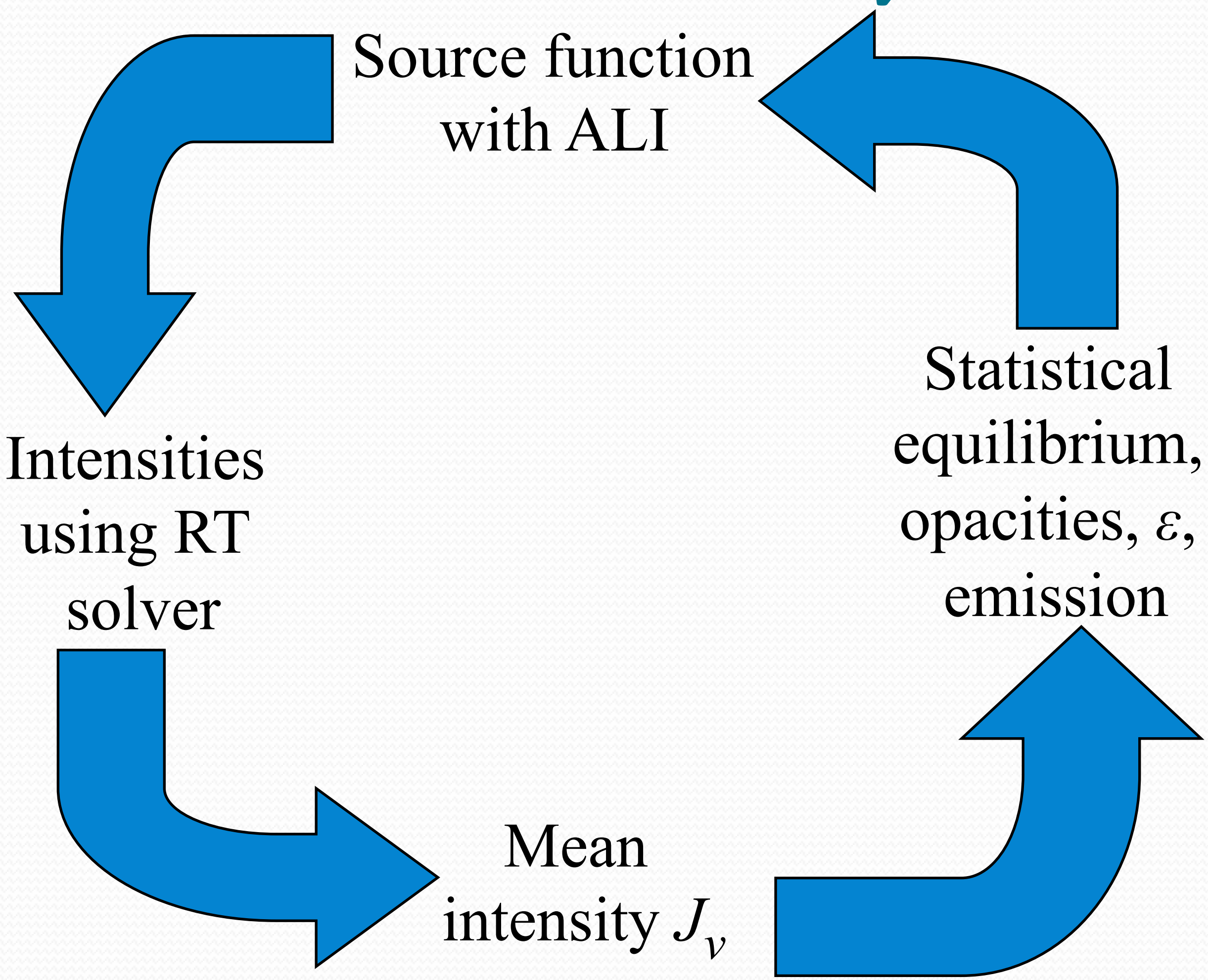
RT with hydrodynamics in multi-dimensions

A few important aspects that we managed to avoid so far

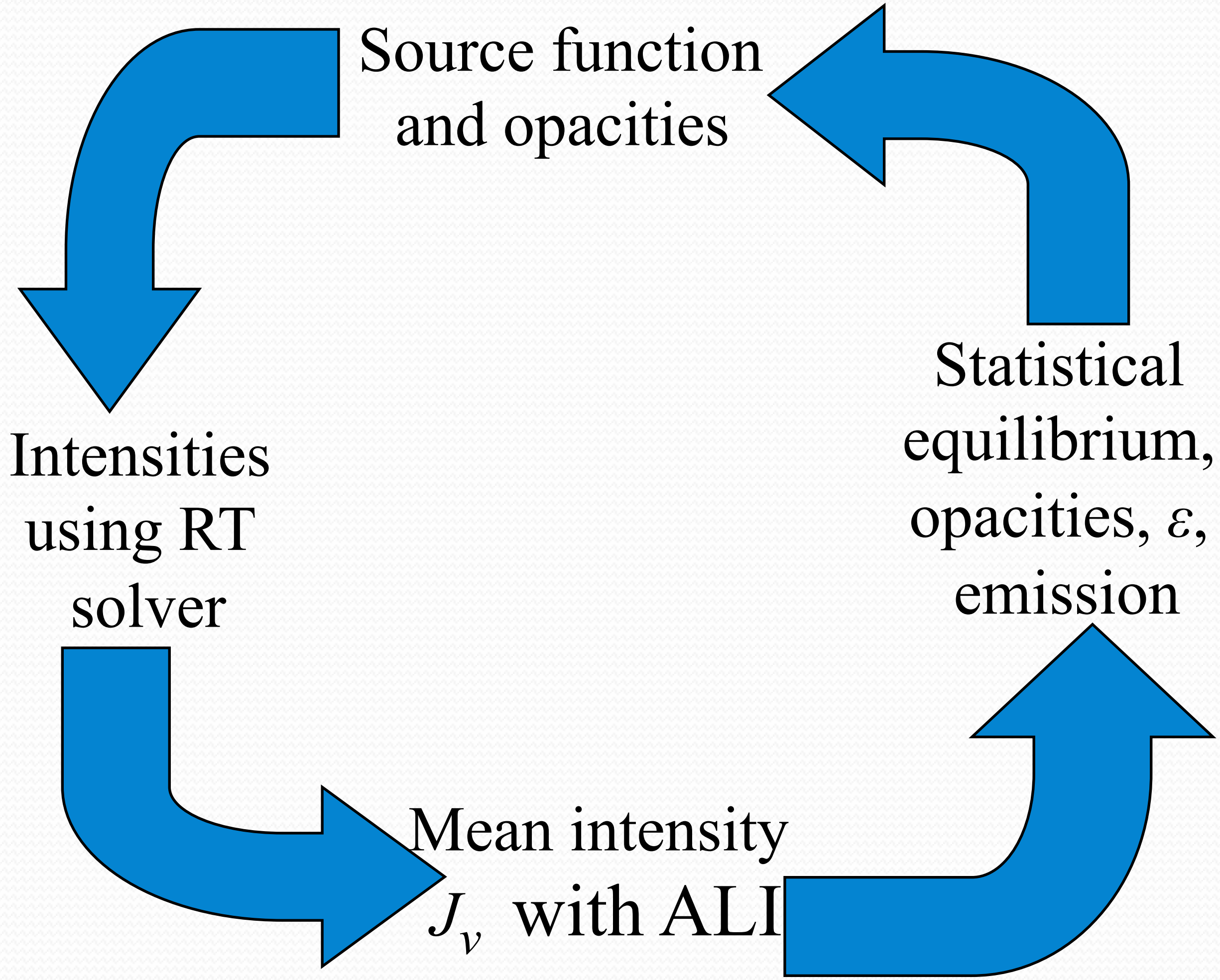
What did we skip?

