Lecture 3:

Microphysics of the radiative transfer

RT in a simple case

- Local Thermodynamic Equilibrium (LTE) -> all microprocesses (radiative, collisional, chemical) are in detailed balance
- Static (no time dependence)
- Geometry: semi-infinite medium
- One dimension

What are the coefficients in RT equation?

Equation of radiative transfer:

$$\frac{dI_\nu}{dx_\nu} = -k_\nu(x)\cdot\rho(x)\cdot I_\nu + k_\nu(x)\cdot\rho(x)\cdot S_\nu(x)$$

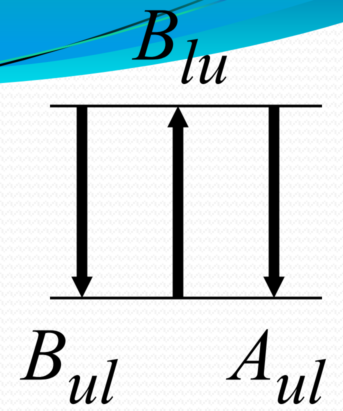
or $$\frac{dI_\nu}{dx_\nu} = -\alpha_\nu(x)\cdot I_\nu + \alpha_\nu(x)\cdot S_\nu(x)$$

or $$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

For the case of semi-infinite medium (e.g. stellar atmosphere) boundary condition is set deep (inside a star):

$$I_\nu(\tau_\infty) = I_\nu^\infty$$

Einstein coefficients



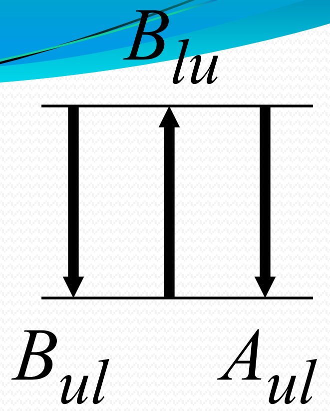
- A_{ul} - spontaneous de-excitation.
- B_{lu} - radiative excitation.
- B_{ul} - stimulated de-excitation.
- Einstein relations connect the probabilities:

$$\frac{B_{lu}}{B_{ul}} = \frac{g_u}{g_l}; \qquad \frac{A_{ul}}{B_{ul}} = \frac{2h\nu^3}{c^2}$$

and that *lu* transition rate match the *ul* rate:

$$n_l B_{lu} I_\nu = n_u A_{ul} + n_u B_{ul} I_\nu$$

Absorption/Emission



- Energy absorbed:

$$k_\nu^{bb}\rho\cdot I_\nu=\frac{h\nu}{4\pi}\cdot n_l\cdot B_{lu}\cdot\varphi(\nu-\nu_0)\cdot I_\nu$$

- Energy emitted:

$$j_\nu^{bb}\rho=\frac{h\nu}{4\pi}\cdot n_u\cdot B_{ul}\cdot\chi(\nu-\nu_0)\cdot I_\nu+$$

$$+\frac{h\nu}{4\pi}\cdot n_u\cdot A_{ul}\cdot\psi(\nu-\nu_0)$$

- Probability profiles are area normalized:

$$\int_0^\infty\varphi(\nu-\nu_0)d\nu=\int_0^\infty\psi(\nu-\nu_0)d\nu=\int_0^\infty\chi(\nu-\nu_0)d\nu=1$$

Absorption/Emission

- Absorption coefficient expressed through Einstein probabilities:

$$k_\nu^{bb}\rho = \frac{h\nu}{4\pi}\left[n_l B_{lu}\varphi(\nu-\nu_0) - n_u B_{ul}\chi(\nu-\nu_0)\right] =$$

$$= \frac{h\nu}{4\pi} n_l B_{lu}\varphi(\nu-\nu_0)\left[1 - \frac{n_u g_l \chi(\nu-\nu_0)}{n_l g_u \varphi(\nu-\nu_0)}\right]$$

- Emission coefficient:

$$j_\nu^{bb}\rho = \frac{h\nu}{4\pi}\cdot n_u \cdot A_{ul}\cdot\psi(\nu-\nu_0)$$

Source function

- Source function:

$$S_\nu^{bb} = \frac{j_\nu^{bb}}{k_\nu^{bb}} = \frac{n_u A_{ul} \psi(\nu - \nu_0)}{\left[n_l B_{lu} \varphi(\nu - \nu_0) - n_u B_{ul} \chi(\nu - \nu_0)\right]}$$

