# Introduction to Polarimetry

Lecture 7

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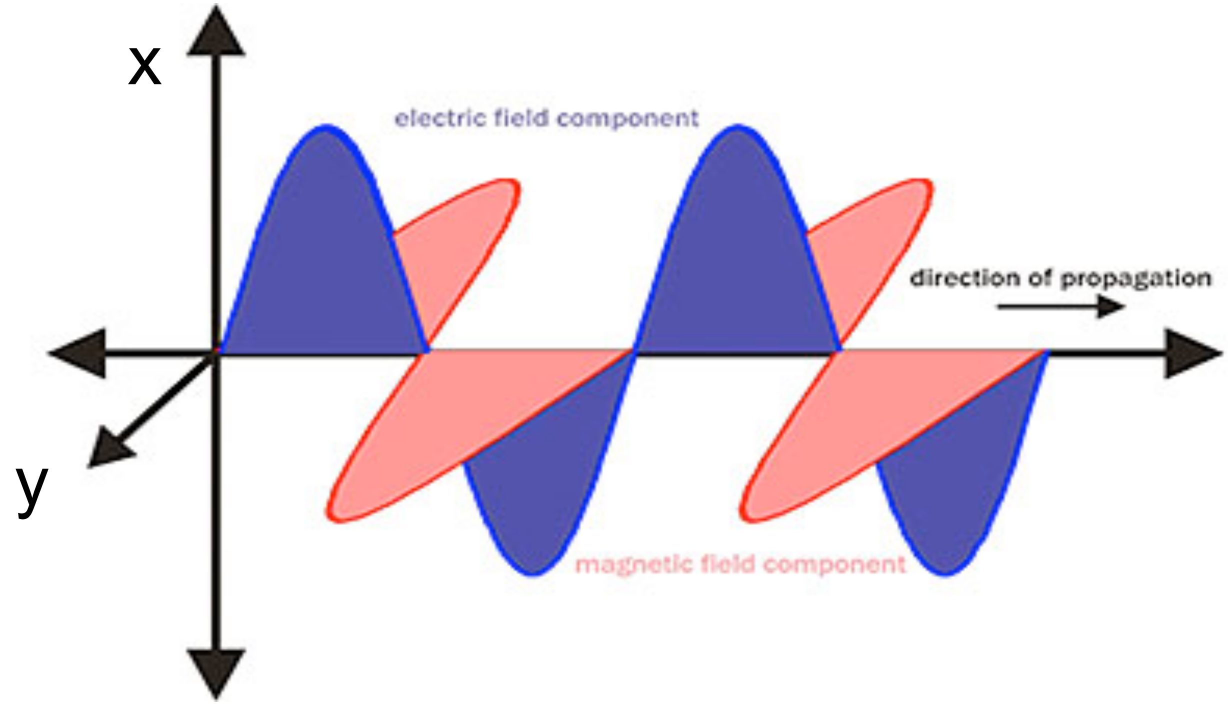
# Outline

- Mathematical description of polarised light
- Physical processes creating polarisation
- Examples of astrophysical polarimetry

# Electromagnetic wave

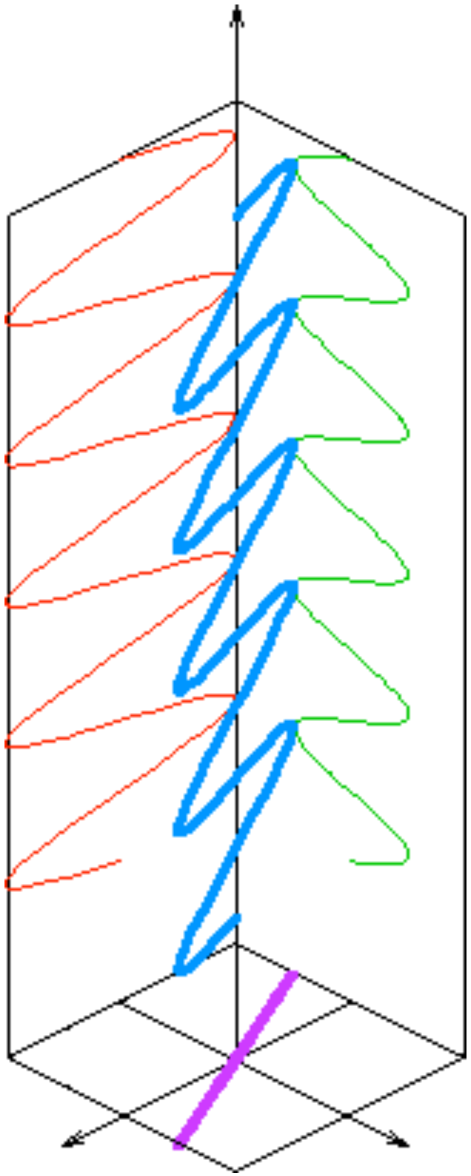



$$\boldsymbol{E}(t) = \{E_x(t), E_y(t)\}$$

$$E_x(t) = E_x(0)\cos\left(\omega t - \phi_1\right)$$
$$E_y(t) = E_y(0)\cos\left(\omega t - \phi_2\right)$$

Polarisation = evolution of ***E(t)***

# Polarised EM waves



Polarisation is defined by

$$E_x(0), E_y(0), \delta \equiv \phi_1 - \phi_2$$

- Linear polarisation:

$$\phi_1 = \phi_2$$

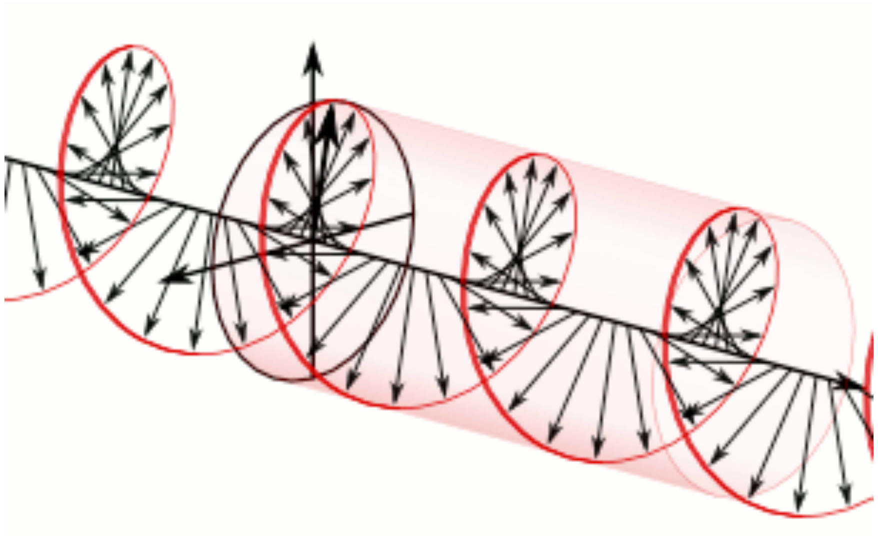
$$E_x(t) = E_x(0)\cos(\omega t)$$

$$E_y(t) = E_y(0)\cos(\omega t)$$

# Polarised EM waves

Polarisation is defined by

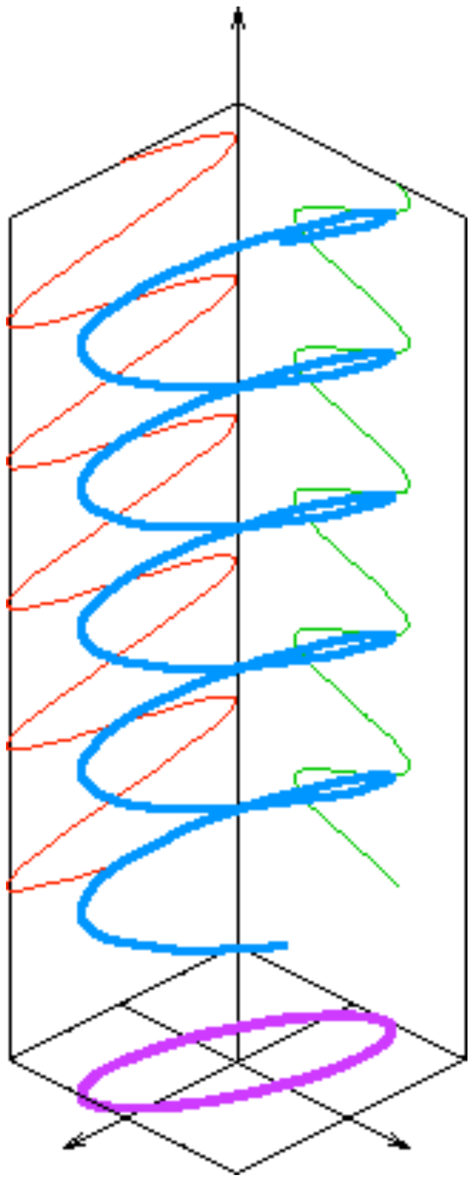
$$E_x(0), E_y(0), \delta \equiv \phi_1 - \phi_2$$



- Circular polarisation:

$$\phi_2 = \phi_1 \pm \pi/2$$
$$E_x(0) = E_y(0) = E_0$$
$$E_x(t) = E_0 \cos(\omega t)$$
$$E_y(t) = \pm E_0 \sin(\omega t)$$

# Polarised EM waves



Polarisation is defined by

$$E_x(0), E_y(0), \delta \equiv \phi_1 - \phi_2$$

- Elliptical polarisation (general case):

$$E_x(0) \neq E_y(0)$$

$$\phi_1 \neq \phi_2 \neq m\pi/2$$

# Stokes parameters

Astronomical observations: average light intensities instead of EM fields

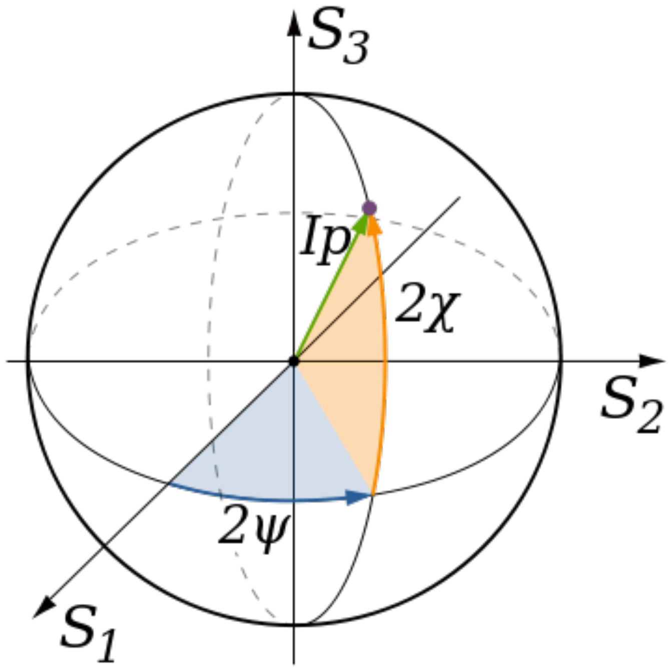
G. Stokes (1852)

$$I = \langle E_x^2 \rangle + \langle E_y^2 \rangle$$

$$Q = \langle E_x^2 \rangle - \langle E_y^2 \rangle$$

$$U = 2\langle E_x E_y \cos\delta \rangle$$

$$V = 2\langle E_x E_y \sin\delta \rangle$$

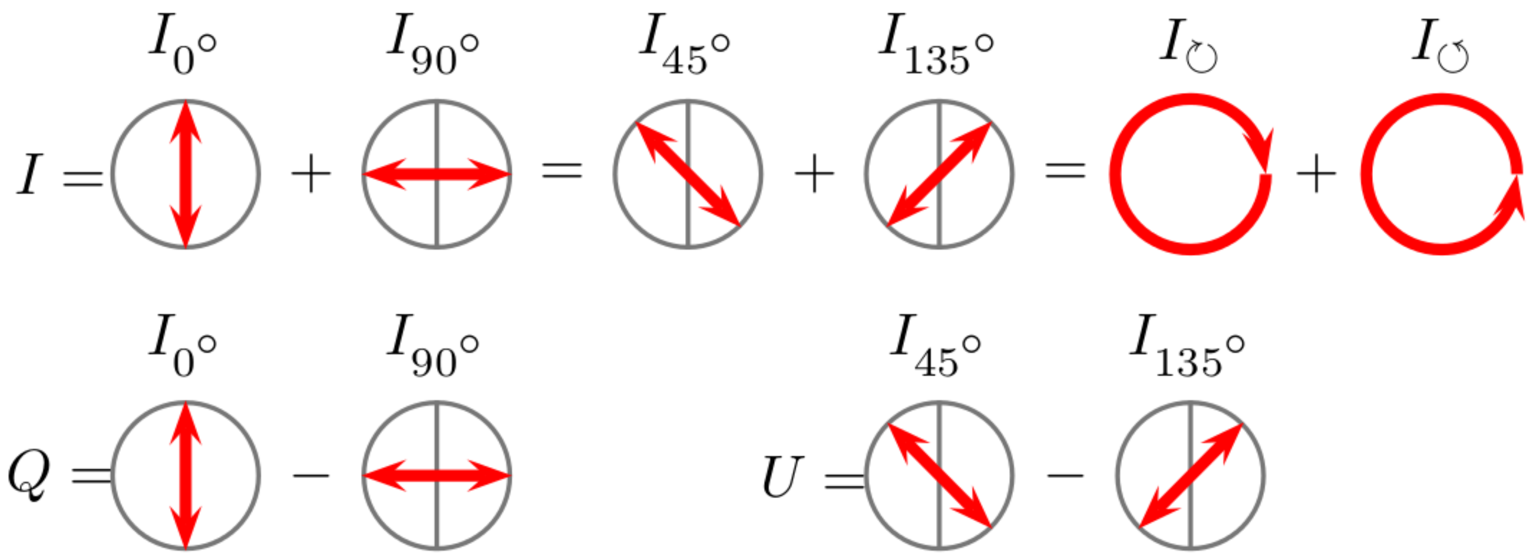


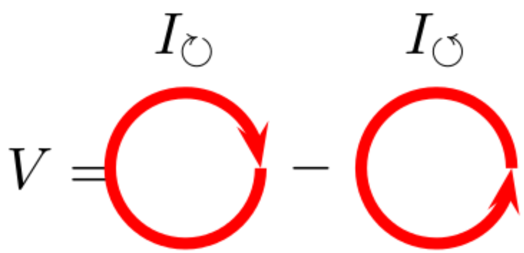

Poincare sphere

$\{S_1, S_2, S_3\} = \{Q, U, V\}$

# Stokes parameters

## Operational definition using ideal polarisers

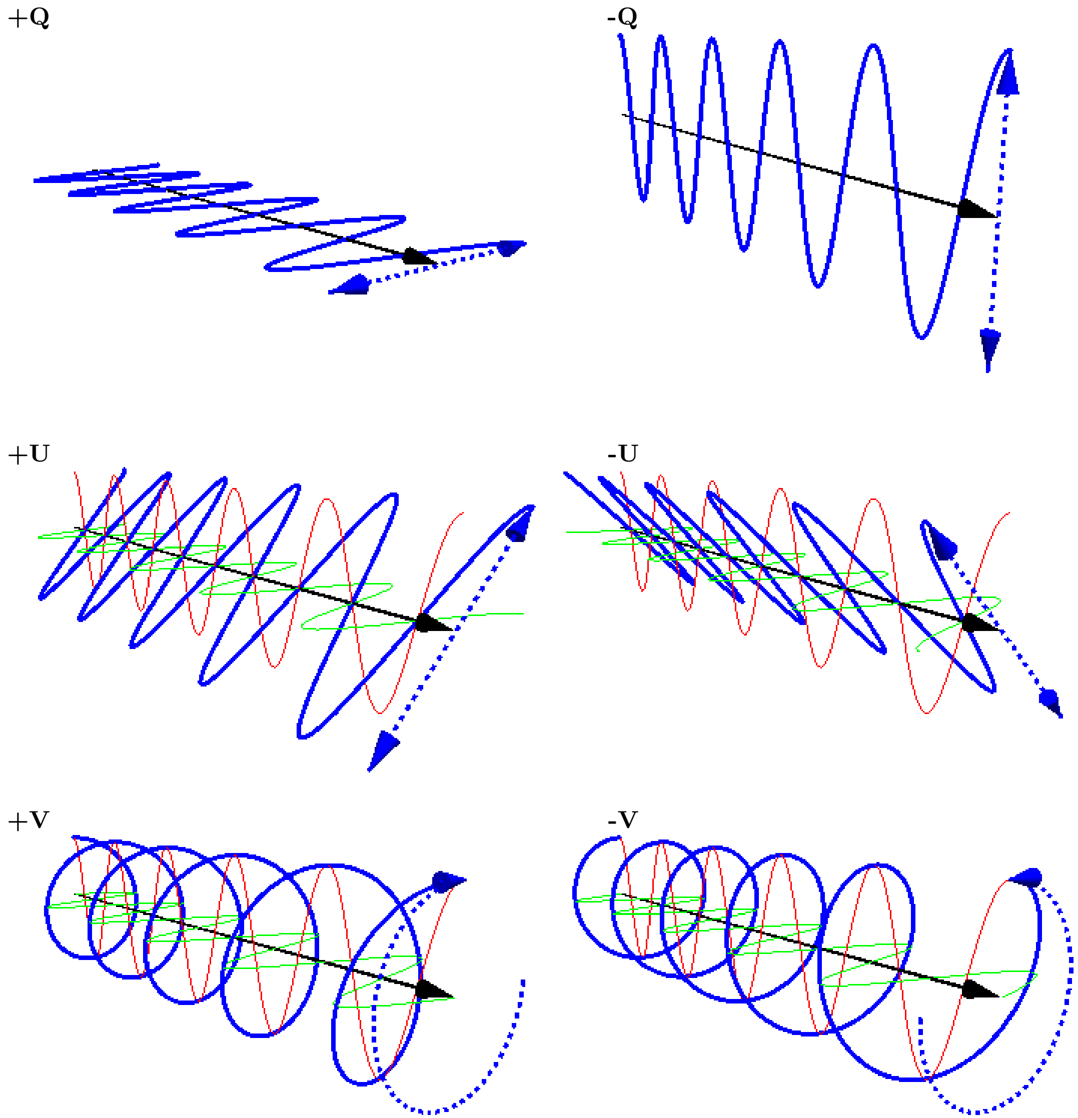



$I_{\alpha^\circ}$ intensity measured through perfect linear polarizer

$I_{\circlearrowright}, I_{\circlearrowleft}$ intensity measured through perfect right-hand and left-hand polarizers