1. A complete ALI loop
2. RT in 3D
3. RT in dynamic environment

The great loop using S_ν

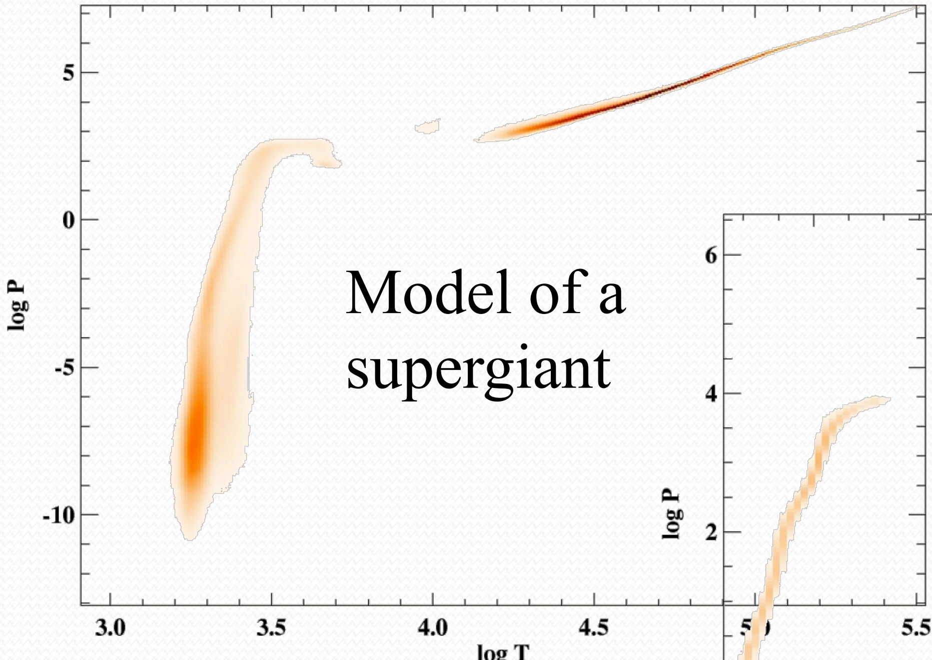


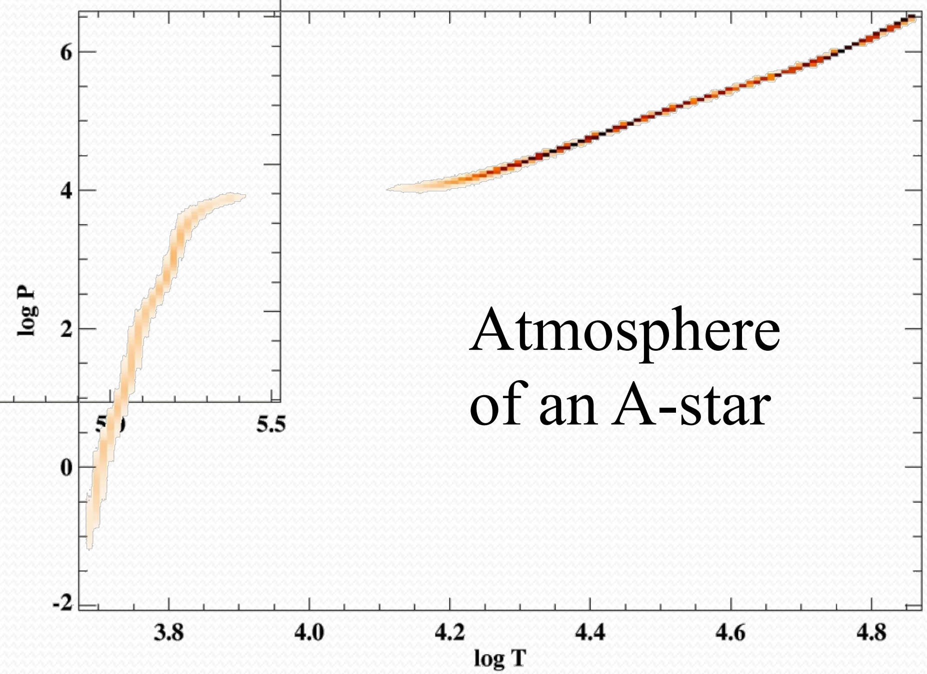
… or a great loop using J_ν



Opacity Tables

For optically thick parts we can assume LTE and pre-compute opacity tables. For stellar atmospheres Temperature-Pressure is sparsely populated:





This is why 1D models work!