- LTE (detailed balance for each frequency) means that probability profiles are the same:

$$\varphi(\nu - \nu_0) = \psi(\nu - \nu_0) = \chi(\nu - \nu_0)$$

- Level population in LTE is described by the Boltzmann distribution:

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-\frac{\Delta E}{\mathrm{k}T}} = \frac{g_u}{g_l} e^{-\frac{h\nu}{\mathrm{k}T}}$$

Source function and absorption coefficient in LTE

- Source function in LTE is the Planck function:

$$S_\nu^{bb} = \frac{n_u A_{ul} \psi(\nu - \nu_0)}{\left[n_l B_{lu} \varphi(\nu - \nu_0) - n_u B_{ul} \chi(\nu - \nu_0)\right]} =$$

$$= \frac{n_u A_{ul}}{\left[n_l B_{lu} - n_u B_{ul}\right]} = \frac{A_{ul}/B_{ul}}{\left(n_l/n_u \cdot B_{lu}/B_{ul} - 1\right)} =$$

$$= \frac{2h\nu^3}{c^2} \cdot \frac{1}{\left(e^{h\nu/kT} - 1\right)} = B_\nu(T)$$

- Absorption coefficient in LTE:

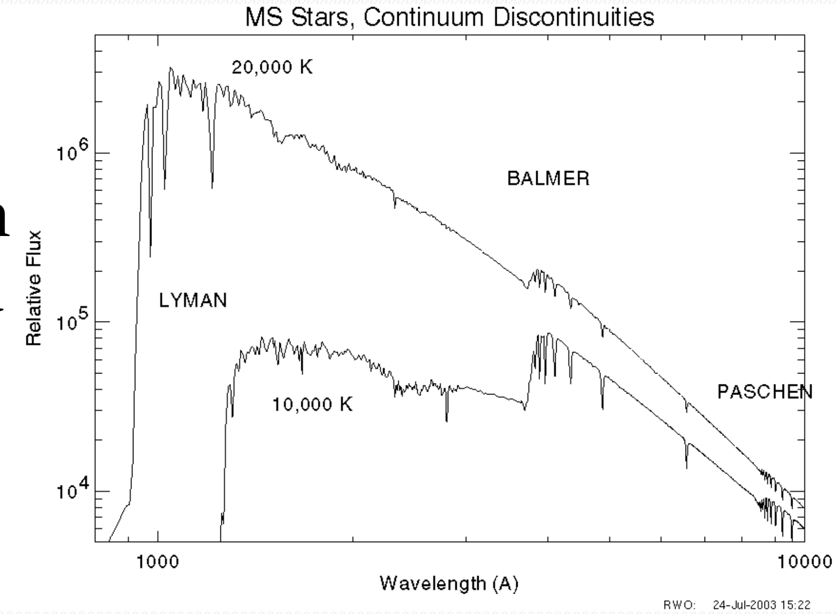
$$k_\nu^{bb} \rho = \frac{h\nu}{4\pi} n_l B_{lu} \varphi(\nu - \nu_0) \cdot \left(1 - e^{-h\nu/kT}\right)$$