# Sign conventions




+V: clockwise rotation of electric vector

+U: counterclockwise rotation by 45$^o$ from +Q

+Q: freely chosen

- Local meridian
- Instrument
- Plane of scattering
- Solar limb

# Normalised and fractional polarisations

- Normalised to Stokes *I*

$$P_X = X/I, X = Q, U, V$$

- Normalised to continuum of Stokes *I*

$$P_X^{(c)} = X/I_c, X = Q, U, V$$

- Fractional linear polarisation

$$P_L = \sqrt{Q^2 + U^2}/I$$

$$\theta = \frac{1}{2}\arctan\left(U/Q\right)$$

# Mueller matrix formalism

Stokes vector changes due to an interaction with matter or astronomical instrument

$$\boldsymbol{S} = \{I, Q, U, V\}^T$$

***M*** is 4 x 4 Mueller matrix

$$\boldsymbol{S}_1 = \boldsymbol{M}\boldsymbol{S}_0 \qquad \boldsymbol{S}_1 = \boldsymbol{M}_n \ldots \boldsymbol{M}_2\boldsymbol{M}_1\boldsymbol{S}_0$$

ideal reflection

$$\boldsymbol{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

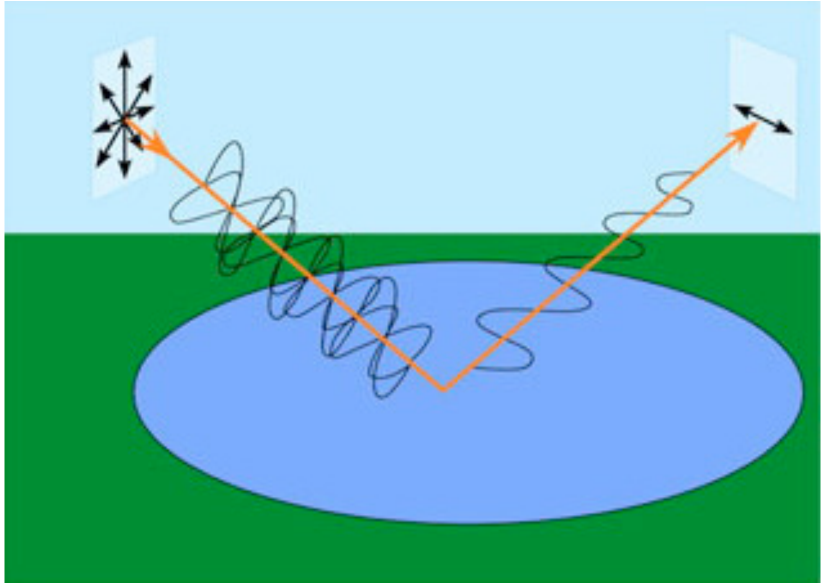
ideal linear polariser

$$\boldsymbol{M} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

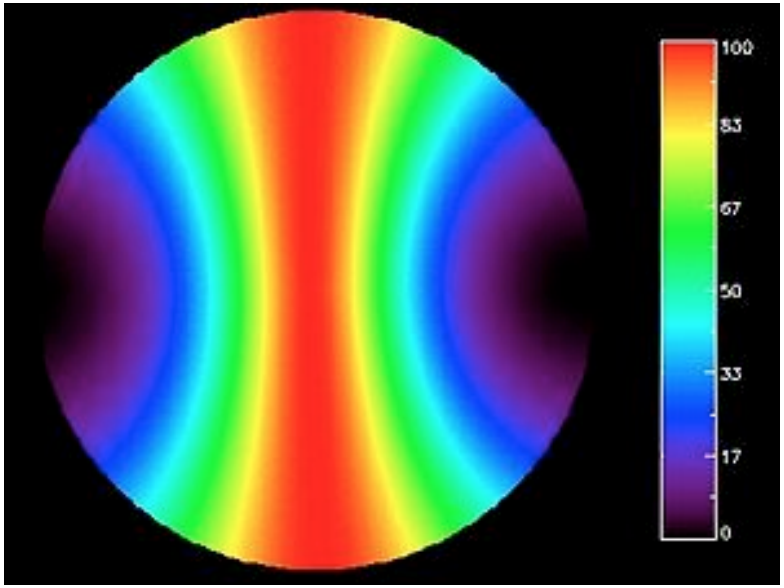
# Macroscopic polarisation

- Every EM wave is intrinsically polarised
- “Natural”, unpolarised light contains a mixture of EM waves of all possible polarisation states
- Macroscopic polarisation signal = statistical preference of a certain polarisation state

# Polarisation in everyday life

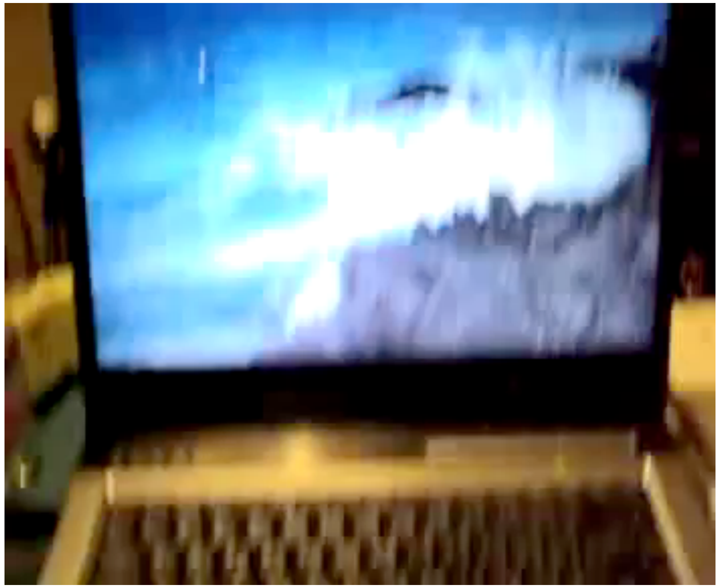


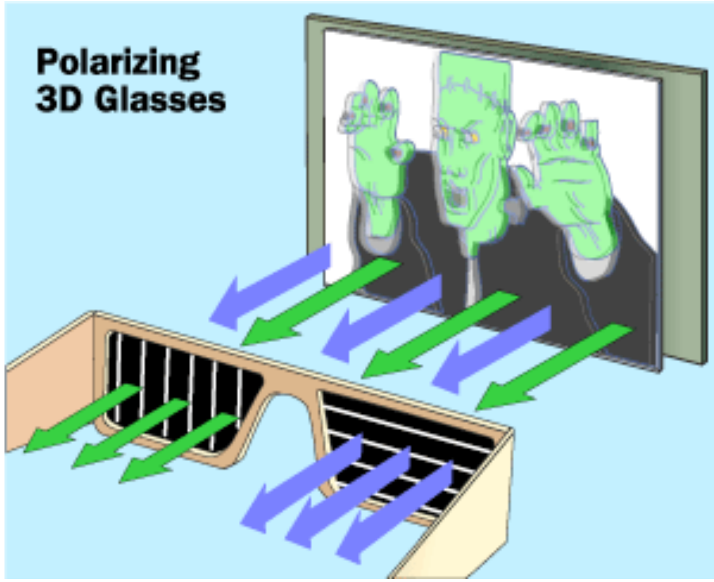
Reflection from water/ice

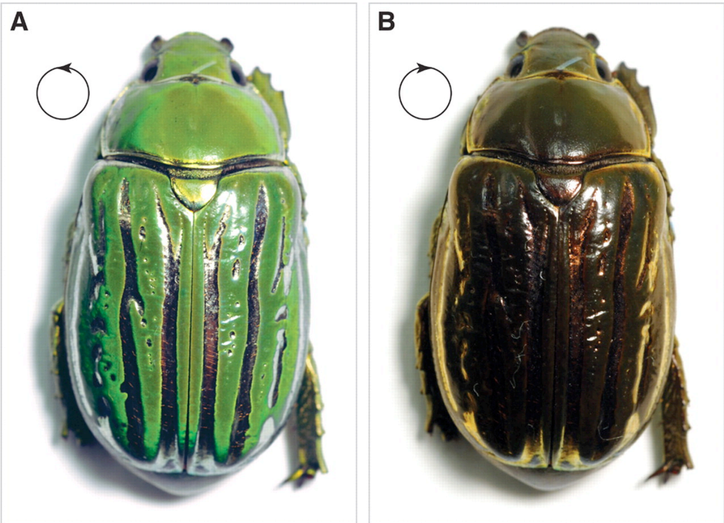



Rayleigh scattering


# Polarisation in everyday life





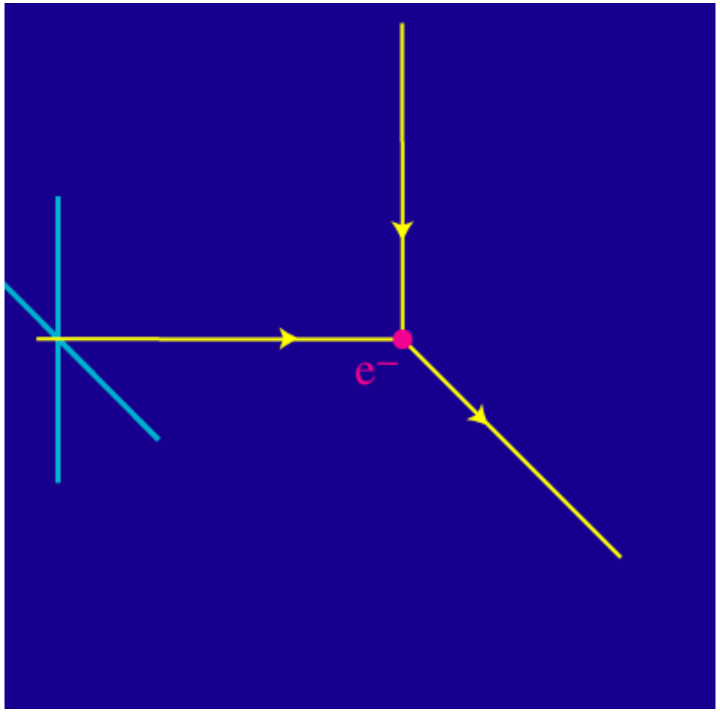



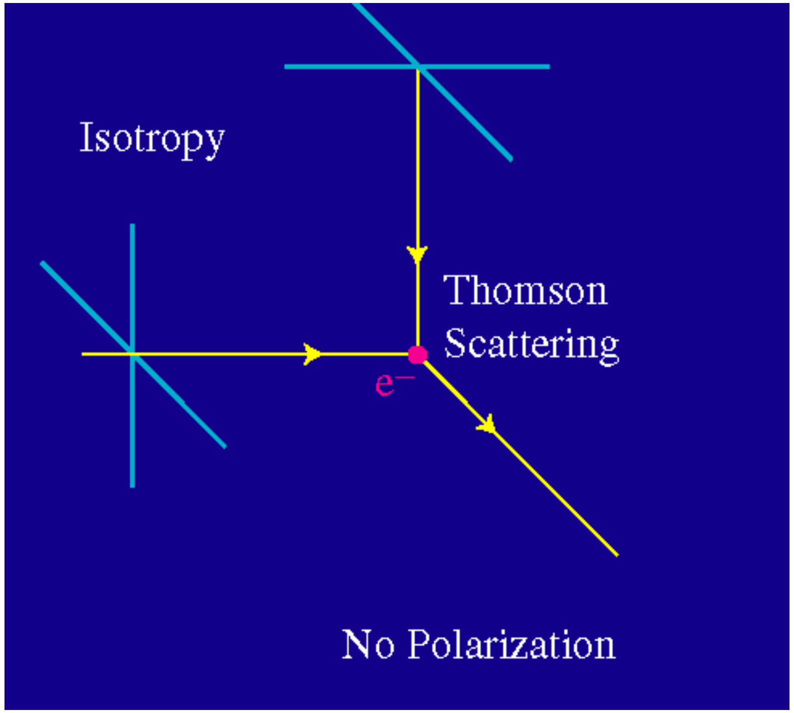

# Polarisation mechanisms

| | |
|---|---|
| Anisotropic scattering/reflection | *continuum* |
| Differential absorption/scattering by aligned non-spherical grains | *continuum* |
| Synchrotron radiation from charged particles in a magnetic field | *continuum* |
| Magnetic line polarisation (Zeeman effect) | *lines* |
| Magnetic depolarisation of continuum radiation (Hanle effect) | *lines* + *continuum* |

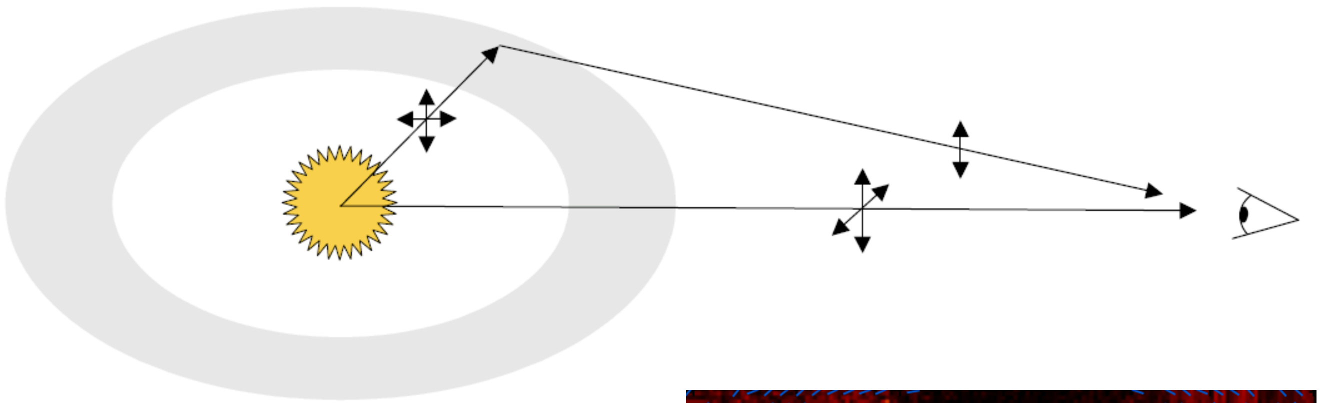
# Scattering polarisation

- Any microscopic scattering (Thomson, Compton, Rayleigh, etc.) creates linear polarisation
- Anisotropy leads to net macroscopic polarisation