Attenuation Operator

- Solution of RT over one grid cell can be written as:

$$I_\nu(\tau_i) = e^{-(\tau_i - \tau_{i-1})} \cdot I_\nu(\tau_{i-1}) + \\ + \int_{\tau_{i-1}}^{\tau_i} S_\nu(t) \cdot e^{-(\tau_i - t)} dt$$

τ is the optical path along the ray, S is a source function

- If S can be approximated by a polynomial in τ we can take the integral above analytically:

$$I_\nu(\tau_i) = I_\nu(\tau_{i-1}) \cdot e^{-(\tau_i - \tau_{i-1})} + \\ + \alpha_{\nu,i} S_{\nu,i-1} + \beta_{\nu,i} S_{\nu,i} + \gamma_{\nu,i} S_{\nu,i+1}$$

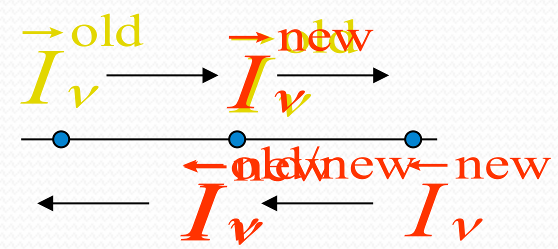
Accelerated Lambda Iterations

Scattering requires Λ-iterations

ALI based on Gauss-Seidel algorithm

(Socas-Navarro & Trujillo Bueno 1997, ApJ 490, p.383)

Main idea:



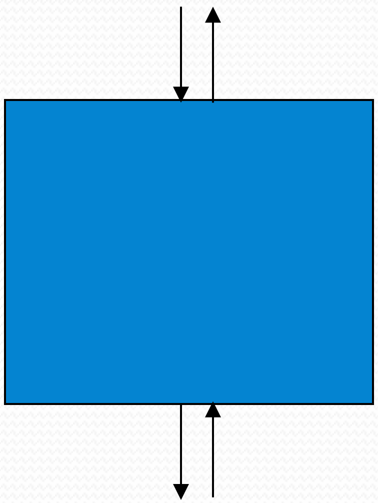
Incoming ray

$$\vec{I}_\nu(\tau_i) = \vec{I}_\nu(\tau_{i-1}) \cdot e^{-\left(\tau_i - \tau_{i-1}\right)} + $$
$$+ \alpha_{\nu,i} S_{\nu,i-1} + \beta_{\nu,i} S_{\nu,i} + \gamma_{\nu,i} S_{\nu,i+1}$$

Outgoing ray

$$\overleftarrow{I}_\nu(\tau_i) = \overleftarrow{I}_\nu(\tau_{i+1}) \cdot e^{-\left(\tau_i - \tau_{i+1}\right)} + $$
$$+ \alpha_{\nu,i} S_{\nu,i+1} + \beta_{\nu,i} S_{\nu,i} + \gamma_{\nu,i} S_{\nu,i-1}$$

Accelerated Lambda Iterations



1) $J \approx \frac{1}{2}\left(I_1^{\rightarrow} + I_1^{\leftarrow}\right)$ $\quad$ $J \approx \frac{1}{4}\left(I_1^{\rightarrow} + I_1^{\leftarrow} + I_1^{\downarrow} + I_1^{\uparrow}\right)$

2) $J \approx \frac{1}{4}\left(I_2^{\rightarrow} + I_2^{\leftarrow} + I_1^{\downarrow} + I_1^{\uparrow}\right)$ $\quad$ $J \approx \frac{1}{4}\left(I_2^{\rightarrow} + I_2^{\leftarrow} + I_2^{\downarrow} + I_2^{\uparrow}\right)$

…

How do we know when to stop?

Our options

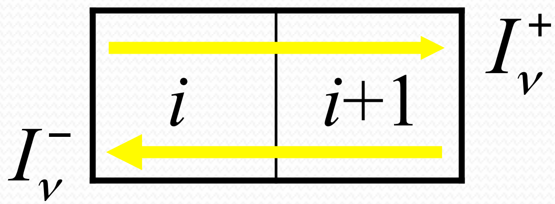
- Mean intensities
- Level populations (best choice)
- Source function
- ...

Short or *long* ?

- In case of distributed sources and optically thick medium both long and short characteristics work fine
- Feautrier readily produces mean intensity needed for statistical equilibrium calculations
- In case of small number of point sources and optically thin medium implementation is more critical the choice of characteristics

Hydro with RT

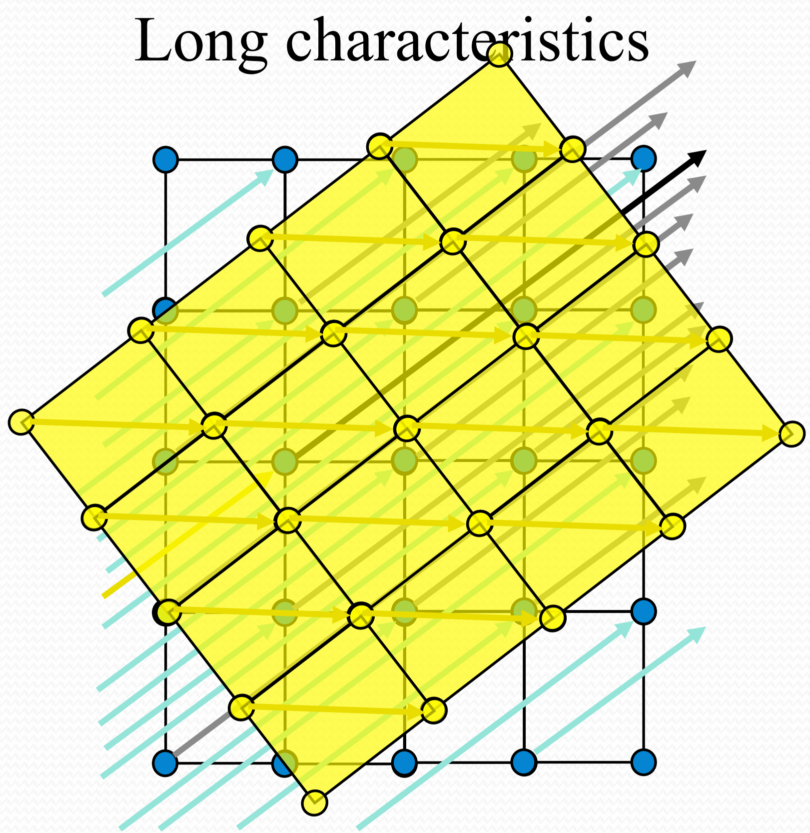
- Hydro needs energy transported by radiation:

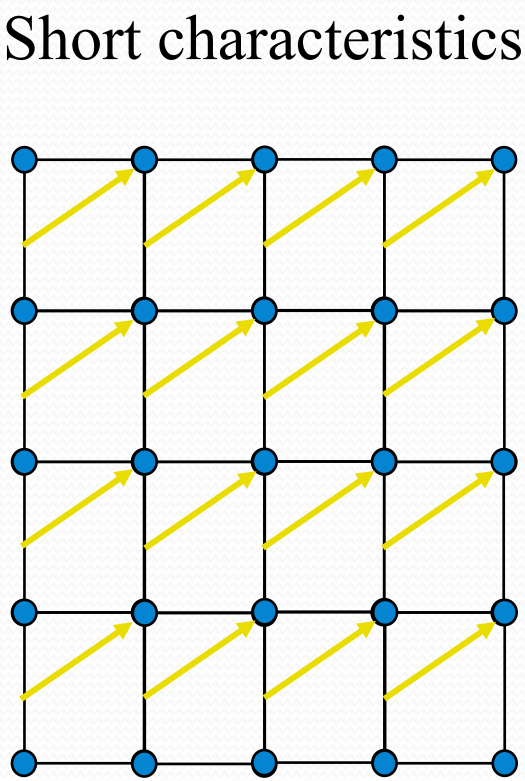


Energy flow between *i* to *i*+1 is $I_\nu^+ - I_\nu^-$ (Feautrier V_ν!)

- We can compute V_ν in two ways:
  - (short char.) integrate I_ν^+ and I_ν^- separately
  - or (long characteristics) …

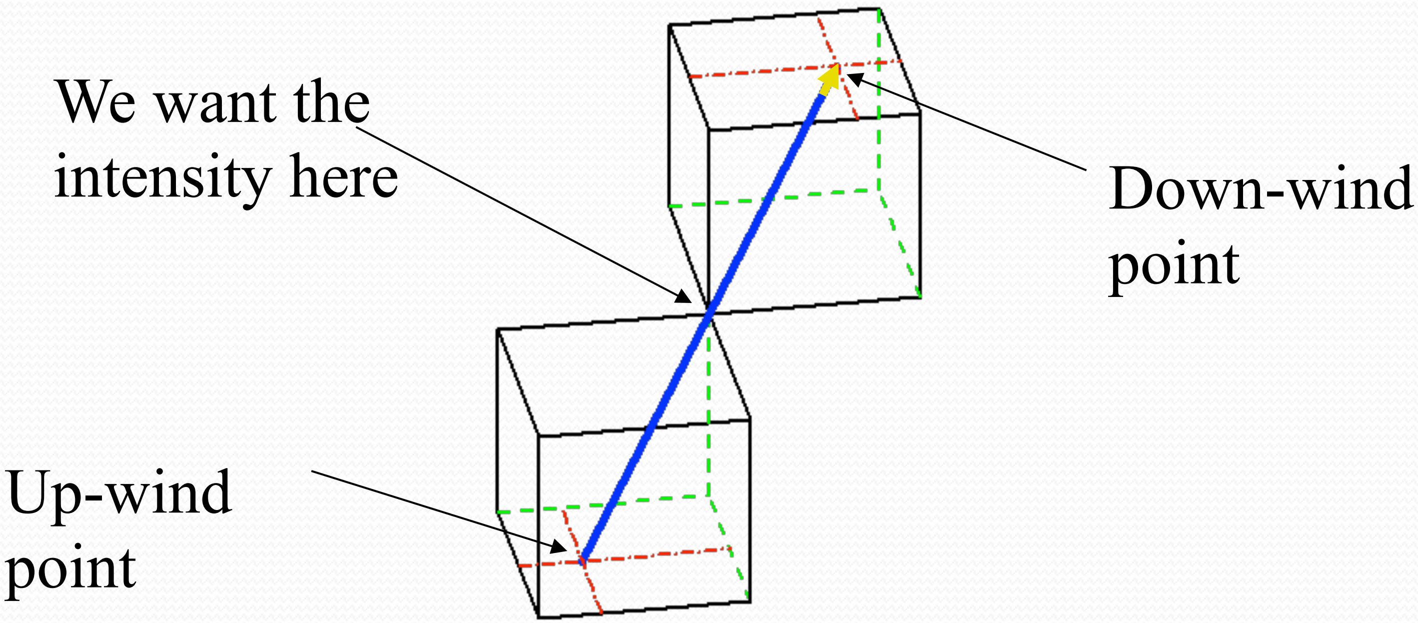
RT Solver: Short Characteristics





Interpolation in 3D

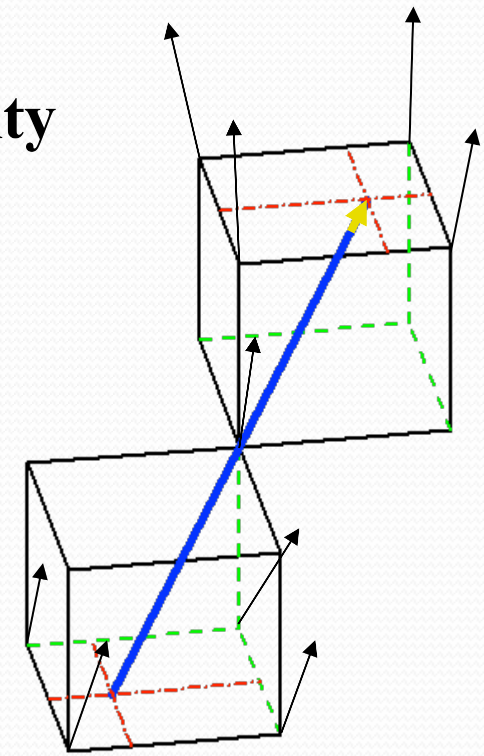
We find up-wind point by interpolating source function, opacity/scattering and intensity between 4 grid nodes