Continuous opacity

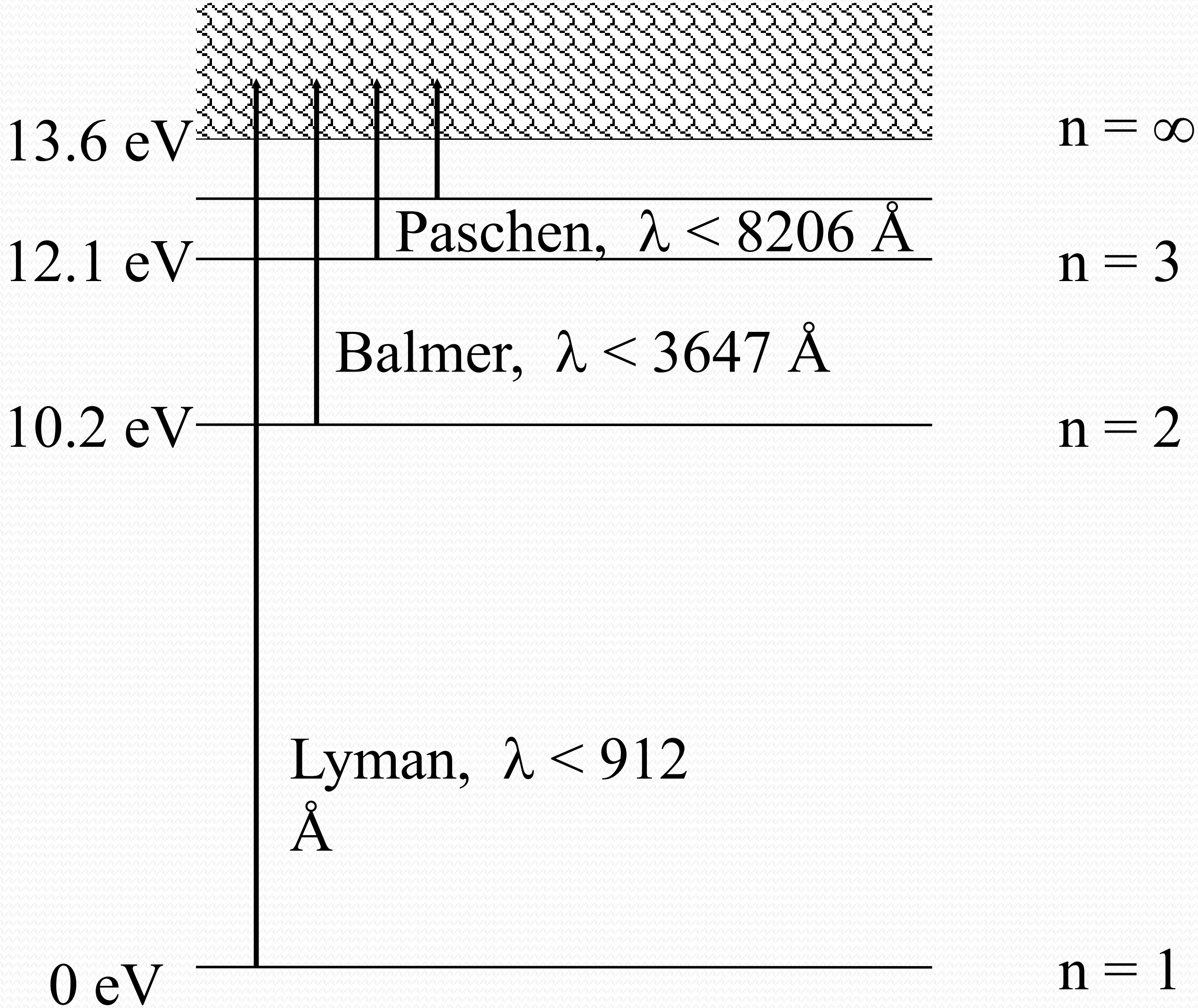
- Continuous opacity includes *b-f* (photo-ionization) and *f-f* transitions.
- Hydrogen is often a dominating source due to its abundance (H, H^-, H_2).
- For *b-f* transitions only photons with energies larger than the difference between the ionization energy and the energy of a bound level can be absorbed:

$$h\nu = hRc/n^2 + m_e v^2/2$$

This produces the absorption edges.



b-f transitions in Hydrogen



Total opacity in LTE

- *b-f* and *f-f* opacities are described by the same expression for opacity coefficient as for *b-b*. Just B_{ul} and the absorption profiles are different.
- The source function is still a Planck function.
- Total absorption:

$$k_\nu = k_\nu^{bb} + k_\nu^{bf} + k_\nu^{ff}$$

Line profile

- The one thing left is the absorption probability profile.
- Spectral lines are not delta-functions due to three effects:

Lorentz
  - Damping by radiation (finite life-time)
  - Perturbation of atomic energy level system by neighboring particles

Gaussian
  - Doppler movements of absorbers/emitters

- The convolution the Lorentz and Doppler profile results in Voigt profile:

$$\mathrm{H}(a,v) = \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{(v-y)^2+a^2}dy$$

Now… we know everything

- RT equation: $\frac{dI_\nu}{dx} = -k_\nu \cdot \rho \cdot I_\nu + k_\nu \cdot \rho \cdot S_\nu$
- Boundary condition: $I_\nu(\tau_\infty) = B_\nu(T_\infty)$
- Absorption coefficient: $k_\nu = k_\nu^{bb} + k_\nu^{bf} + k_\nu^{ff}$
- Absorption profile: $\mathrm{H}(a, v)$
- Source function $B_\nu(T)$