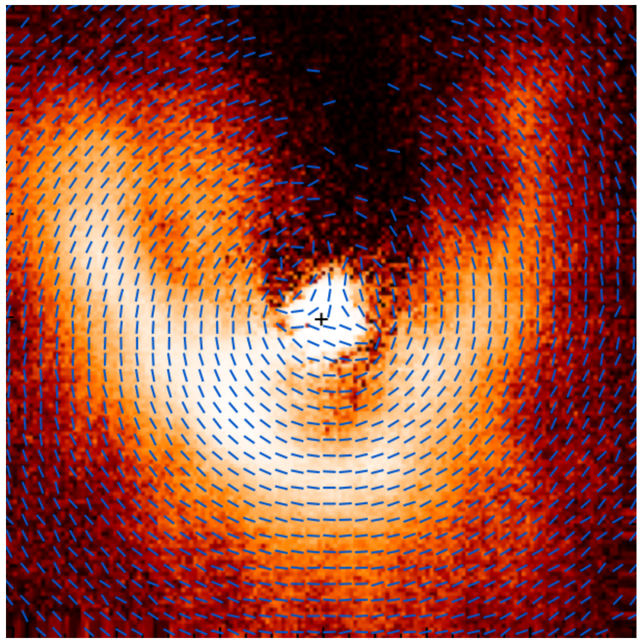


# Polarised view of stellar environments

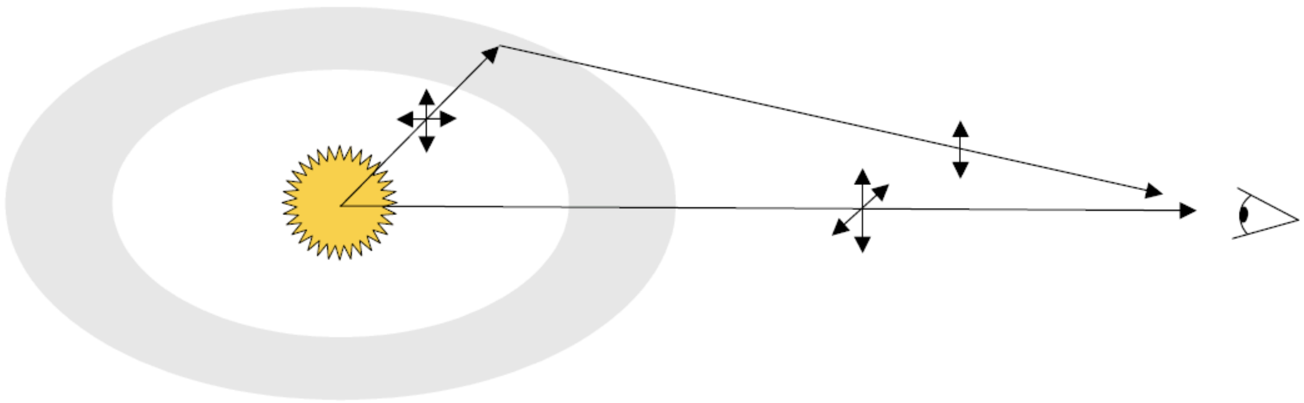


Massive disk around a young star

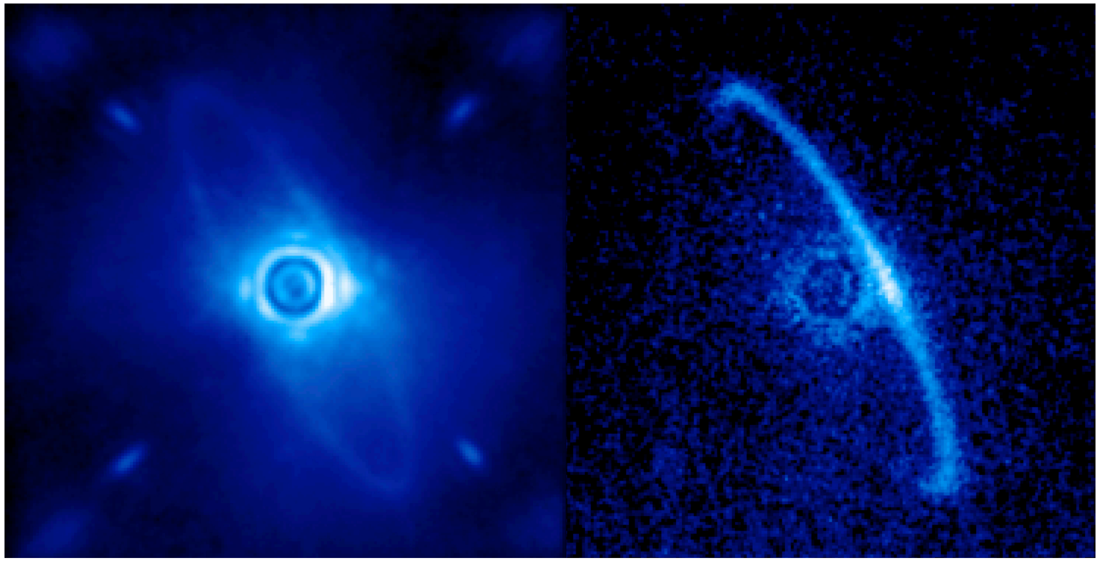


Intensity + linear polarisation

# Polarised view of stellar environments

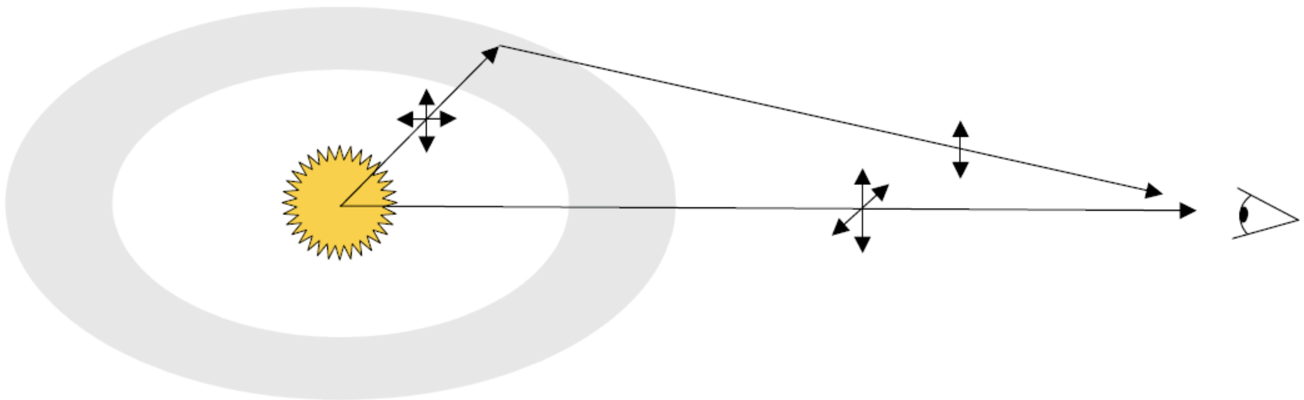


Ring around
a young star

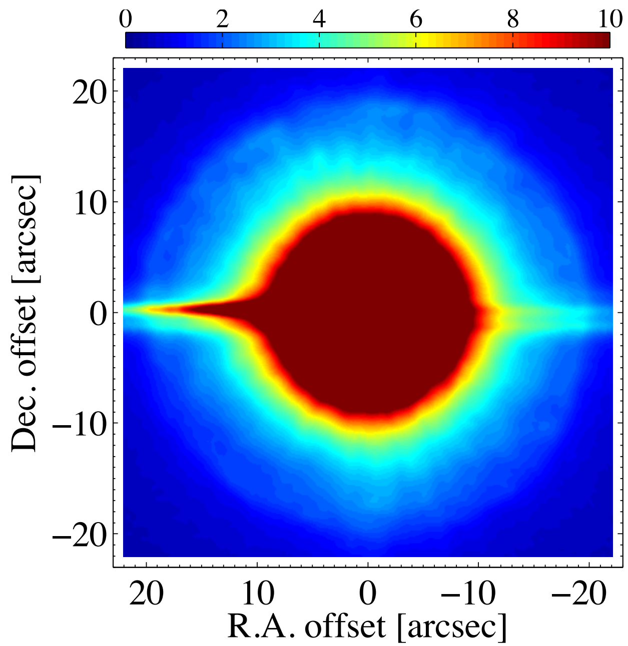


Intensity Linear polarisation

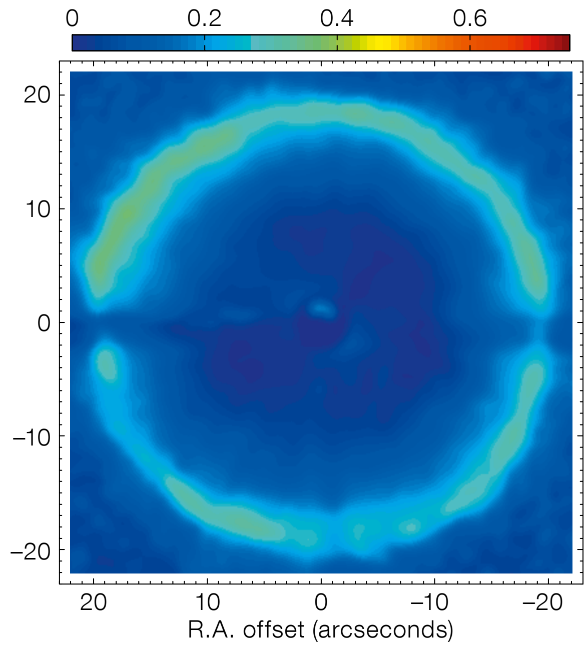
# Polarised view of stellar environments



Shell around a dying star

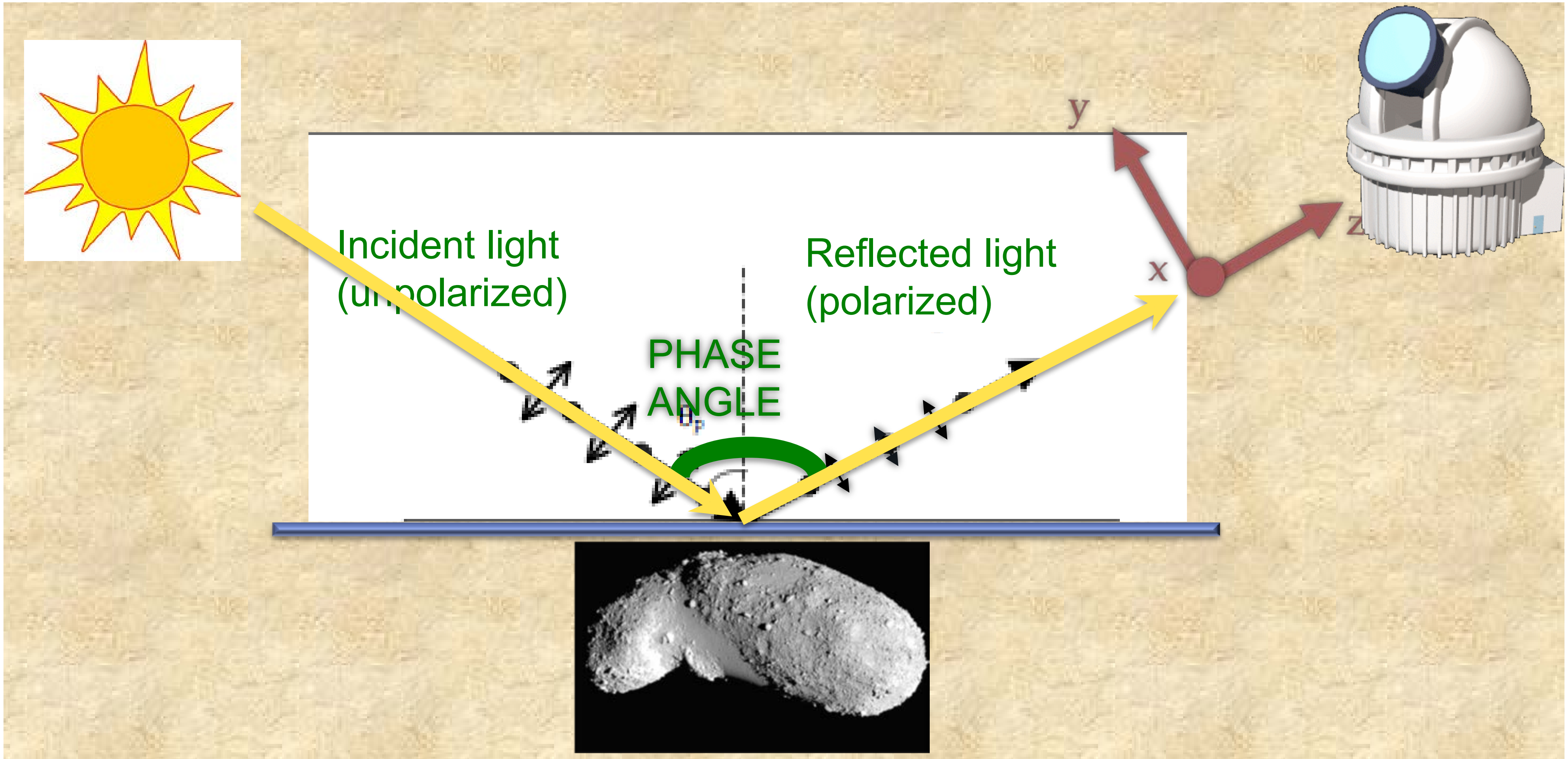



Intensity

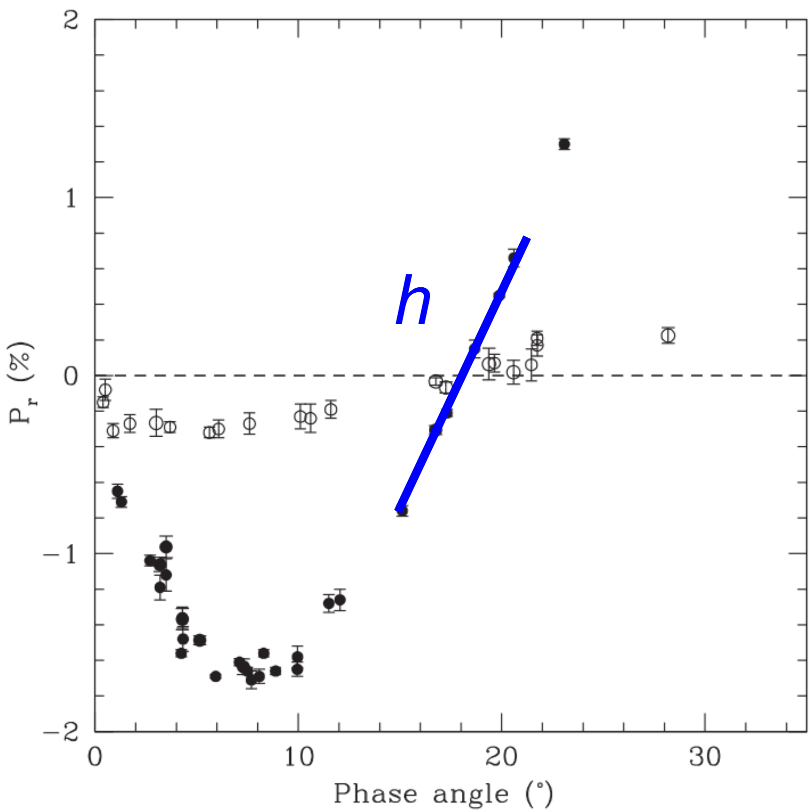



Linear polarisation

# Polarimetry of solar system bodies

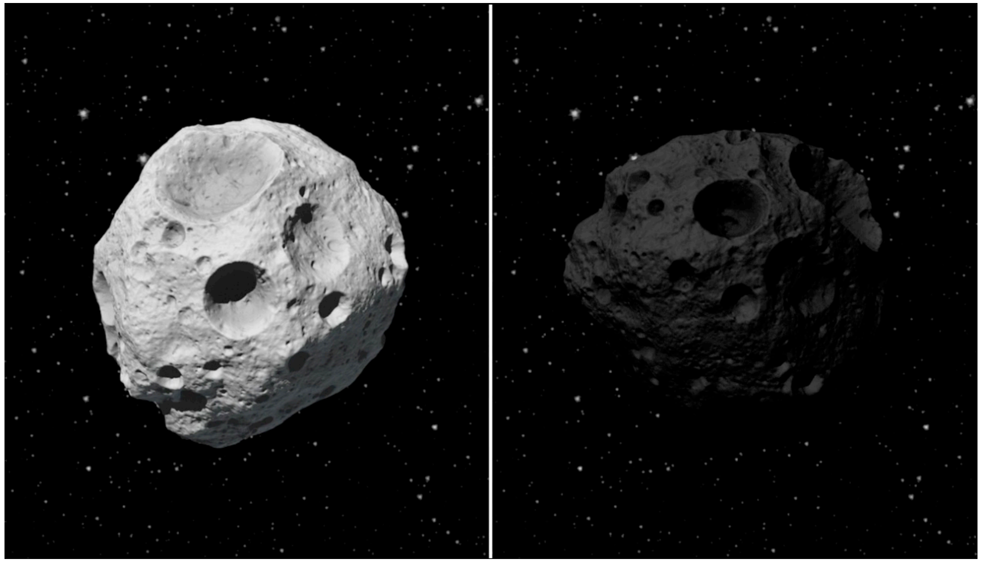


# Polarimetry of solar system bodies




Cellino & Bagnulo (2015)