Velocities in 3D

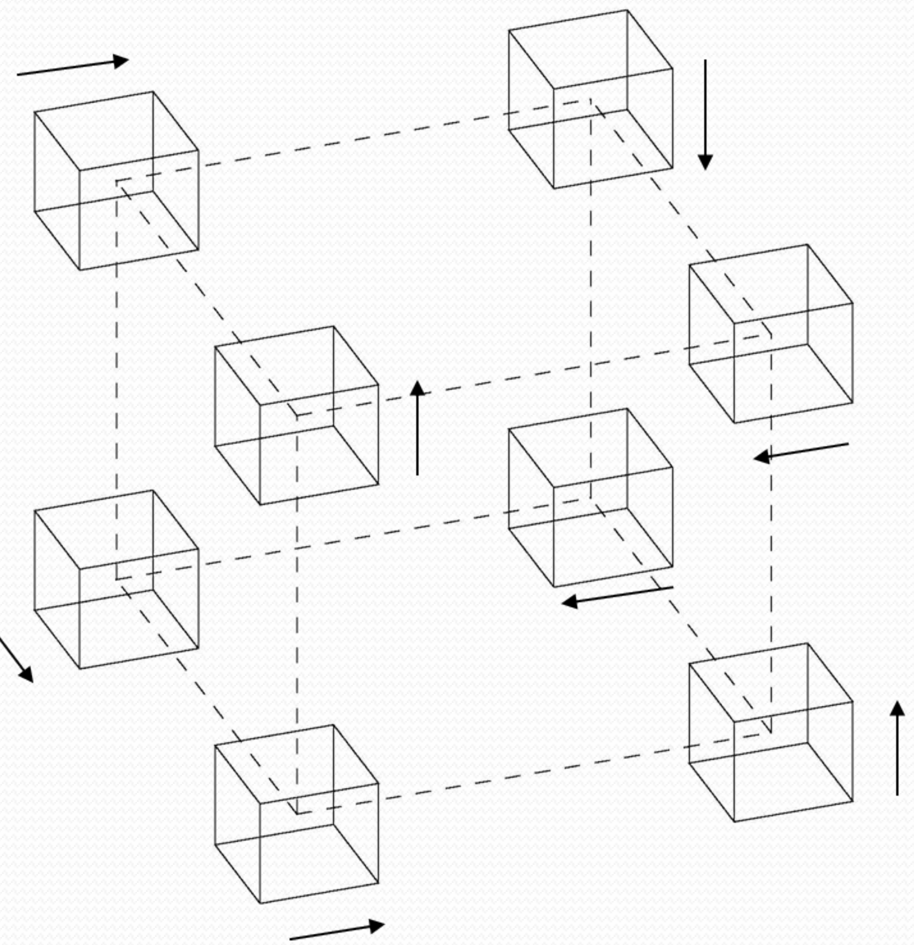
Before interpolation source function and opacities must be converted to the laboratory reference frame

Velocity field



ALI in 3D and in parallel…

The most time consuming part is the source function integration but it can be done in all sub-cube in parallel! Propagation of intensity is serial but is just: $I = aI + b$

