind equation

$$\left(1-\xi^{2(1-\gamma)}\frac{\psi_0^{(\gamma-1)/2}}{\psi^{(\gamma+1)/2}}\right)\frac{d\psi}{d\xi}=-2\xi^2\frac{d}{d\xi}\left(\xi^{-2\gamma}\left(\frac{\psi_0}{\psi}\right)^{\frac{\gamma-1}{2}}\right)-\frac{\lambda}{\xi^2}. \tag{15}$$

This equation can be transformed into

$$\frac{d}{d\xi}\left\{\psi+\frac{2\gamma}{\gamma-1}\left(\frac{\psi_0}{\psi\xi^4}\right)^{\frac{\gamma-1}{2}}\right\}=-\frac{\lambda}{\xi^2} \tag{16}$$

or

$$\frac{d}{d\xi}\left\{\psi+\frac{2\gamma}{\gamma-1}\left(\frac{\psi_0}{\psi\xi^4}\right)^{\frac{\gamma-1}{2}}-\frac{\lambda}{\xi}-C\right\}=0 \tag{17}$$

where C is an integration constant. The solution is

$$\psi+\frac{2\gamma}{\gamma-1}\left(\frac{\psi_0}{\psi\xi^4}\right)^{\frac{\gamma-1}{2}}-\frac{\lambda}{\xi}=C' \tag{18}$$

The integration constant is determined by the initial condition

$$\psi(1)=\psi_0 \tag{19}$$

and the general solution assumes the form

$$\psi=\psi_0+\frac{2\gamma}{\gamma-1}\left\{1-\left(\frac{\psi_0}{\psi\xi^4}\right)^{\frac{\gamma-1}{2}}\right\}-\lambda\frac{\xi-1}{\xi}. \tag{20}$$

This is an implicit equation for $\psi(\xi)$ (with ψ_0 as a parameter) which can be solved numerically for a given combination of M, r_0, T_0 ($\rightarrow \lambda$), γ and ψ_0. Among the possible solutions there exists only one wind solution, i.e. a flow with a transition from subsonic to supersonic velocities, and the corresponding value of ψ_0 can be found iteratively.

Once the run of $\psi(\xi)$ for the wind solution is known the structure of the velocity and the temperature can be calculated using

$$u(r) = \sqrt{\frac{\psi \, T_0 \, k}{\mu \, m_u}} \tag{21}$$

and

$$T(r) = T_0 \cdot \xi^{2(1-\gamma)} \left(\frac{\psi_0}{\psi} \right)^{\frac{\gamma-1}{2}} \tag{22}$$

The equation of continuity can be used to determine the density structure

$$\rho(r) = \rho_0 \cdot \xi^{-2} \left(\frac{\psi_0}{\psi} \right)^{\frac{1}{2}} \tag{23}$$

once the value ρ_0 (which is not required to construct the wind solution) has been chosen (e.g. by specifying the mass loss rate).

3 Radiation-driven Winds

In order to drive a substantial stellar wind by radiation pressure usually two requirements have to be fulfilled: a high stellar luminosity and the presence of efficient absorbers (high opacity). These conditions can be met under various circumstances and different types of radiation-driven winds exist, depending both on the stellar parameters like mass, luminosity and effective temperature, and the dominant absorbing species (atoms, molecules, dust grains).

Compared to a pressure-driven wind as discussed in the previous section, the equation of motion for a radiation-driven wind contains an additional term accounting for the radiation pressure. To calculate the radiation pressure we must know the frequency-dependent radiative flux at every point in the wind. The computation of the flux, however, can only be performed for a known spatial structure of density, temperature and velocity fields (Doppler shifts). Therefore, the modeling of radiation-driven winds in general requires a simultaneous (or iterative) solution of both, the equations of hydrodynamics and the equations of radiative transfer. In certain cases, i.e. if molecules or dust grains are important sources of opacity, it may even be necessary to include a (non-equilibrium) description of chemistry or a (time-dependent) treatment of dust formation. In contrast to the simple pressure-driven wind model which we studied in the preceding section, a general analytical solution of the problem of radiation-driven winds is not possible, and one has to resort to numerical simulations.