Linear polarisation as a function of phase angle for low-albedo (Ceres, filled symbols) and high-albedo (Nysa, open symbols)

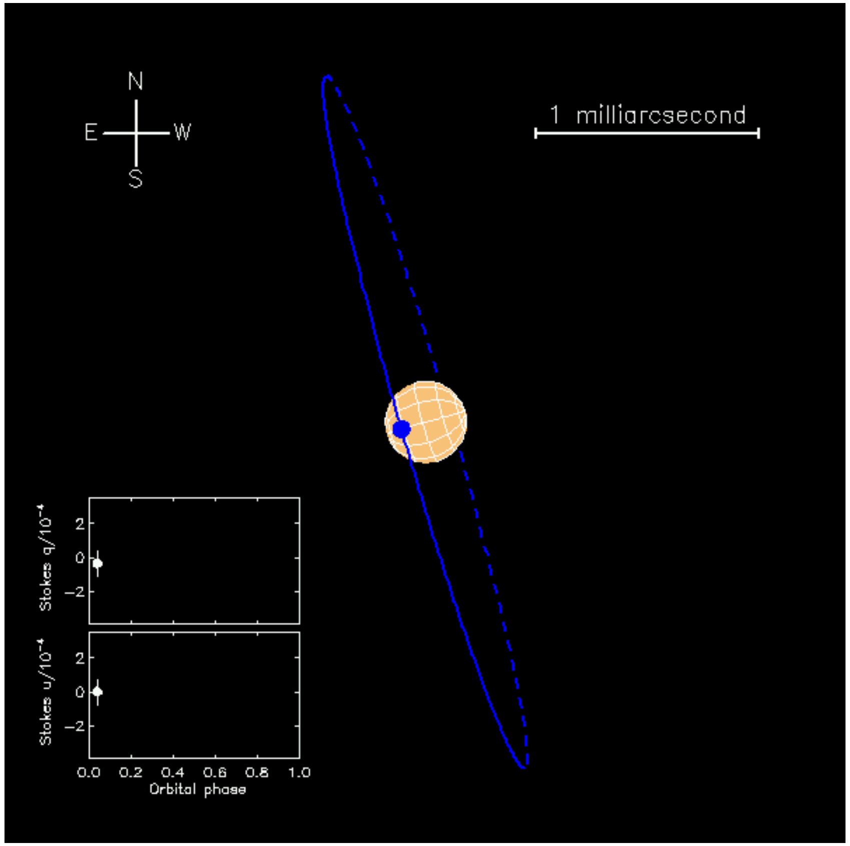


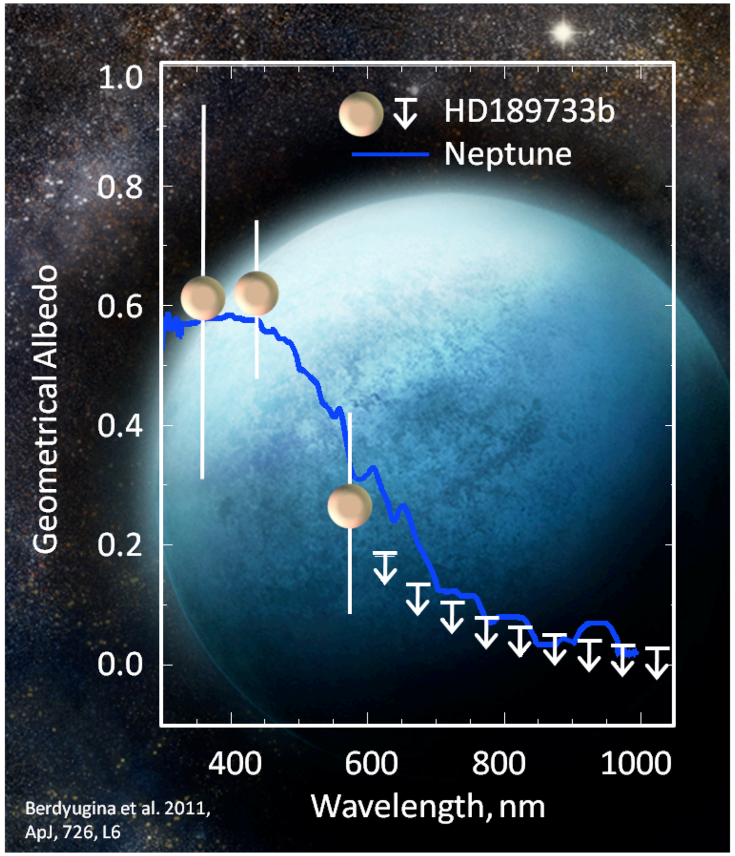
Albedo from polarimetric slope

$$\log A = C_1 \log h + C_2$$

# Polarimetry of exoplanets

Scattering polarisation modulated by phase angle

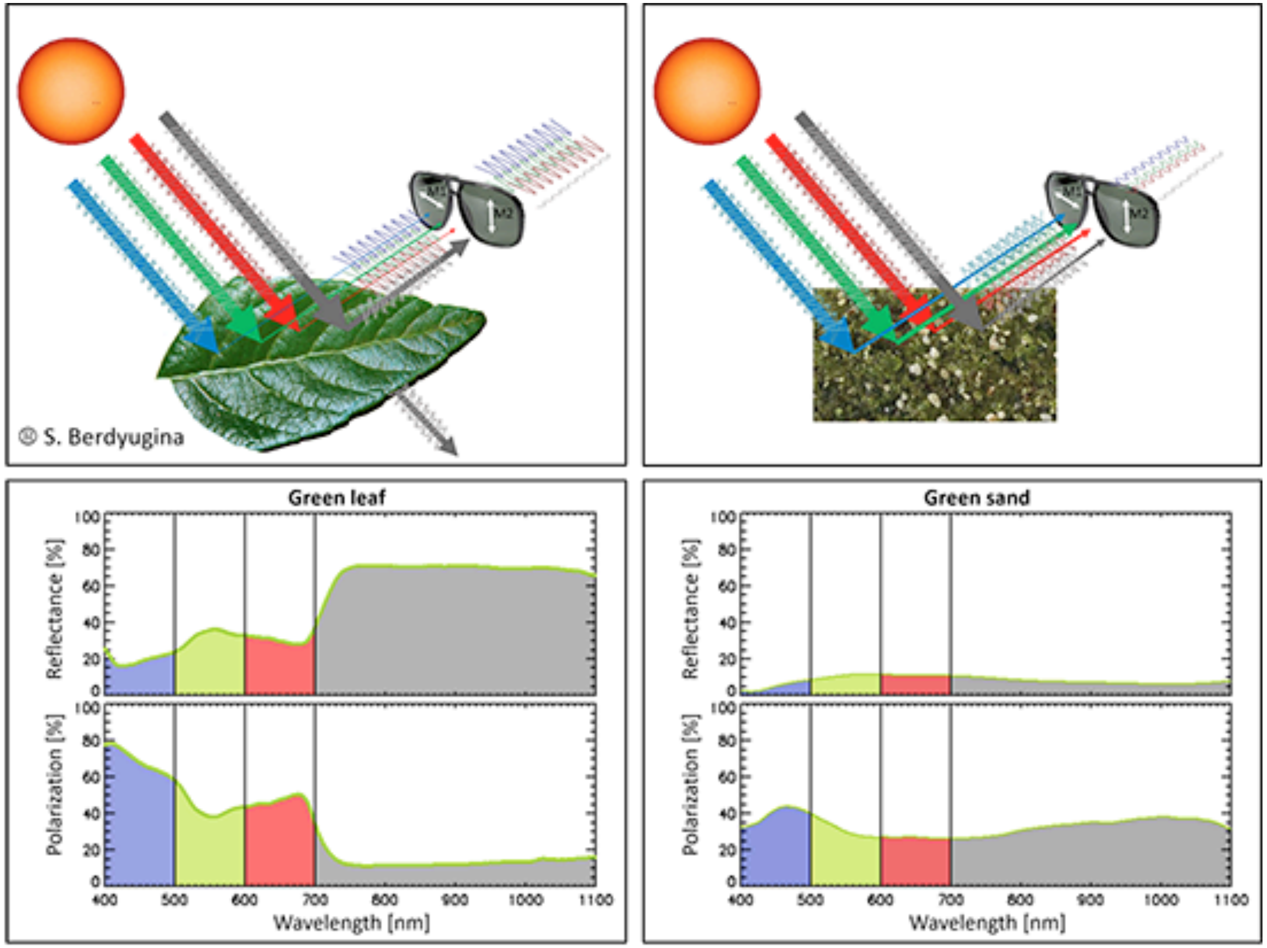



Berdyugina et al. (2011)

# Polarimetry of biospheres

Red edge polarisation signature of chlorophyll




Berdyugina et al.

# Polarimetry of biospheres

## Red edge polarisation signature of chlorophyll

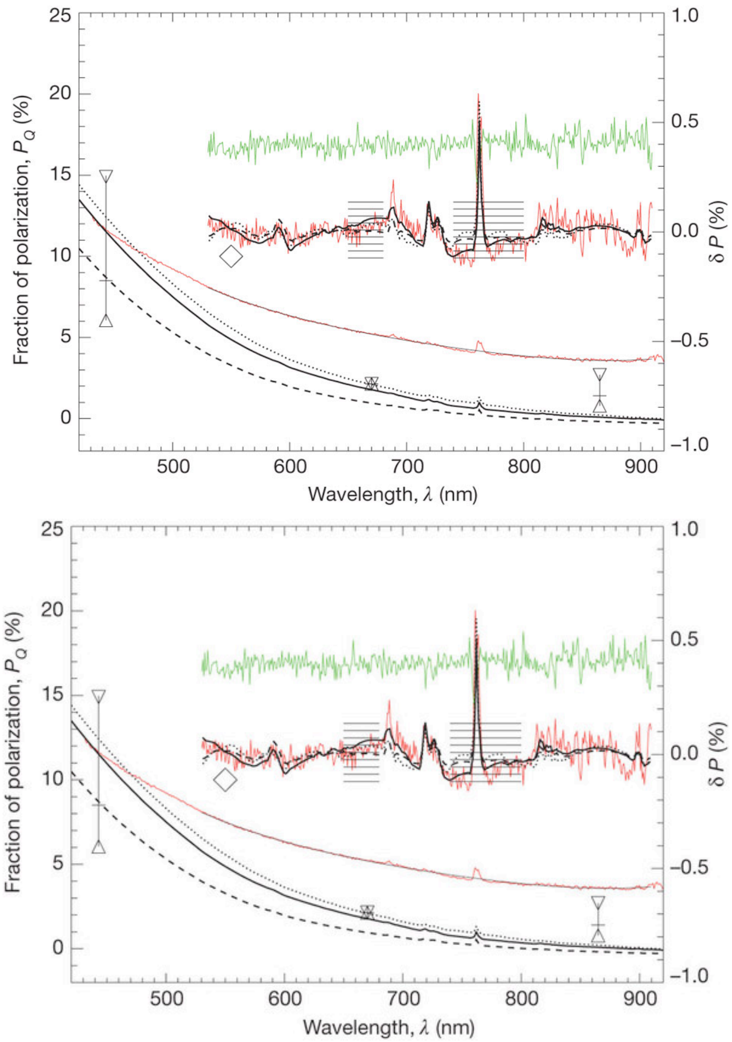



Table 1: Earth observations

| Observing Date | 25-Apr-2011:UT09 | 10-Jun-2011:UT01 |
|---|---|---|
| View of Earth as seen from the Moon |  |  |
| Sun-Earth-Moon phase | 87 deg | 102 deg |
| ocean fraction in ES | 18% | 46% |
| vegetation fraction in ES | 7% | 3% |
| tundra, shrub, ice and desert fraction in ES | 3% | 1% |
| total cloud fraction in ES | 72% | 50% |
| cloud fraction $\tau > 6$ | 42% | 27% |

Earthshine polarisation spectra

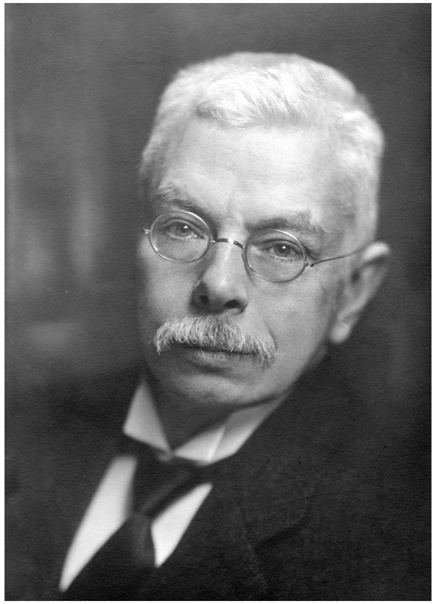
Sterzik et al. (2012)

# Zeeman effect

## Atomic Hamiltonian in the presence of a magnetic field

$$H = -\frac{\hbar}{2m}\nabla^2 + V(r) + \xi(r)\mathbf{L}\cdot\mathbf{S} + \left[-\frac{e}{2mc}\mathbf{B}\cdot(\mathbf{L}+2\mathbf{S}) + \frac{e^2}{8mc^2}B^2 r2\sin^2\theta\right]$$

(1) (2) (3) (4) (5)