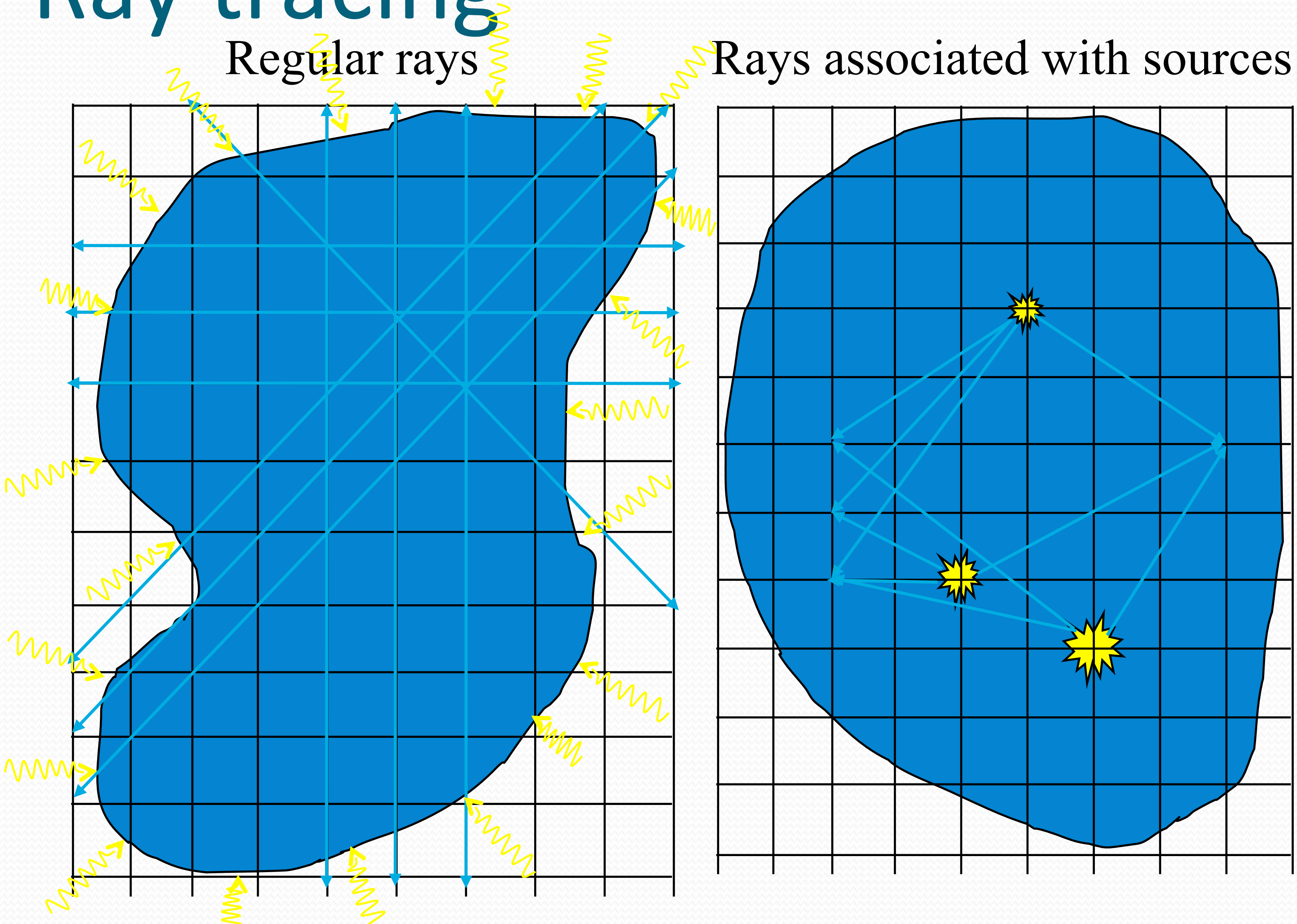


Ray sequence

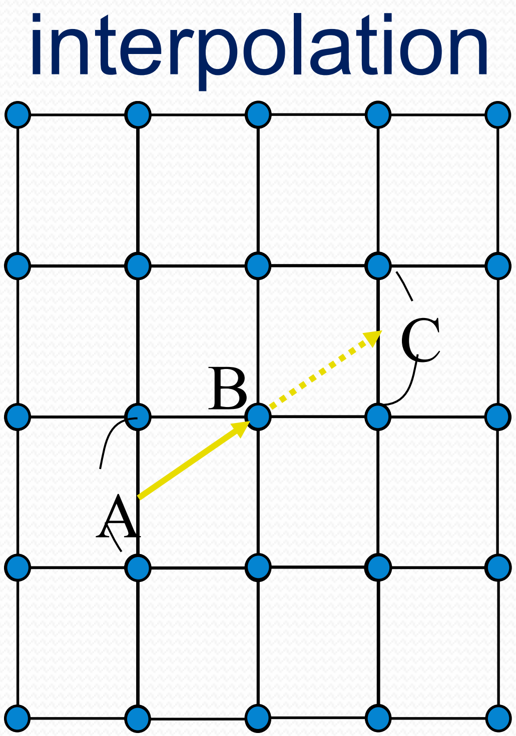
Optically THIN versus optically **thick**

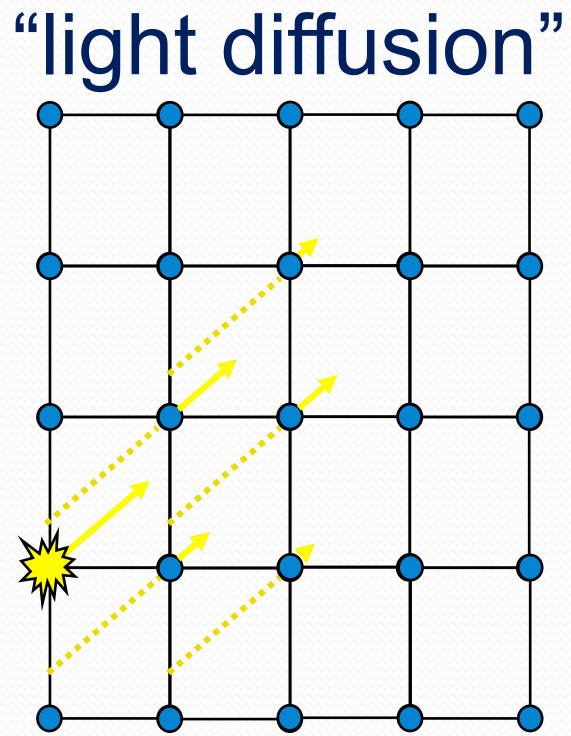
- Long characteristics: somewhat more expensive – limit the number of rays
- Short characteristics: + and – directions are needed anyways
- Two regimes: optically thick/thin cells
- Problems with point sources

Ray tracing



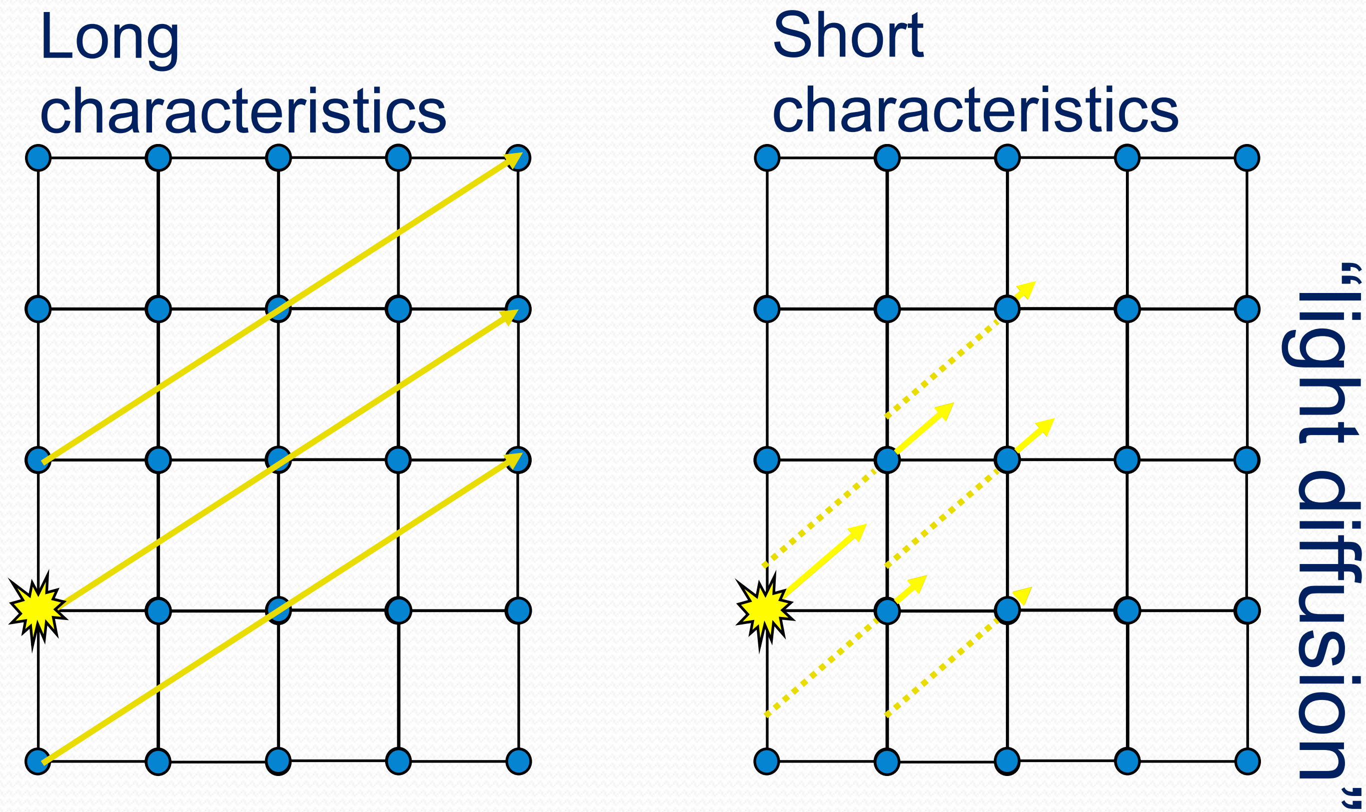
Optically thin





When using short characteristics collimated beams diverge.