Hot Star Winds

The winds of luminous hot stars (e.g. O stars) are usually so-called line-driven winds, i.e. the opacity is due to atomic lines. They are characterized by high outflow velocities (on the order of 1000 km/s) and therefore large velocity gradients in the acceleration zone of the wind. During the acceleration the lines are Doppler-shifted considerably, such that the effective opacity becomes larger. The high outflow velocities are necessary to overcome the large surface gravity.

Cool Star Winds

Possible sources of opacity in winds of evolved cool stars are molecules and dust grains which form in the upper layers of the atmospheres. The wind velocities of cool giant stars are very low, typically on the order of $10-30$ km/s, corresponding to the low surface gravity of these extended objects. Mass loss rates can reach up to $10^{-4}\, M_\odot$/yr, dominating the further evolution of the star. If dust grains provide the main source of opacity the outflow is called a dust-driven wind.

For stars on the asymptotic giant branch (AGB) observations suggest a correlation of pulsation and mass loss. Most of the stars are known to be large-amplitude pulsators (long-period variables, Mira stars). The stellar pulsation creates shock waves in the atmospheres which lead to a levitation of the outer atmospheric layers, i.e. relatively high densities at low temperatures. In this environment molecules and dust grains can form efficiently and provide the necessary opacity to drive a massive slow wind. These complex interacting processes require a consistent simultaneous modeling of time-dependent hydrodynamics, frequency-dependent radiative transfer and non-equilibrium dust formation.

4 Dynamics of Dust-driven Winds: A Toy Model

In order to illustrate the basic dynamical properties of a dust-driven wind originating from a pulsating AGB star, the following simple model is adopted: We consider the dynamical evolution of a single fluid element from an instant when it has just been accelerated outwards by a passing atmospheric shock wave to the point where it is accelerated beyond the escape velocity, or starts falling back towards the stellar surface. Assuming that gravity and radiation pressure are the only relevant forces, the equation of motion (in a co-moving frame of reference) can be written as

$$\frac{du}{dt} = -g_0 \left(\frac{r_0}{r}\right)^2 (1-\Gamma) \qquad \text{with} \qquad \Gamma = \frac{\kappa_H L_*}{4\pi c G M_*} \tag{24}$$

where r is the distance from the stellar center, $u = dr/dt$ the radial velocity, $g_0 = GM_*/r_0^2$ denotes the gravitational acceleration at r_0, κ_H the flux mean opacity, and M_* and L_* are the stellar mass and luminosity, respectively (c = speed of light, G = gravitation constant). For given stellar parameters, the quantity Γ (ratio of radiative to gravitational acceleration) is determined by the flux-mean opacity κ_H which is a function of distance from the star. In our simple toy model we can distinguish two regimes: (a) the zone of hot dust-free gas close to the photosphere where κ_H is relatively low and, consequently, $\Gamma \ll 1$, and (b) the cooler regions further away from the star where κ_H may be dominated by dust opacities and Γ may exceed the critical value of 1, leading to an outward acceleration of the fluid element (see below).

Atmospheric shocks caused by stellar pulsation or large-scale convective motions enter the model by setting the initial velocity $u_0 = u(r_0)$ of the fluid element in the photosphere. After the passage of a shock, the fluid element first follows a ballistic trajectory (since $\Gamma \ll 1$ in the hot dust-free gas close to the photosphere), slowing down gradually due to gravity. If no (or very little) dust is forming, Γ will stay close to zero and the matter will reach a maximum distance $r_{\rm max}$ according to its initial velocity (kinetic energy),[1] given by

$$\frac{r_0}{r_{\rm max}} = 1 - \left(\frac{u_0}{u_{\rm esc}}\right)^2 \qquad \text{where} \qquad u_{\rm esc} = \sqrt{\frac{2GM_*}{r_0}} \tag{25}$$

($u_{\rm esc}$ = escape velocity at r_0) and then fall back towards the stellar surface.