Practical implementation

- Maxwellian velocity distribution:

$$P(v_x) = \frac{1}{\sqrt{\pi} \cdot \bar{v}} \cdot e^{-\left(\frac{v_x}{\bar{v}}\right)^2} \; ; \quad \bar{v} = \sqrt{\frac{2kT}{m_A}}$$

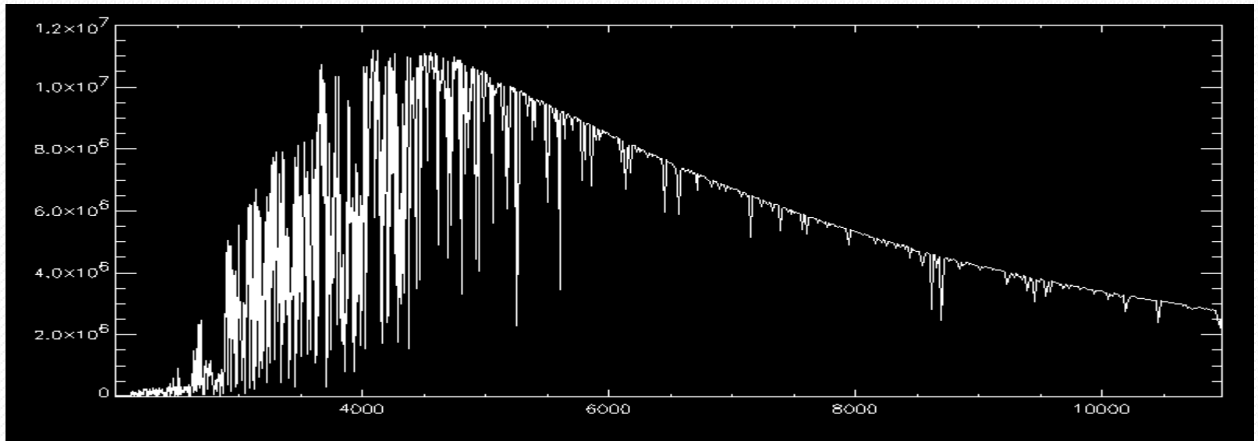
- Boltzmann level population

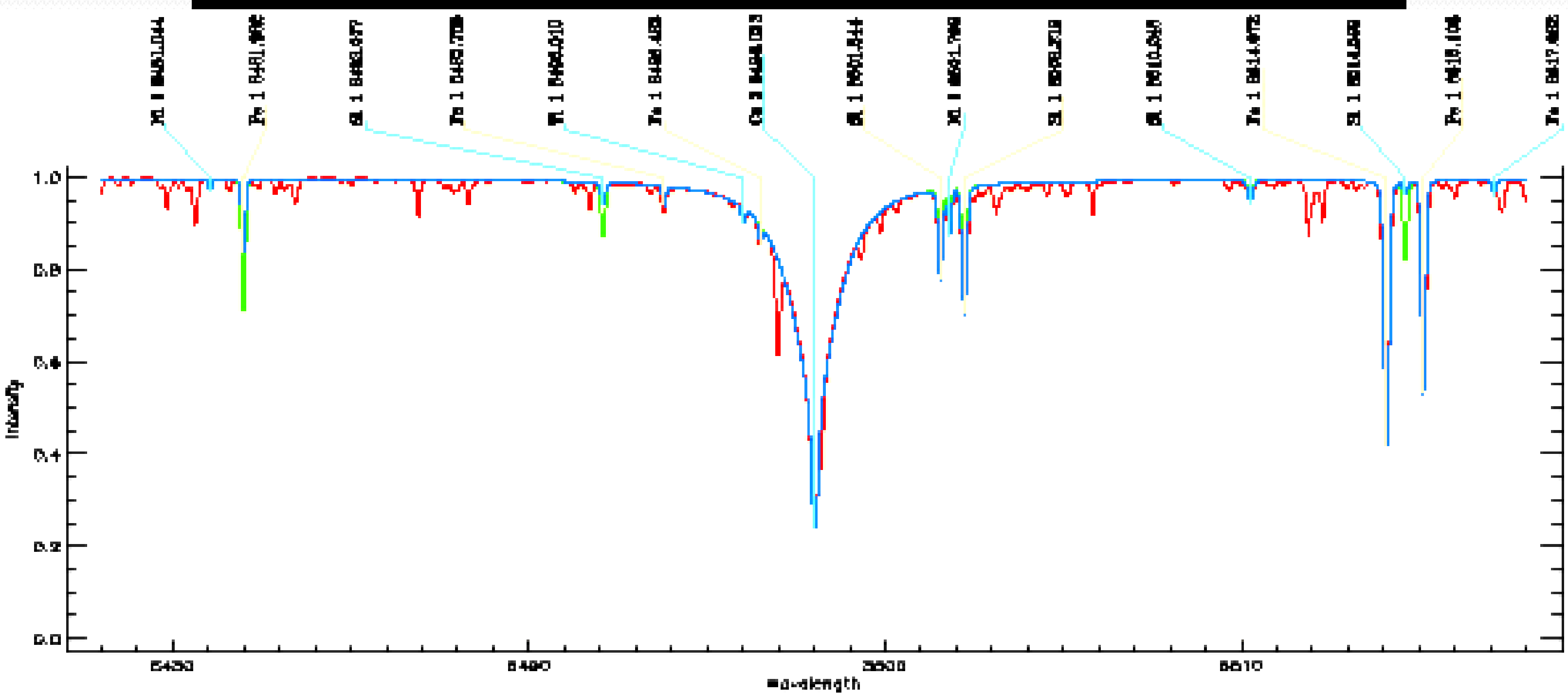
$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-\frac{(E_u - E_l)}{kT}}$$