Optically thick

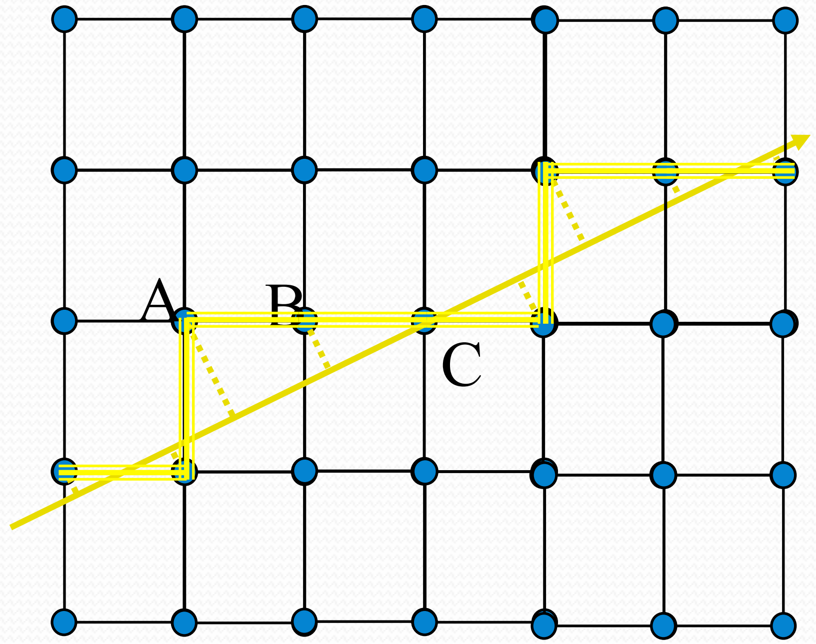


"light diffusion" can actually be good!

Optically thin

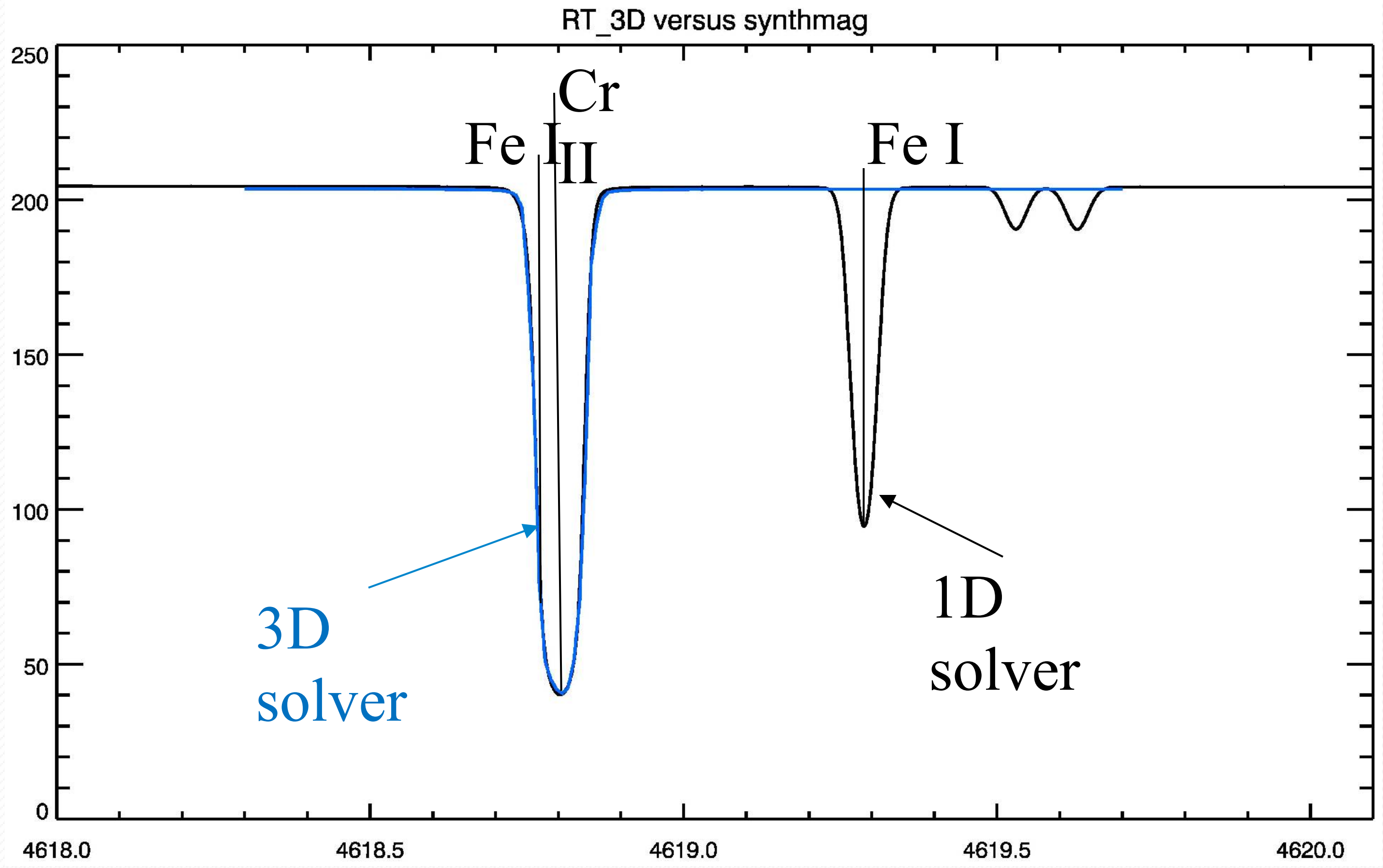
Nearest neighbor or “Fat rays”:

Lower order but no light diffusion



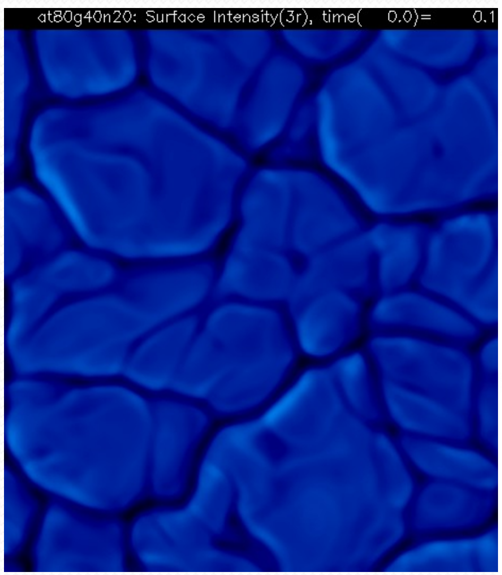
Testing

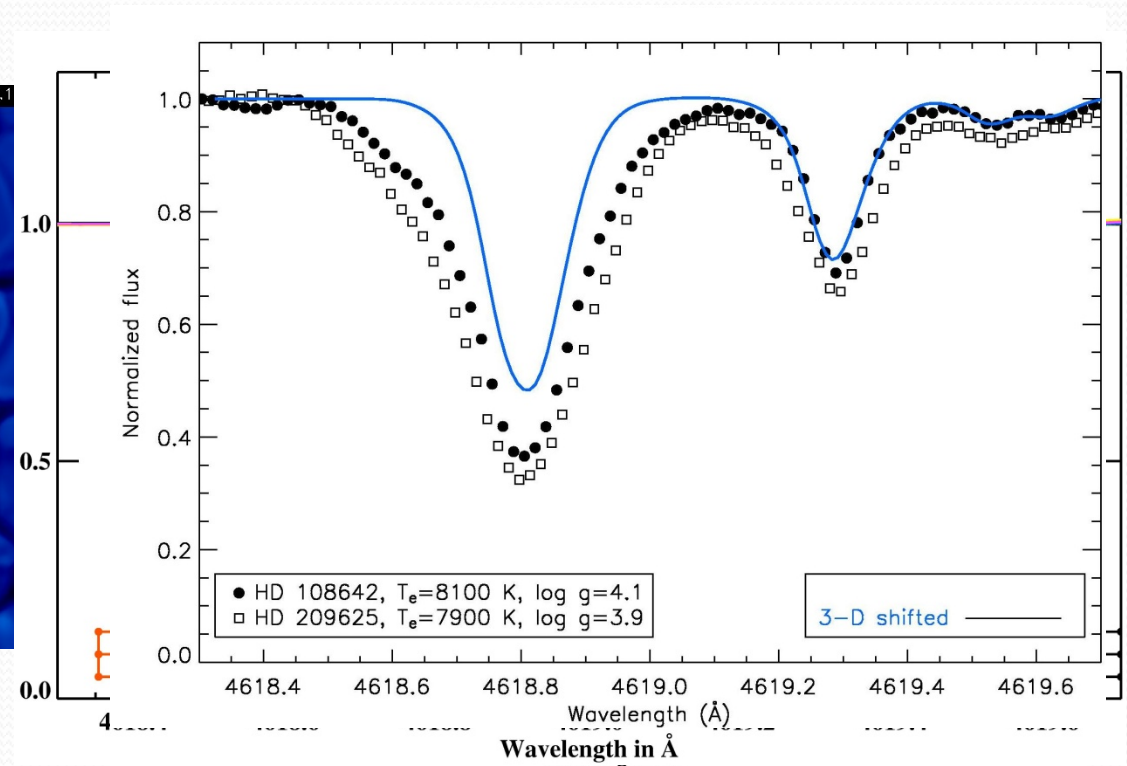
1D model replicated in a 3D cube



Applications: A-stars

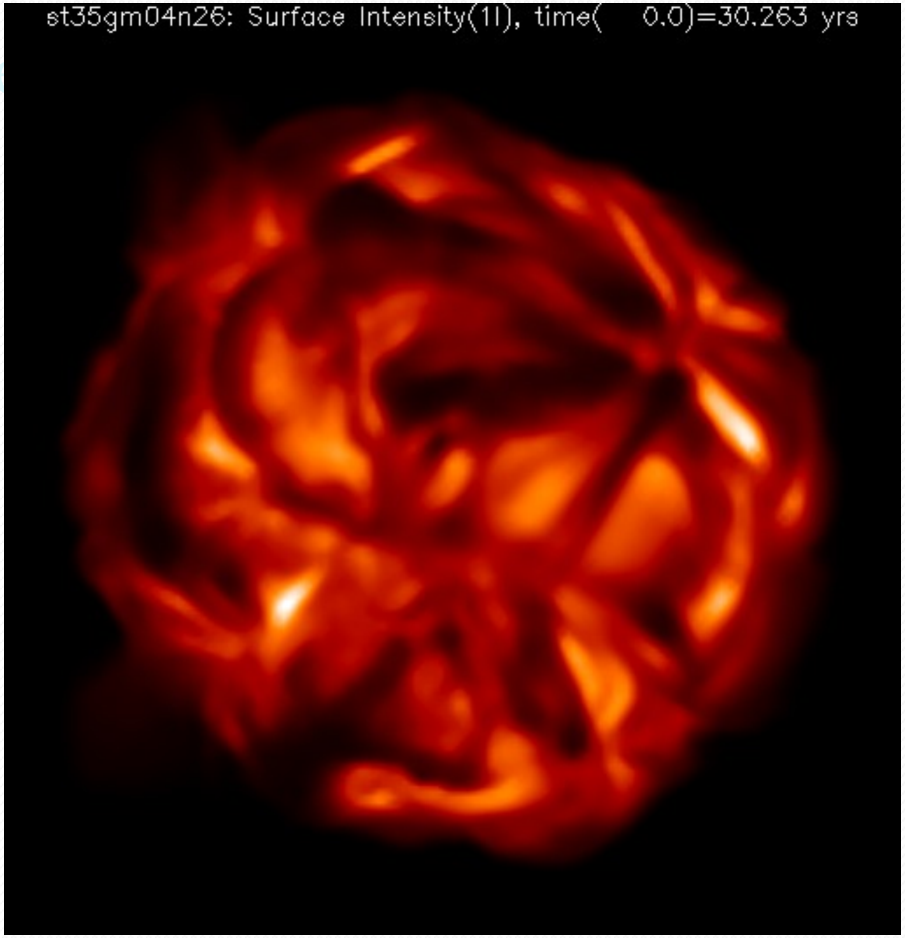
- Convective zone is very thin but detectable in lines
- CO[5]**BOLD** model: “box-in-a-star”
- Predicted line asymmetry was *wrong*…at first





Applications: Betelgeuse

- COBOLD model: "star-in-a-box"
- Star in differe



Applications: Betelgeuse

- COBOLD model: "star-in-a-box"
- Star in different wavelengths