P. Zeeman (1896)

***B***: magnetic field vector

**L**: orbital angular momentum

**S**: spin angular momentum

(1): kinetic energy

(2): potential (Coulomb) energy

(3): spin-orbit coupling energy

(4): linear magnetic term

(5): quadratic magnetic term

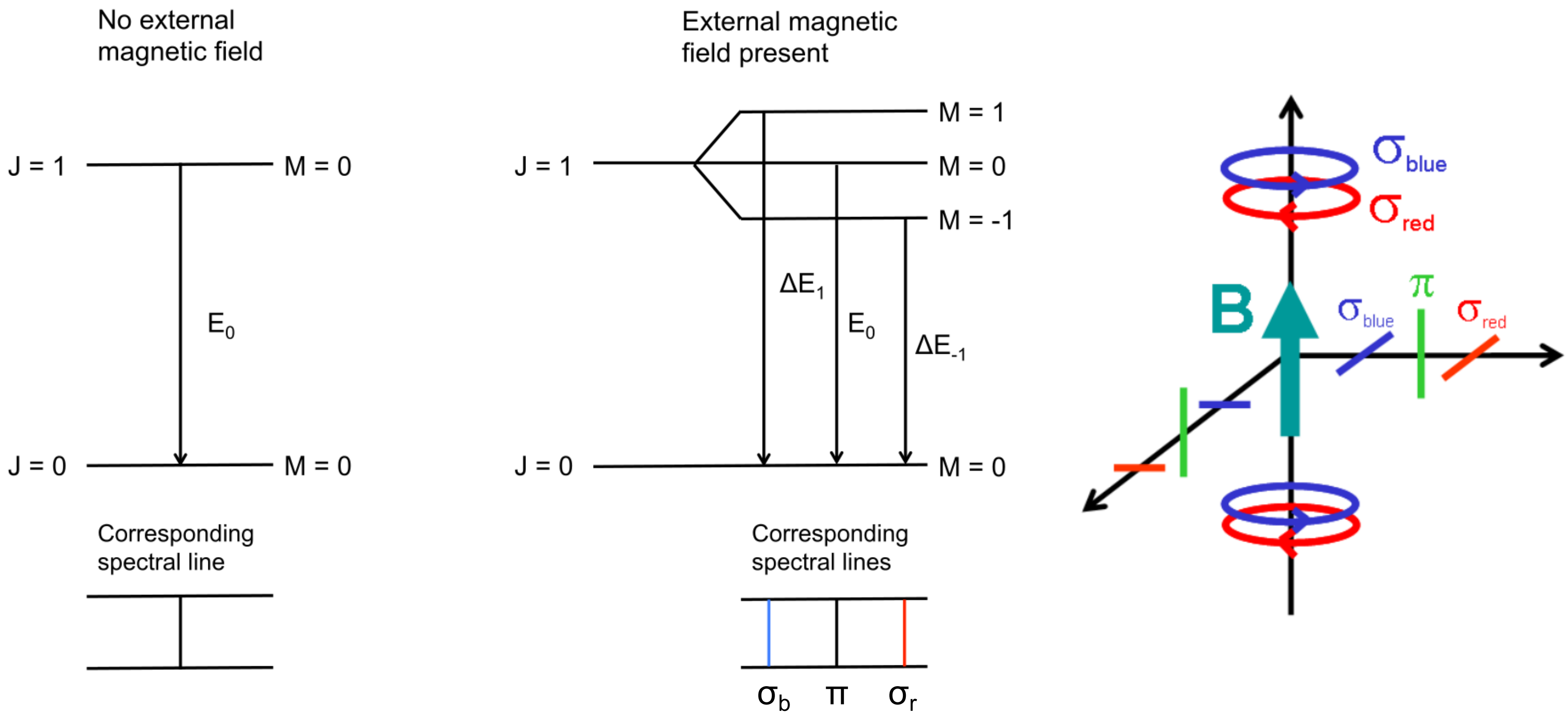
(5) << (4) << (3): linear Zeeman effect

(5) << (4) & (3) << (4): Paschen-Back effect

(4) << (5) & (3) << (5): quadratic Zeeman effect

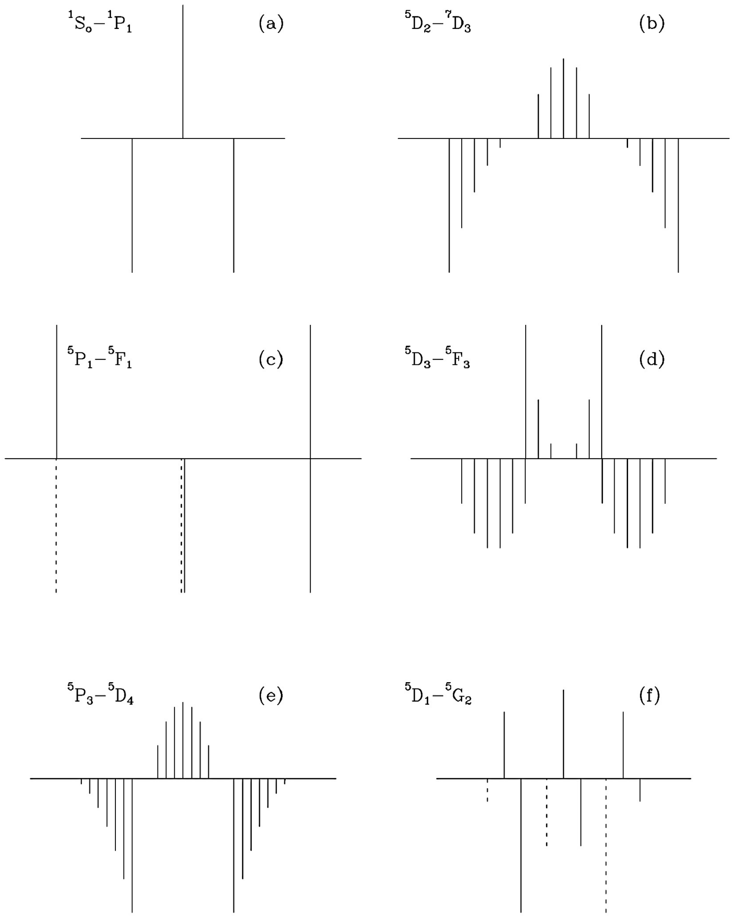
# Zeeman effect

## Splitting and polarisation of spectral lines



# Zeeman effect

Zeeman splitting patterns




- Zeeman splitting [G, Å]

$$\Delta\lambda_{\sigma-\pi} = 4.67 \times 10^{-13} \bar{g} \lambda_0^2 B$$

1G=$10^{-4}$ T

1kG=0.1 T

= 0.7 km $s^{-1}$ / kG for

$$\lambda_0 = 5000 \text{ Å}, \bar{g} = 1$$

- Polarisation in weak field

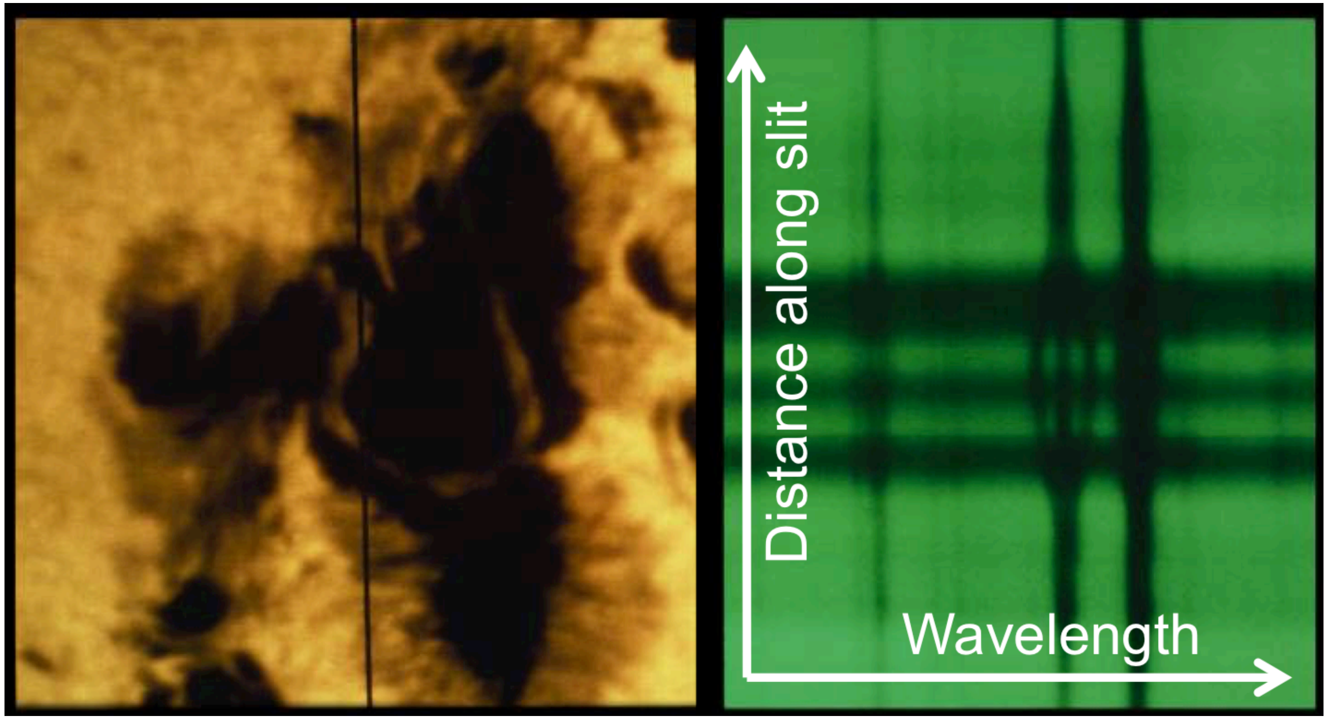
$$I \approx I_0, Q \approx 0, U \approx 0$$

$$V \approx -4.67 \times 10^{-13} \bar{g} \lambda_0^2 B \cos\theta \frac{\partial I}{\partial \lambda}$$

# Solar magnetism

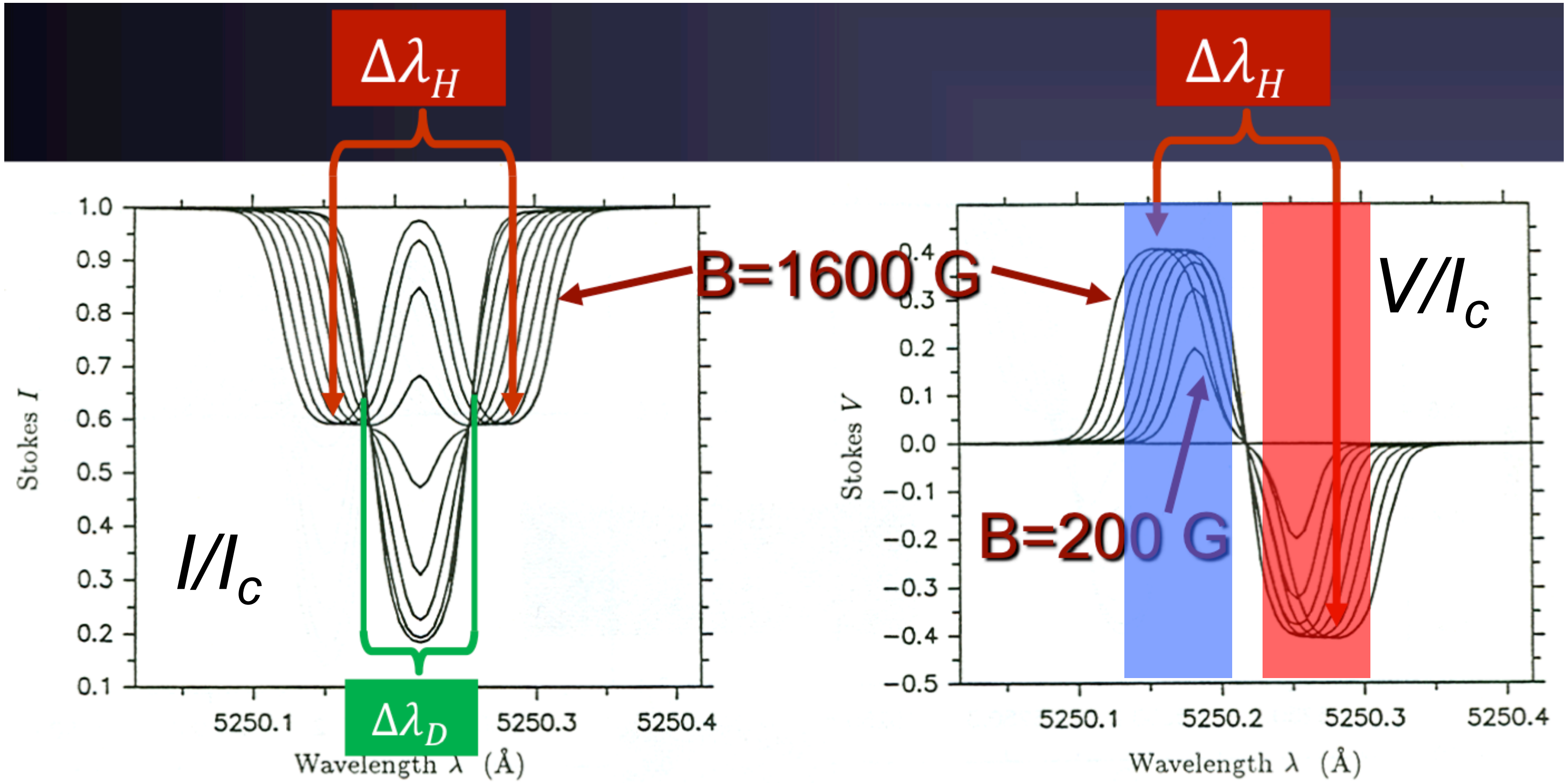
Sunspots are magnetic

G. Hale (1908)

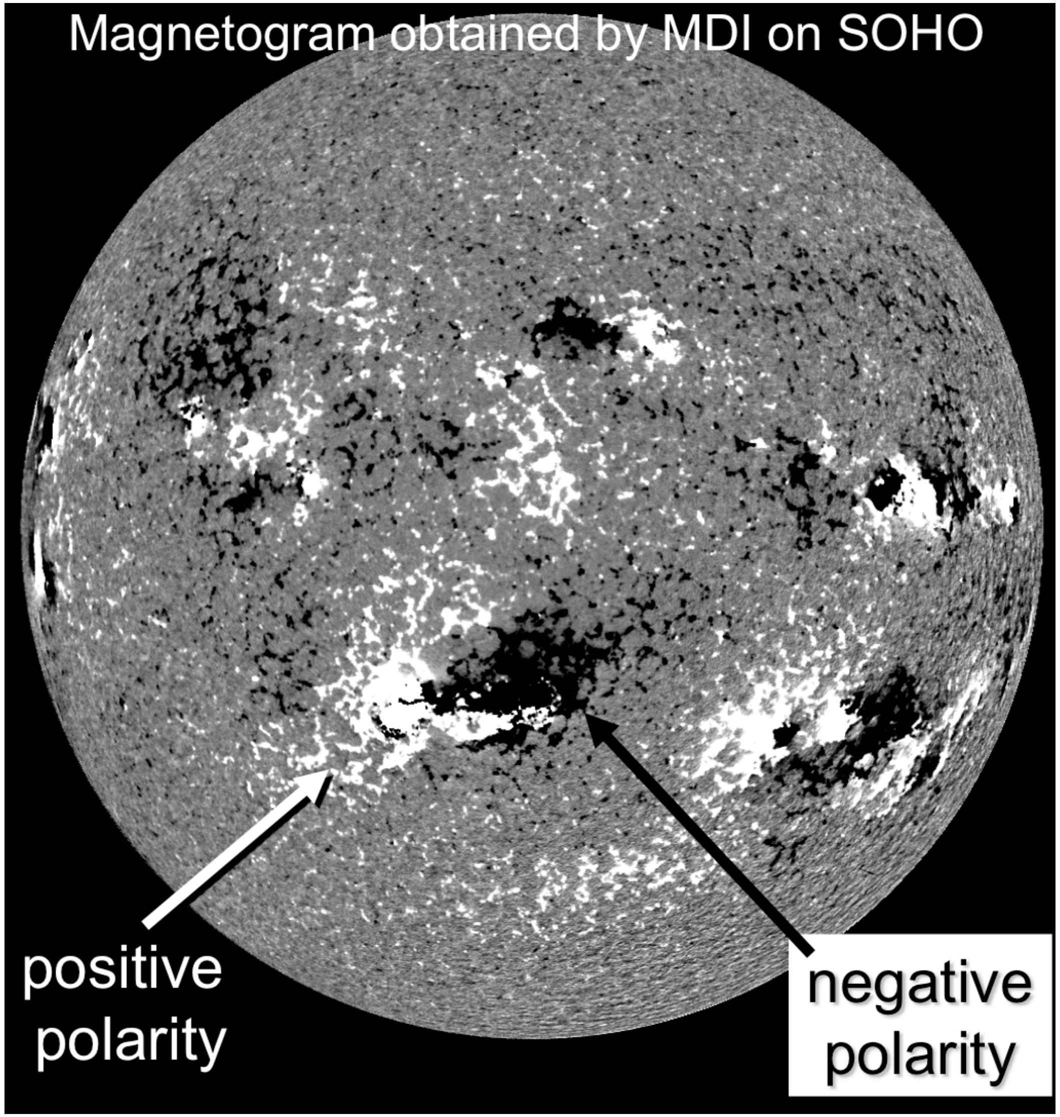


# Zeeman diagnostics

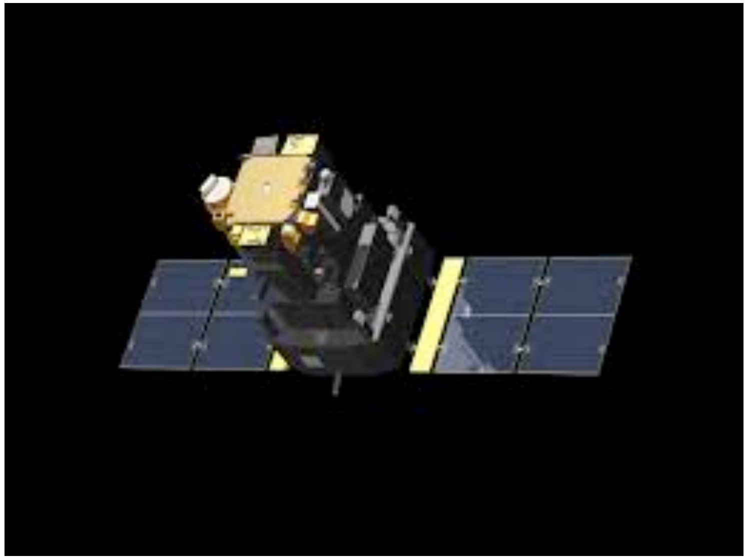
- Strong fields: Zeeman slitting and Stokes *QUV*
- Weak fields: Stokes *V*
- Polarisation amplitudes: $10^{-2} – 10^{-3}$