- Saha ionization balance

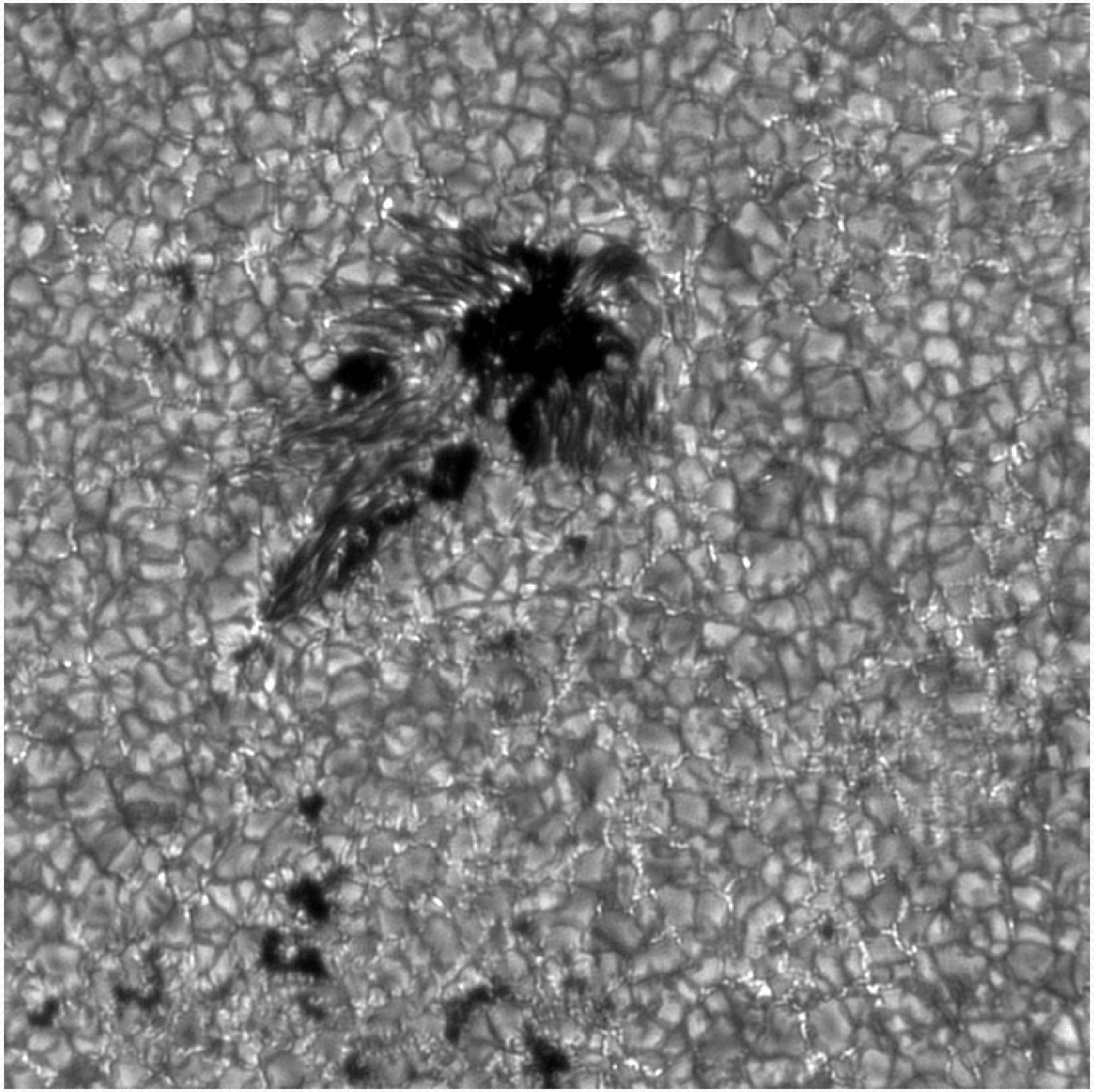
$$\frac{n_{i+1}}{n_i} = \frac{1}{N_e} \cdot \frac{2\,U_{i+1}}{U_i} \cdot \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \cdot e^{-\frac{E_i}{kT}}$$