Dust Formation and Wind Acceleration

The growth and survival of dust particles requires grain temperatures below the stability limit of the respective condensate. The temperature of a grain will be mostly determined by its interaction with the radiation field, and, consequently, a condensation distance R_c can be defined as the distance from the center of the star where the radiative equilibrium temperature of a grain is equal to the condensation temperature T_c of the material, i.e. the closest distance where such grains can exist. Assuming a Planckian radiation field, geometrically diluted with distance from the stellar surface, and a power law for the grain absorption coefficient $\kappa_{\rm abs}(\lambda)$ in the relevant wavelength range (around the flux maximum of the stellar photosphere) the condensation distance can be expressed as

$$\frac{R_c}{R_*} = \frac{1}{2}\left(\frac{T_c}{T_*}\right)^{-\frac{4+p}{2}} \qquad \text{where} \qquad \kappa_{\rm abs} \propto \lambda^{-p} \tag{26}$$

and R_* and T_* are the radius and effective temperature of the star (see, e.g., Lamers & Cassinelli, 1999). The condensation temperature T_c and the power law index p depend on the grain material. As an example, small amorphous carbon grains have a power law index of the absorption coefficient $p \approx 1$ and a condensation temperature $T_c \approx 1500\,{\rm K}$ which, together with a typical stellar temperature $T_* \approx 3000\,{\rm K}$, leads to $R_c \approx 2-3\,R_*$ (i.e., such dust grains can form at distances of about $1-2$ stellar radii above the photosphere, and they are probably driving the winds of C-rich AGB stars).

If the initial velocity u_0 of the fluid element (caused by a passing atmospheric shock) is high enough that the material reaches distances beyond R_c, grain formation and growth may set in, causing κ_H and thereby Γ to increase. In this case, we can distinguish several scenarios:

[1] In the discussion here we assume that the velocity of the mass element is below the escape velocity before the material reaches the dust condensation zone. Otherwise it would be misleading to talk about a dust-driven wind.

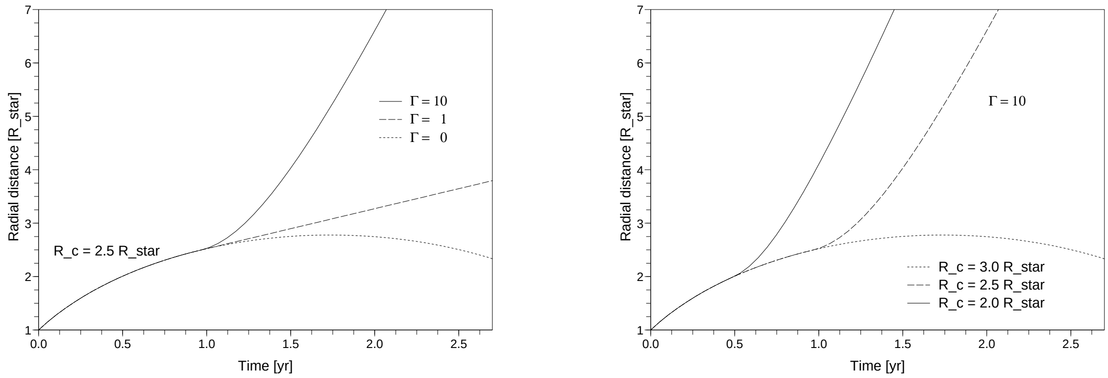

Figure 1: A toy model for the dynamics of a dust-driven wind: Both panels show the location of a fluid element (i.e., its distance from the stellar center) as a function of time. The velocity is determined by assuming a distance-dependent Γ (ratio of radiative acceleration to gravitational acceleration) in the equation of motion, i.e., $\Gamma = 0$ for $r < R_c$ (below the condensation distance, no dust), and $\Gamma =$ constant at distances beyond R_c. Both, the value of Γ (varied in left panel, for constant R_c) and the condensation distance R_c (varied in right panel, for constant Γ) play a crucial rule for the onset of a stellar wind. See text for details.