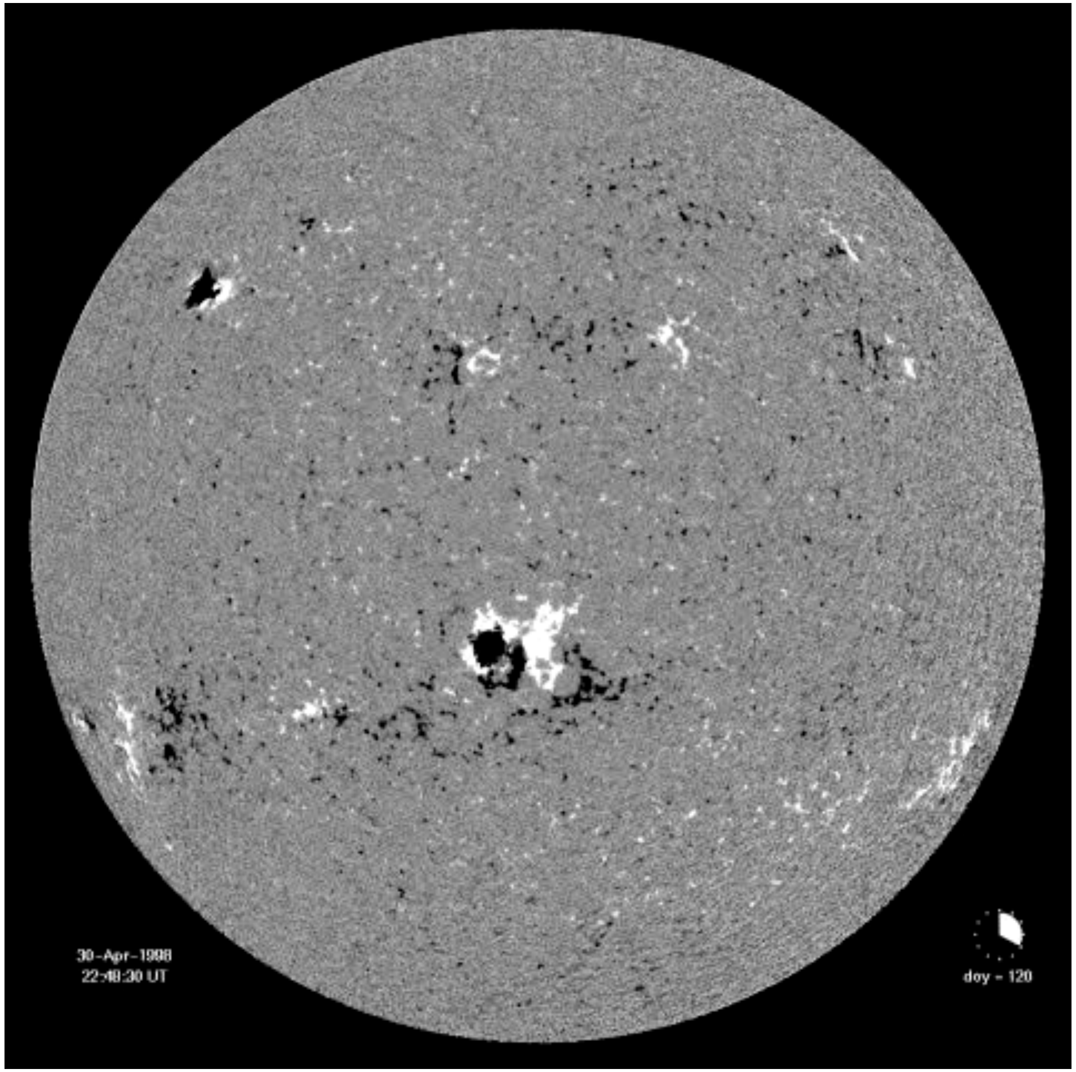
# Magnetograms



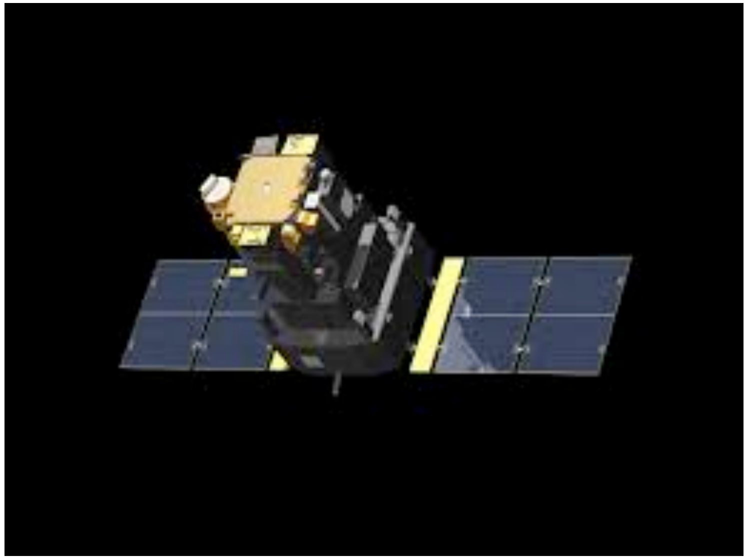

Line of sight field component estimated from a single line



# Magnetograms



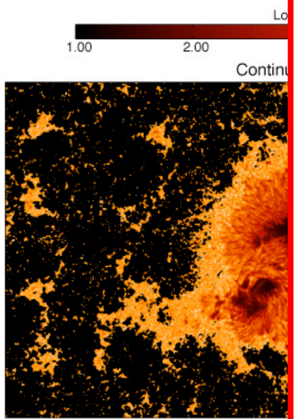

Line of sight magnetic field component estimated from wings of a single line

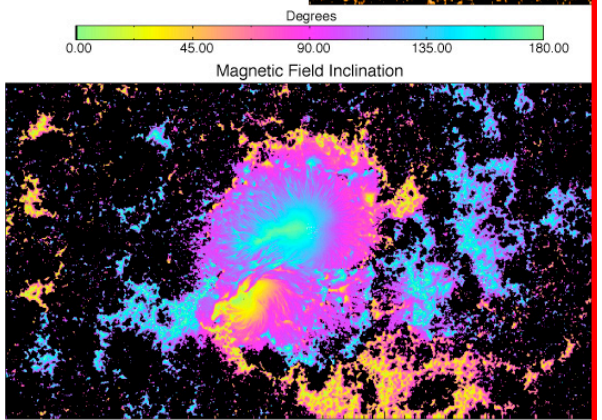


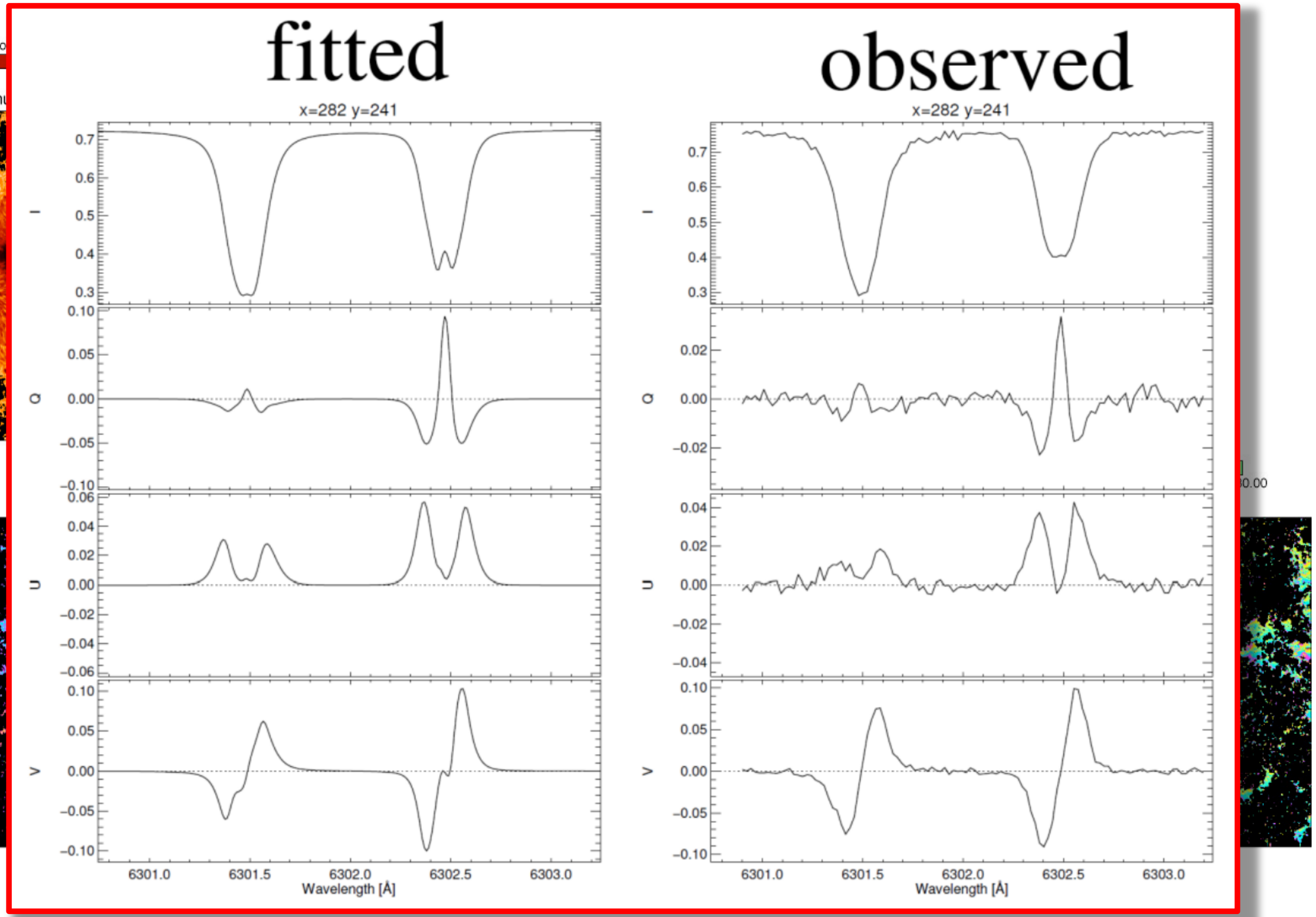
# Vector magnetograms

Full magnetic field vector from modelling of *IQUV* parameter profiles of one / few spectral lines for each image pixel