How good is LTE for solving RT?





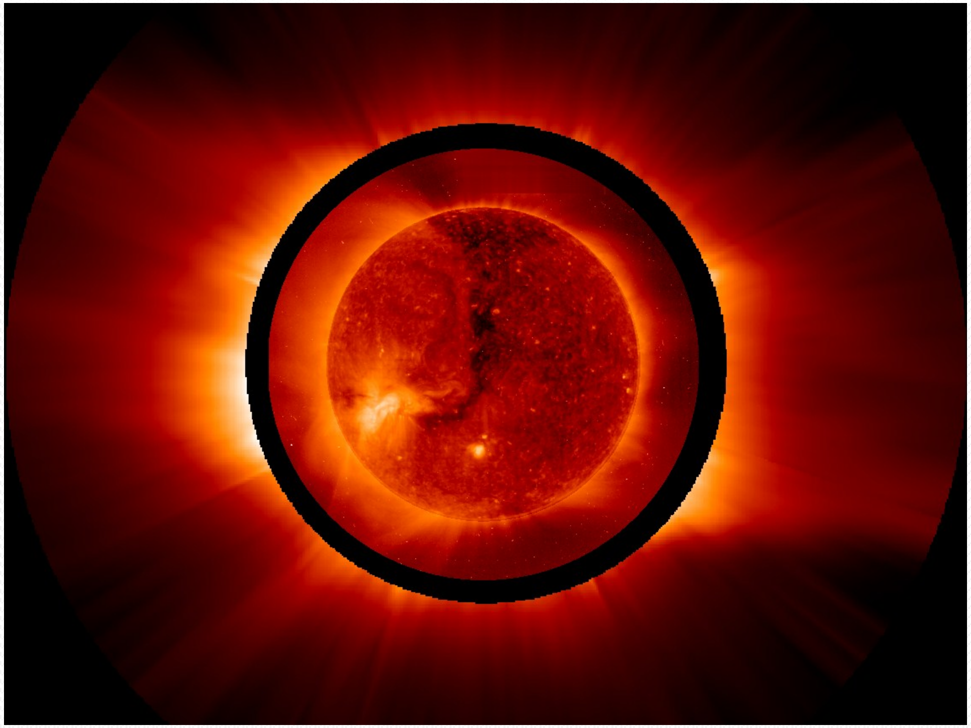
Stellar surfaces



Courtesy of Göran Scharmer/Swedish Solar Vacuum Telescope

Energy transport in stars

- Radiation
- Convection
- Particle ejection
- Waves

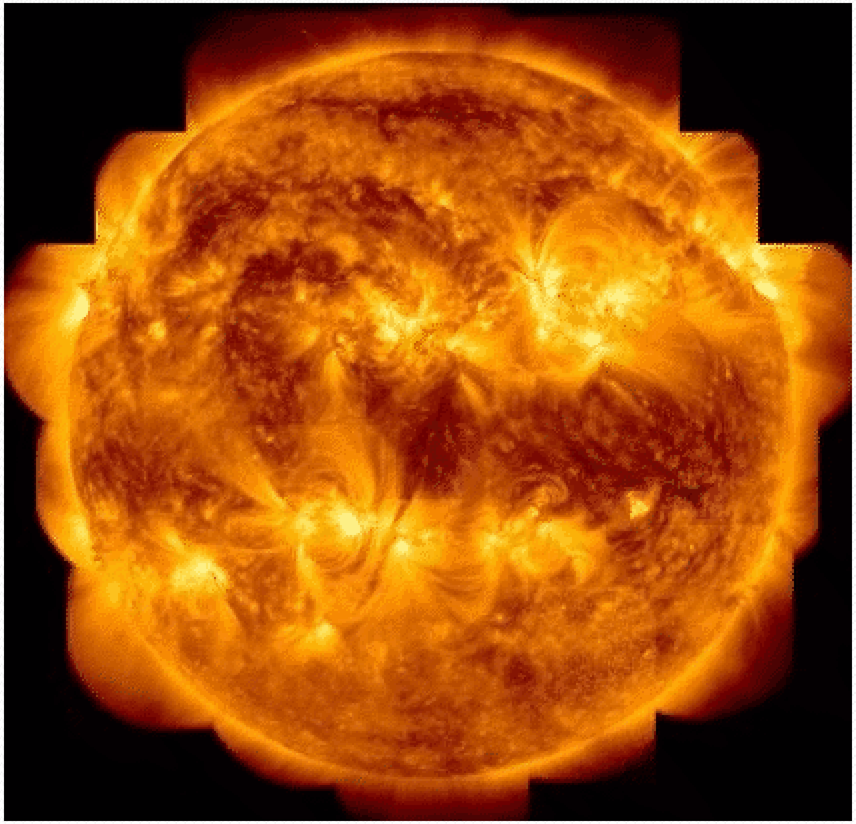