- If $\Gamma > 1$, du/dt is positive, the matter is accelerated away from the star.
- If $\Gamma = 1$, the accelerations due to gravity (inwards) and radiative pressure (outwards) cancel each other, and the fluid element continues to move at a constant velocity.
- If $0 < \Gamma < 1$, the final fate of the matter element (escape or fall-back) depends on the velocity at which it is moving when it reaches the dust formation zone. As a positive value of Γ can be interpreted as a reduction of the effective gravitational acceleration, the corresponding local escape velocity is smaller than in the dust-free case, leading to a narrow regime where the combined momentum transferred by the shocks and the radiation pressure on dust may drive the matter away from the star.

Figure 1 illustrates the different scenarios (for stellar parameters $M_* = 1\,M_\odot$, $L_* = 7000\,L_\odot$, $T_* = 2700\,\mathrm{K}$, and initial conditions $r_0 = R_*$, $u_0 = 0.8\,u_{\mathrm{esc}}$), assuming that $\Gamma = 0$ for $r < R_c$ (no dust), and $\Gamma =$constant at distances beyond R_c. This is, of course, a simplification of the real situation where dust formation and grain growth are spread out over a range of distances, and, in addition, the resulting flux mean opacity may vary due to other factors than the changing amount of material condensed into grains. The simple toy model, however, makes it easy to demonstrate that both, the total opacity of the dust grains (i.e. the resulting Γ) *and* the condensation distance R_c play a crucial rule for the onset of a stellar wind (see Fig. 1) Even a small difference in R_c for different grain materials can have a dramatic effect on their potential as a wind drivers.

Dust Opacities

While grain temperatures and, consequently, condensation distances are determined by the respective absorption cross sections, the opacities relevant for radiative pressure are a more complex topic. The corresponding grain cross sections are given by a combination of absorption and scattering, i.e.

$$C_{\rm rp} = C_{\rm ext} - g_{\rm sca}\,C_{\rm sca} = C_{\rm abs} + (1 - g_{\rm sca})\;C_{\rm sca} \tag{27}$$

where $C_{\rm abs}$, $C_{\rm sca}$ and $C_{\rm ext}$ denote the cross sections for absorption, scattering and extinction, respectively, $g_{\rm sca}$ is the asymmetry factor describing deviations from isotropic scattering ($g_{\rm sca} = 0$ for the isotropic case, $g_{\rm sca} = 1$ corresponding to pure forward scattering, i.e., no momentum being imparted on the grain due to scattering; see, e.g., Krügel 2003), and $C_{\rm rp}$ may depend strongly on grain size, as well as on material properties.

In order to compute the total flux mean opacity κ_H which determines the radiation pressure and, consequently, Γ, it is necessary to know the combined opacity (cross section per mass) of all grains of a certain size (i.e., radius $a_{\rm gr}$, for spherical particles) in a matter element (with a total mass density ρ),

$$\kappa_{\rm rp}(\lambda, a_{\rm gr}) = C_{\rm rp}(\lambda, a_{\rm gr})\,n_{\rm gr}\,\frac{1}{\rho} \tag{28}$$