SDO
HMI

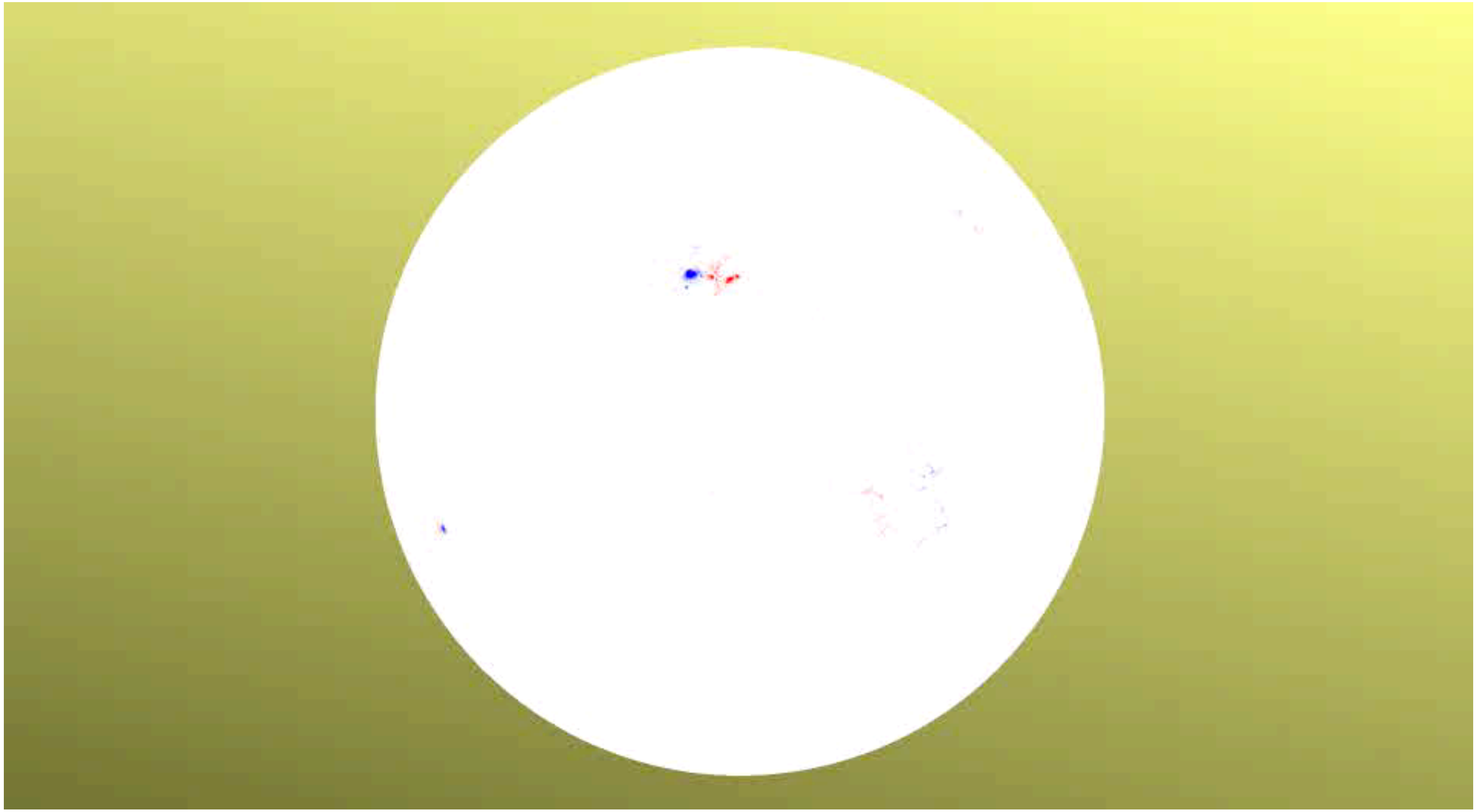


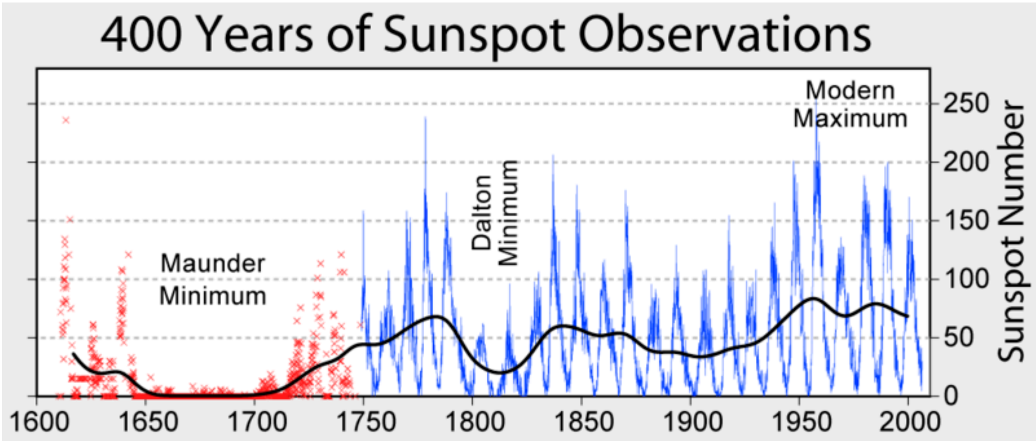
# Vector magnetograms

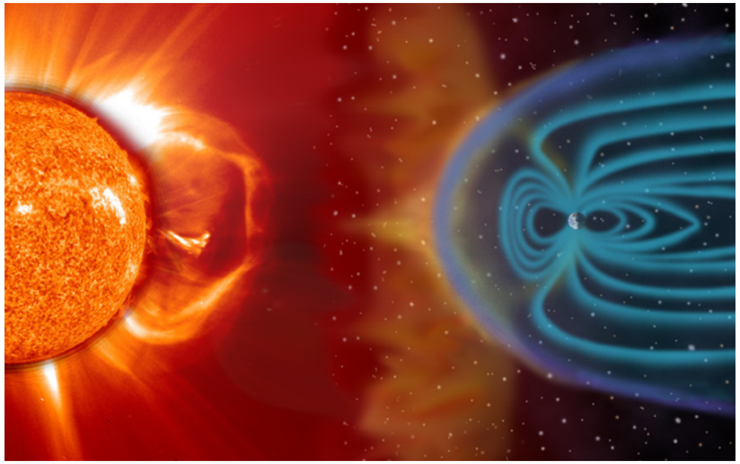
Results obtained with HMI SDO data

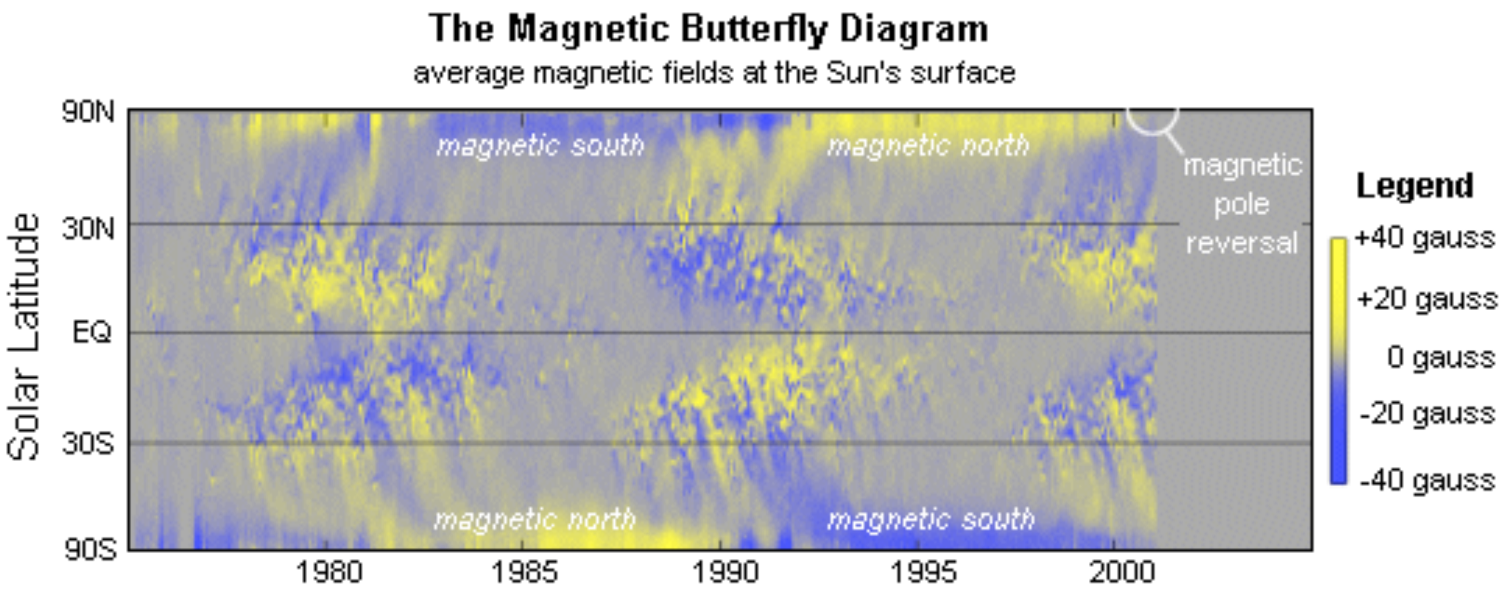


# Solar activity cycle

Cyclic evolution of interior and surface magnetic fields

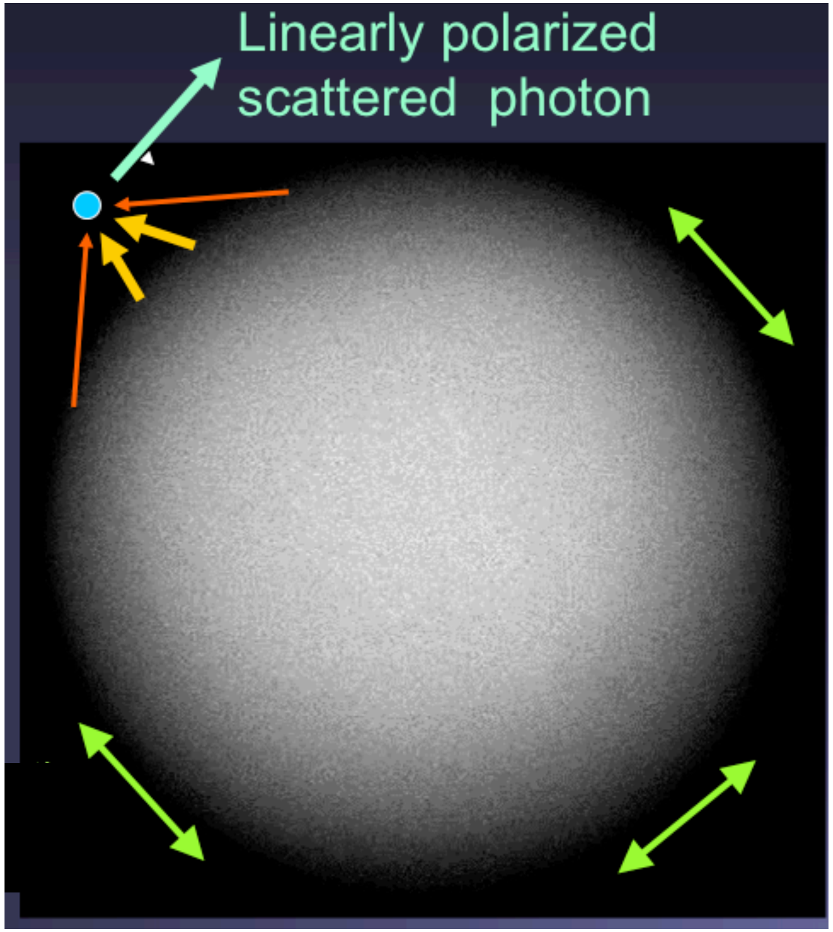


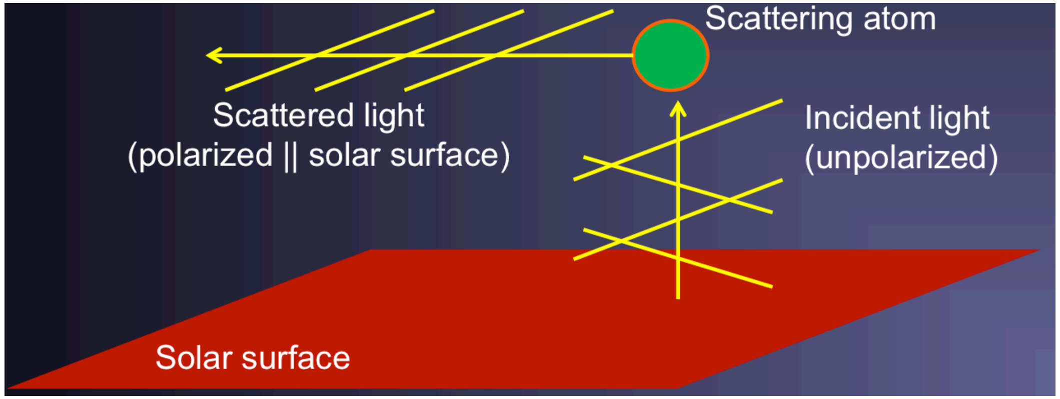





# Scattering polarisation

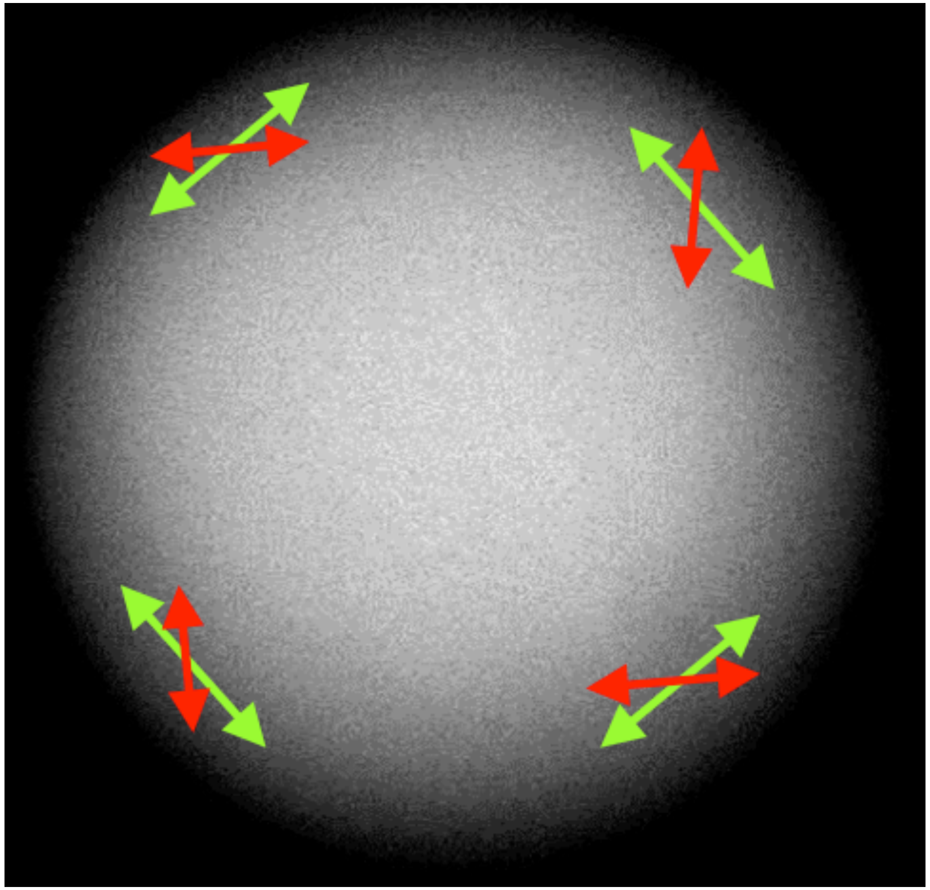
- Solar limb => anisotropic illumination + scattering => linear polarisation



# Hanle effect

- Solar limb => anisotropic illumination + scattering => linear polarisation
- Weak (0.1-100 G) field => depolarisation and rotation of polarisation plane

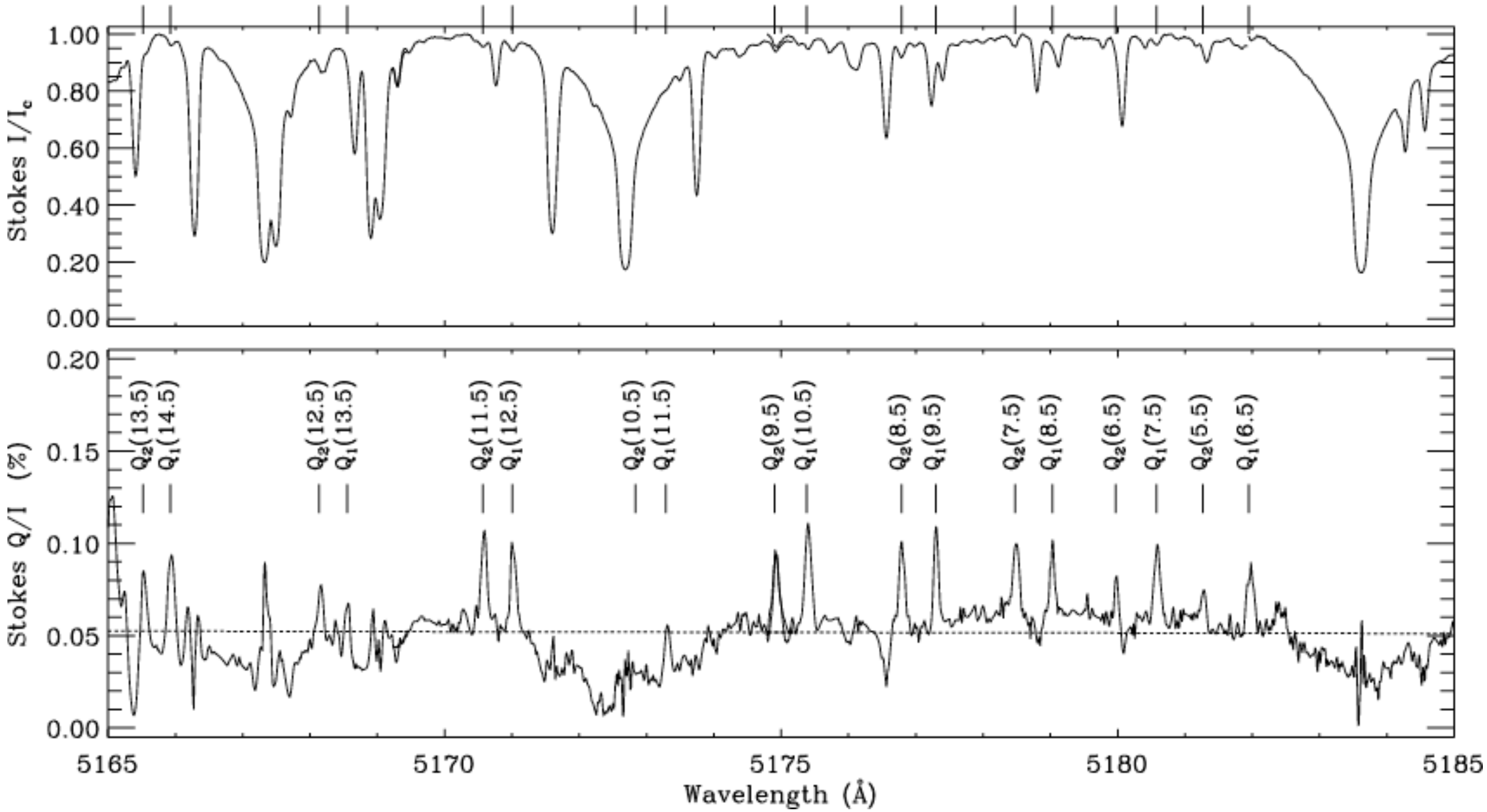


$\boldsymbol{B} = 0$

$\boldsymbol{B} \neq 0$

# “Second” solar spectrum

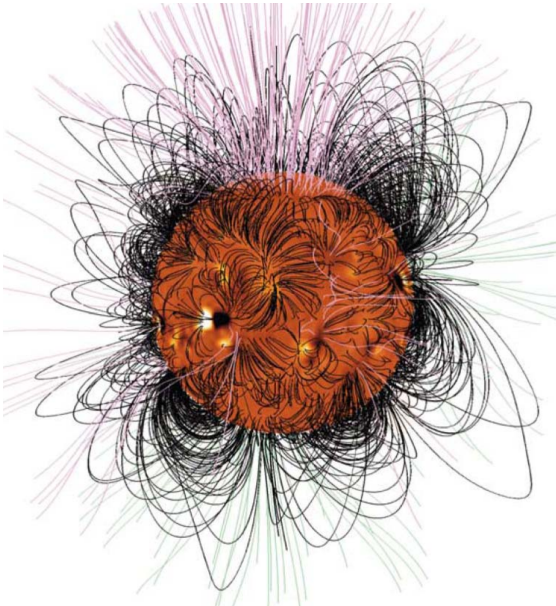
Stokes *Q* spectrum at the solar limb



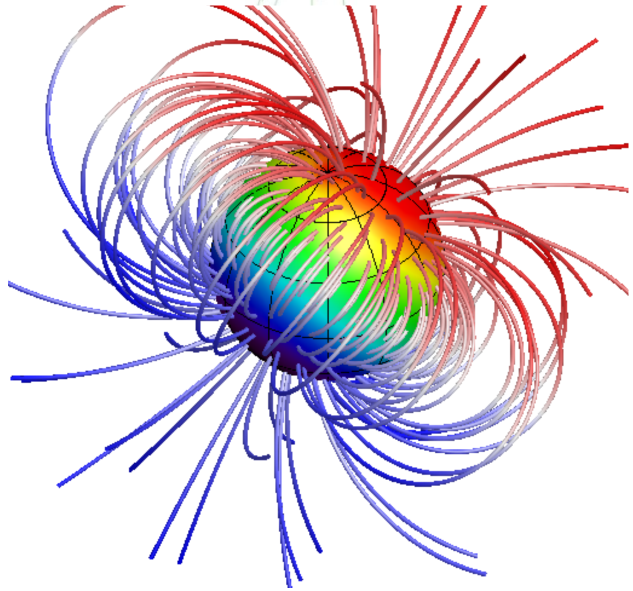

MgH

# Stellar magnetism

- Sun and cool stars:
  - fields are generated by dynamo
  - fields are weak, complex, evolving