Space above surface is not empty!

Courtesy of SOHO

Solar corona

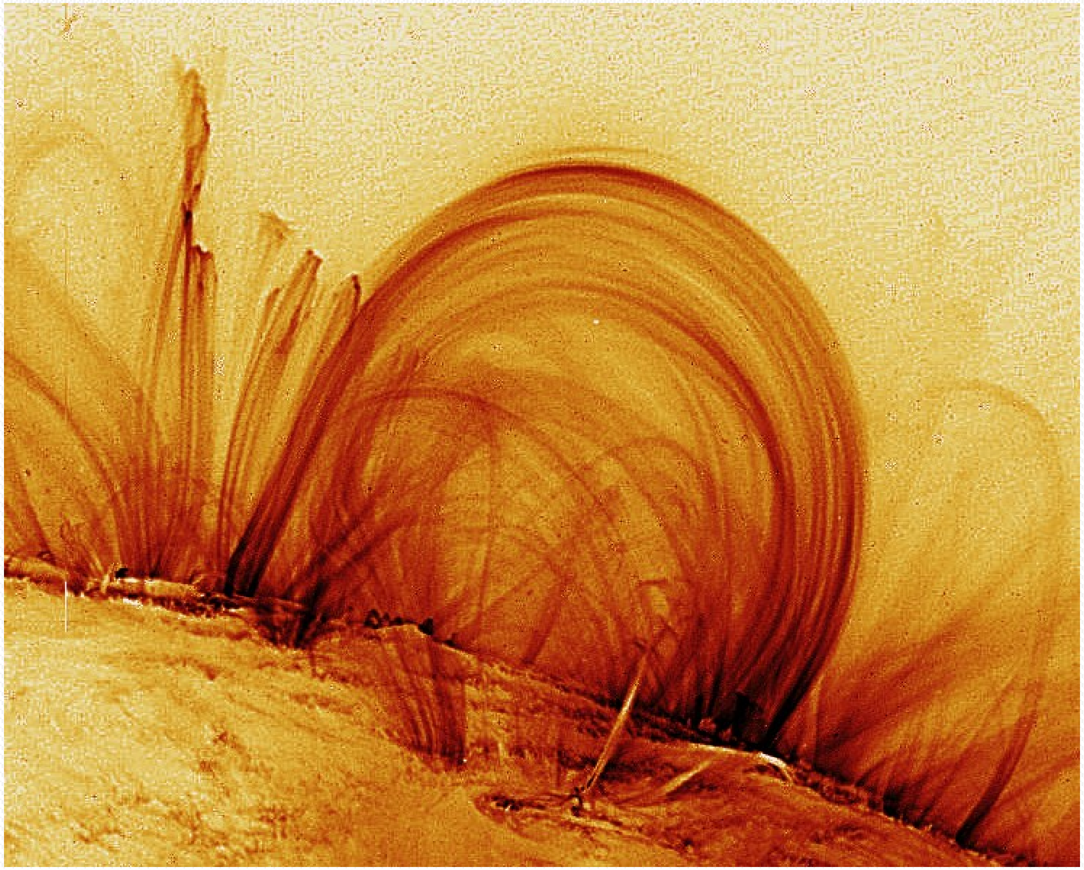
- Low density: $<10^7$ particle/cm^3
- Hot: $>10^6$ K
- Optically thin
- Temperature of radiation is very different from the kinetic temperature