($n_{\rm gr}$ = number density of grains of radius $a_{\rm gr}$) at all relevant wavelengths λ. Introducing the efficiency $Q_{\rm rp} = C_{\rm rp}/\pi a_{\rm gr}^2$ (radiative cross section divided by geometrical cross section) which can be calculated from refractive index data using Mie theory the dust opacity defined above can be reformulated as

$$\kappa_{\rm rp}(\lambda, a_{\rm gr}) = \pi\,a_{\rm gr}^2\,Q_{\rm rp}(\lambda, a_{\rm gr})\;n_{\rm gr}\,\frac{1}{\rho} = \frac{\pi}{\rho}\,\frac{Q_{\rm rp}(\lambda, a_{\rm gr})}{a_{\rm gr}}\;a_{\rm gr}^3\,n_{\rm gr} \tag{29}$$

where $a_{\rm gr}^3\,n_{\rm gr}$ represents the volume fraction of a stellar matter element which is occupied by grains, apart from a factor $4\pi/3$. This quantity can be rewritten in terms of the space occupied by a monomer (basic building block) within the condensed material, $V_{\rm mon}$, times the number density of condensed monomers in the matter element under consideration, $n_{\rm mon}$, i.e.,

$$a_{\rm gr}^3\,n_{\rm gr} = \frac{3}{4\pi}\,V_{\rm mon}\,n_{\rm mon} = \frac{3}{4\pi}\,\frac{A_{\rm mon} m_p}{\rho_{\rm grain}}\;f_c\,\varepsilon_c\,n_{\rm H} \tag{30}$$

where the monomer volume $V_{\rm mon}$ has been expressed in terms of the atomic weight of the monomer $A_{\rm mon}$ and the density of the grain material $\rho_{\rm grain}$ (m_p = proton mass). Assuming, for simplicity, that all grains at a given location have the same size, the number density of condensed monomers $n_{\rm mon}$ has been rewritten as the product of the abundance of the key element of the condensate ε_c, the degree of condensation of this key element f_c, and the total number density of H atoms $n_{\rm H}$. Using $n_{\rm H} = \rho/(1 + 4\,\varepsilon_{\rm He})\,m_p$ (where $\varepsilon_{\rm He}$ denotes the abundance of He), we obtain

$$\kappa_{\rm rp}(\lambda, a_{\rm gr}) = \frac{3}{4}\;\frac{A_{\rm mon}}{\rho_{\rm grain}}\;\frac{Q_{\rm rp}(\lambda, a_{\rm gr})}{a_{\rm gr}}\;\frac{\varepsilon_c}{1 + 4\,\varepsilon_{\rm He}}\;f_c\,. \tag{31}$$

This way of writing the dust opacity which may seem unusual at first has the advantage of distinguishing between material properties (constants and optical properties) and the chemical composition of the atmosphere/wind, and allows to take into account that $Q_{\rm rp}(\lambda, a_{\rm gr})/a_{\rm gr}$ becomes independent of grain size for particles much smaller than the wavelength λ. For a given grain material and stellar element abundances, the only unknown quantity in this expression of the opacity is the degree of condensation f_c (i.e. the fraction of available material actually condensed into grains). As dust formation in AGB stars usually proceeds far from equilibrium, f_c, in general, needs to be calculated with detailed non-equilibrium methods. Assuming a reasonable value for f_c, however, this equation allows to estimate which grain types can, in principle, drive an outflow, and which alternatives can be ruled out for this purpose based on too small radiative pressure.

Taking amorphous carbon as an example ($A_{\rm mon} = 12$), $Q_{\rm rp}(\lambda, a_{\rm gr})/a_{\rm gr} \approx 2 \cdot 10^4$ [1/cm] at $\lambda \approx 1\mu$m (near the stellar flux maximum) in the small particle limit, with a corresponding density of the grain material $\rho_{\rm grain} = 1.85$ [g/cm^3]. Assuming C/O = 1.5 at solar metallicity, leading to $\varepsilon_c = 3.3 \cdot 10^{-4}$ (abundance of excess carbon not bound in CO), together with complete condensation of all available carbon ($f_c = 1$), $L_* = 5000\, L_\odot$ and $M_* = 1\, M_\odot$, results in $\Gamma \approx 10$. This is clearly more than enough for driving the wind of a typical AGB star, and even a small fraction of the available carbon being condensed into grains (about 10 − 20 percent in this case) will be sufficient to cause an outflow.

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