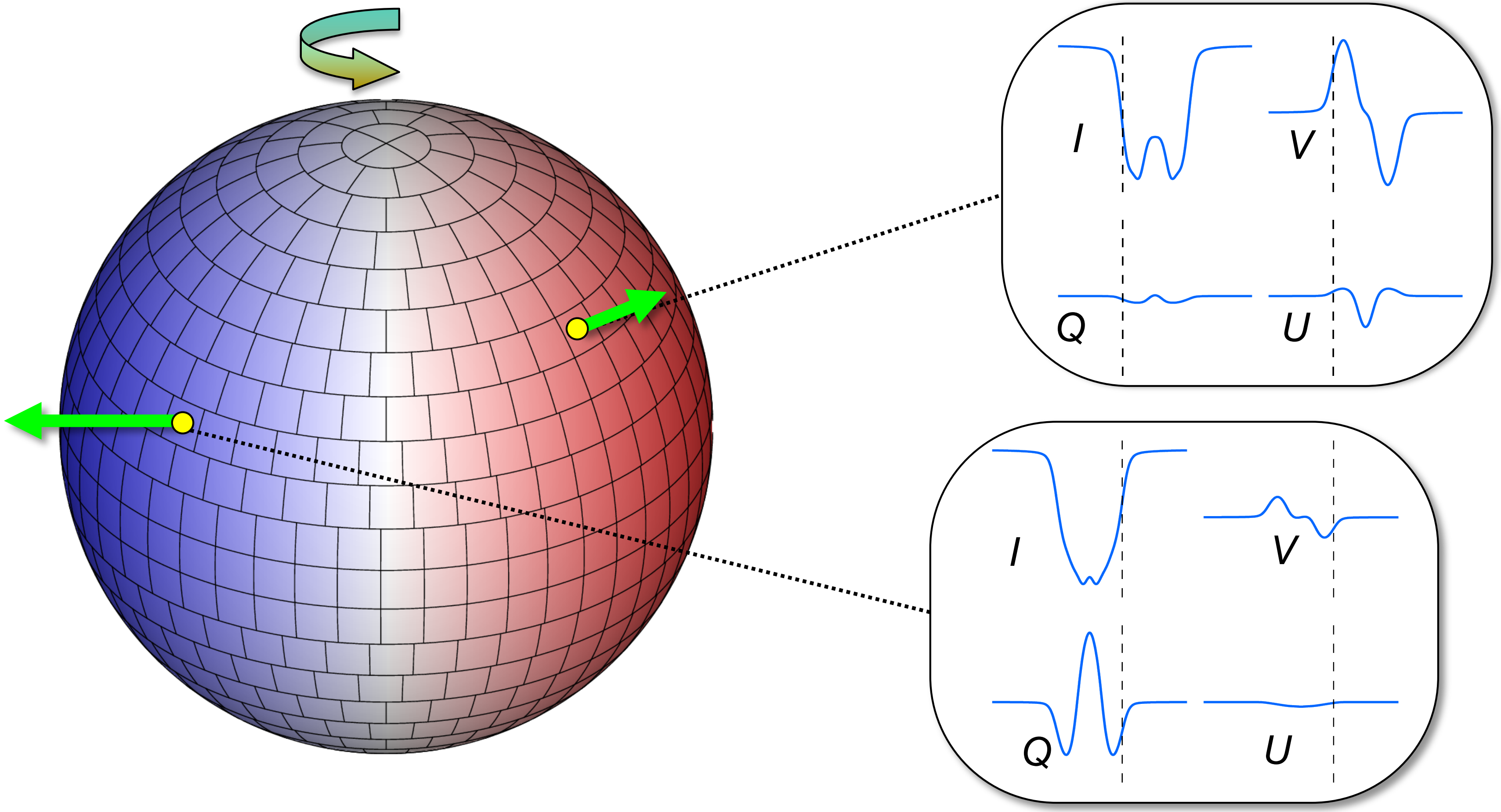


- Hot stars:
  - fields are fossil remnants
  - fields are strong, globally organised, stable



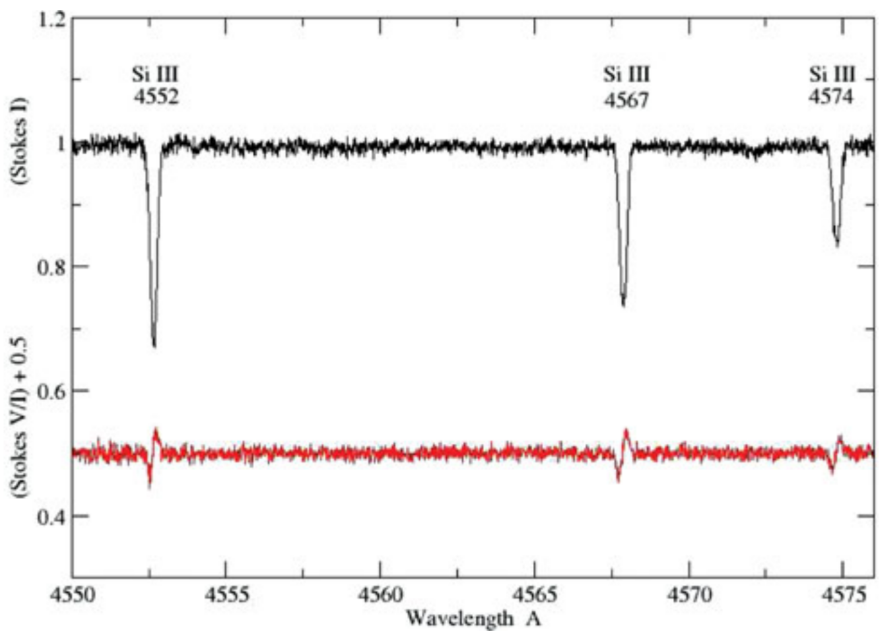
# Stellar Zeeman signatures

Disk-integrated Stokes spectra: a sum of Doppler-shifted contributions from the entire stellar disk

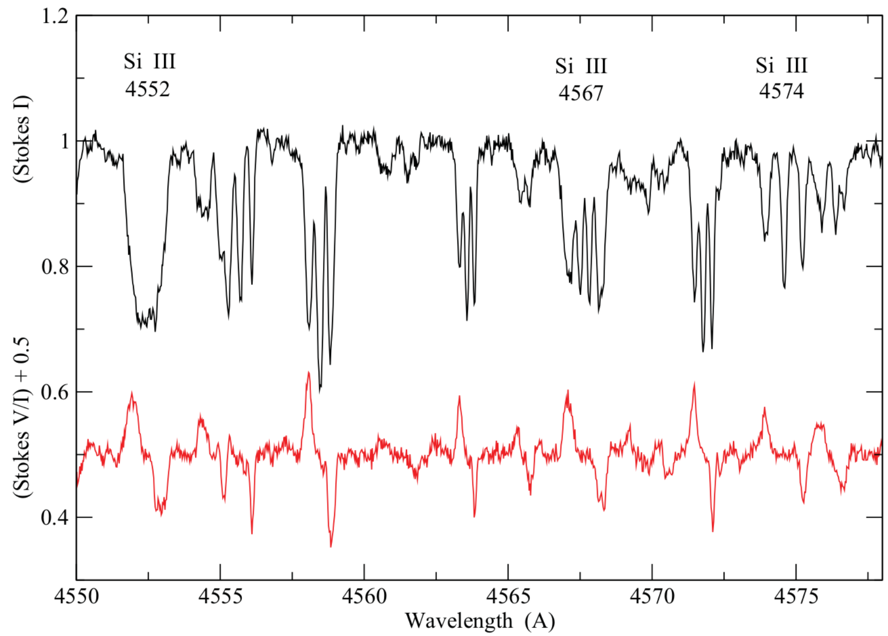


# Strong-field stars

Zeeman splitting in Stokes *I*; Stokes *V* signatures in individual lines in high-resolution spectra



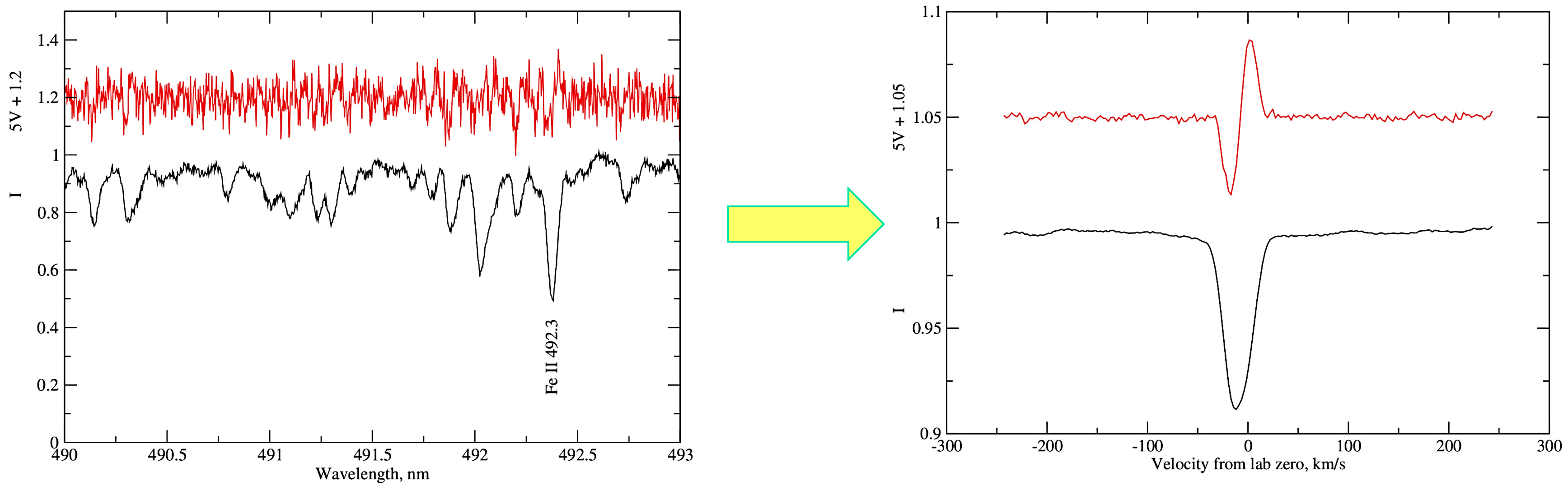

moderately strong field




very strong field

# Weak-field stars

- Stellar surfaces are unresolved => spatial resolution can be traded for wavelength coverage
- Multi-line techniques (Least-Squares Deconvolution)

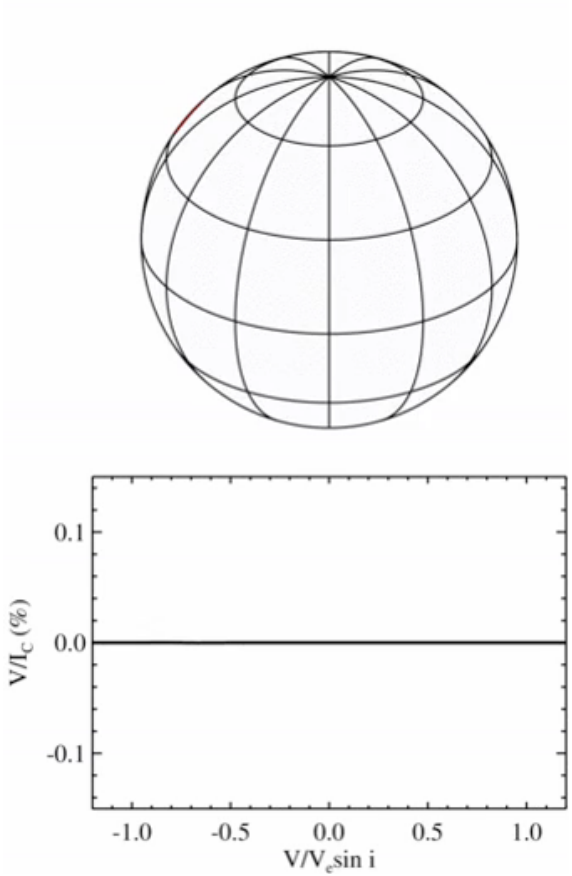



- Polarimetric sensitivity $10^{-5}$ – $10^{-6}$ for bright stars; possible to detect fields ~0.5 G

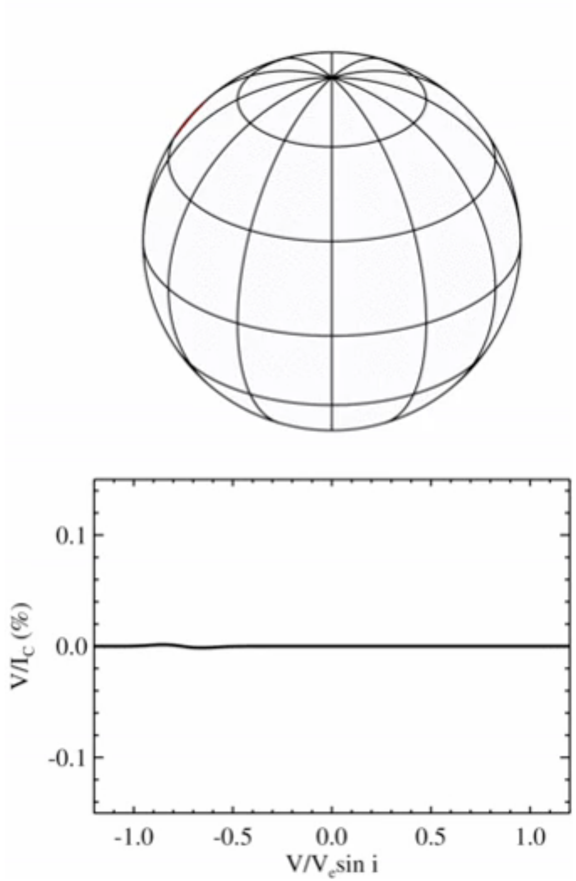
# Zeeman Doppler imaging

Detailed maps of vector fields from modelling of Stokes *IQUV* time series of individual/average lines

Radial field

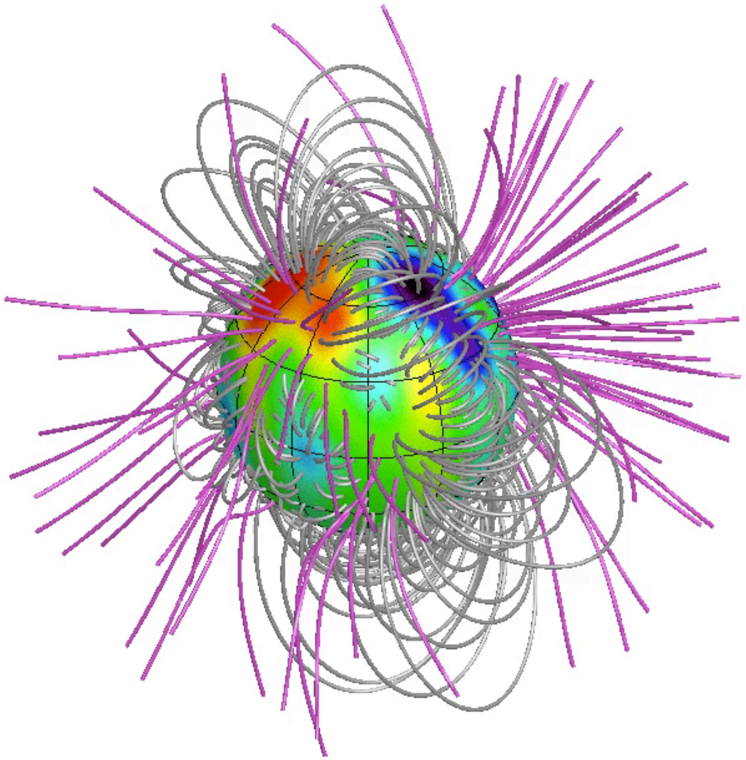


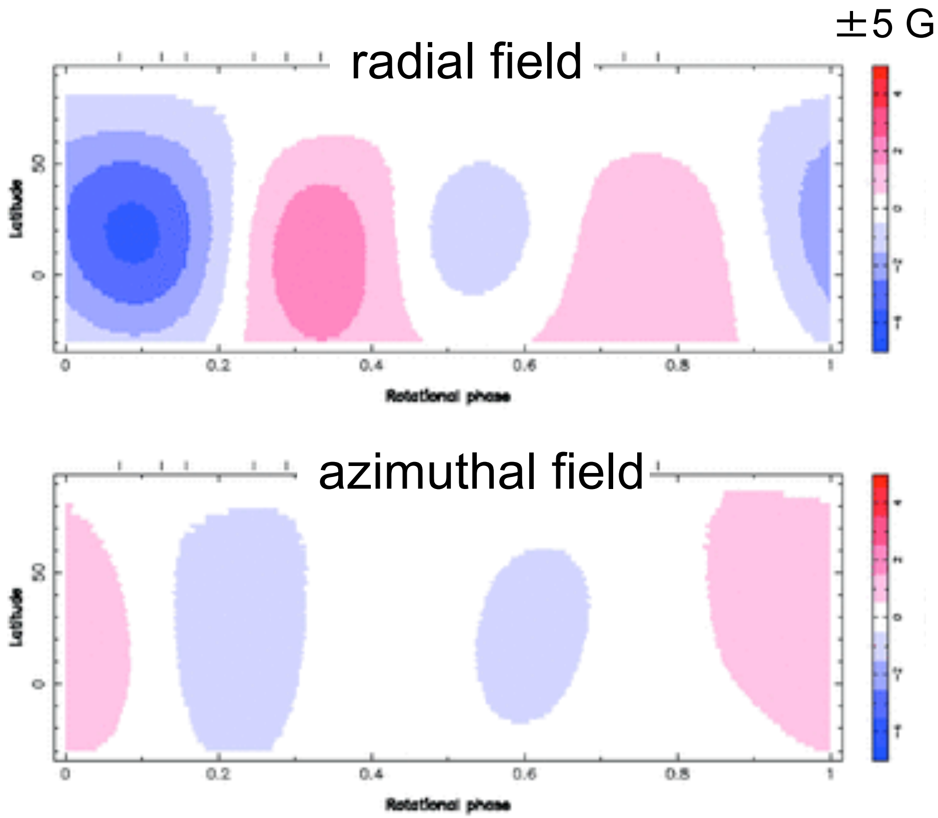

Horizontal field



# Zeeman Doppler imaging

Detailed maps of vector fields from modelling of Stokes *IQUV* time series of individual/average lines






II Peg (active, rapidly rotating)

18 Sco (solar twin)

# Observational requirements

- Sensitivity to weak signals
- Tradeoff between sensitivity/image size/spectral resolution/wavelength coverage/time resolution