Courtesy of SOHO

Coronal arcades on the Sun

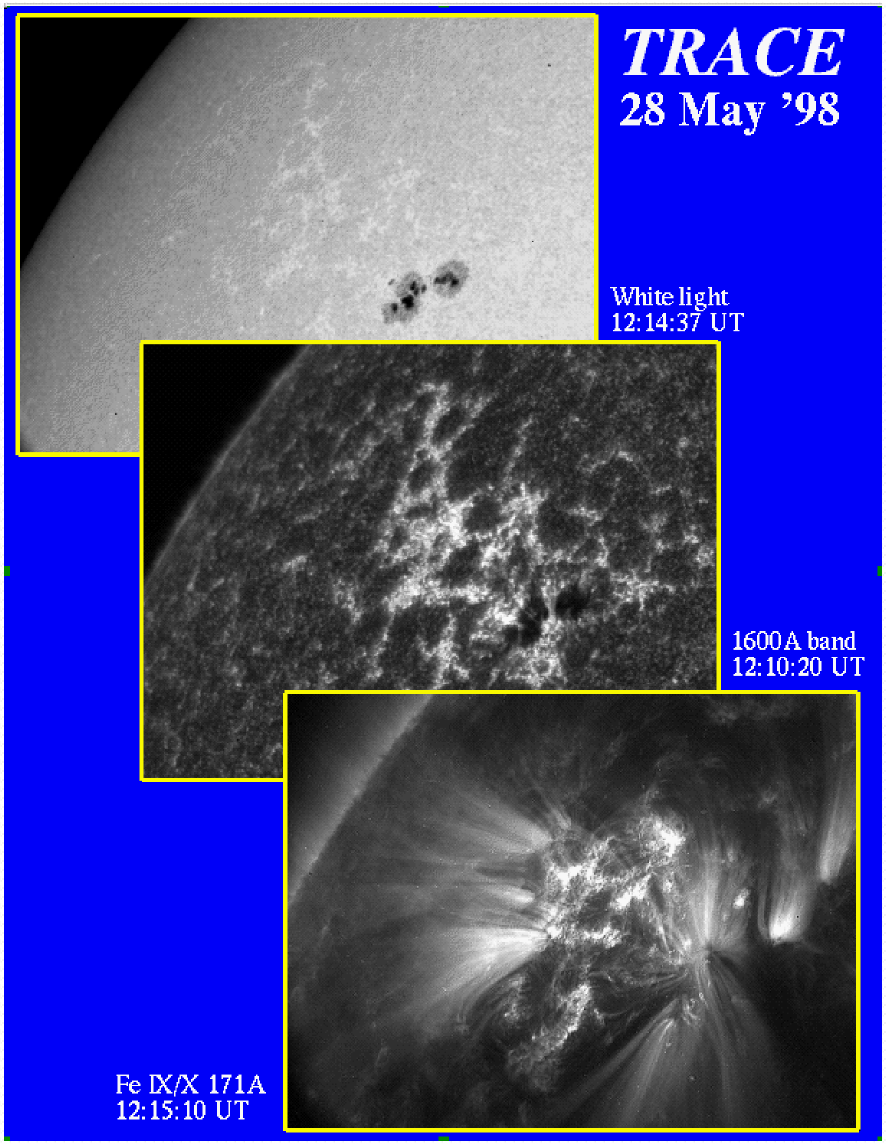
- Heating of the outer layers is part of the energy transport
- Magnetic fields play a major role
- Coherent motions at the photosphere layers dissipate in the corona making it hot



Courtesy of Karel Schrijver/TRACE

How can observe space above solar surface?

- At visual spectral range photosphere dominates the total flux
- UV lines allow to see chromospheric structures
- Going to X-ray is required to observe solar corona



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