Expansion of HII regions

- **Simple model for HII region evolution**
  - Massive star forms within HI cloud
  - Ionizing radiation suddenly "switched on"
  - Sphere of ionized gas with sharp boundary forms
  - Boundary = "Ionization front"
  - IF moves outward from the star
  - Calculate radius as a function of time
    → velocity of IF
- **Initial stages of HII region evolution**
  - Gas around star at rest
    with neutral hydrogen number density n_0 m^{-3}
  - Ionizing flux per unit area at IF is J m^{-2}s^{-1}
  - In time dt, IF moves a distance dR
  - 1 photon ionizes 1 hydrogen atom
    → number of photons arriving during dt
    = number of atoms within dR

$$J\mathrm{d}t = n_0\mathrm{d}R$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{J}{n_0}$$

  - What is J?
    Ionizing flux at stellar surface S_* reduced by
    * geometrical effect ($\propto 1/R^2$)
    * absorption by neutral atoms created by recombination

    → with ionization balance

$$J = \frac{S_* - 4\pi/3 \cdot R^3 \cdot \dot{\mathcal{N}}_\mathrm{R}}{4\pi R^2} \qquad \dot{\mathcal{N}}_\mathrm{R} = n_0^2\beta_2(T_\mathrm{e})$$

- **Conclusions for initial stages**
  - Strömgren radius R_S only reached after a long time (a few times the recombination timescale t_R), but R close to R_S at time $t \approx t_R$
  - IF velocity $\gg$ sound speed ($c_i \approx 10$ km s^{-1}) $\rightarrow$ gas has no time to move $\rightarrow$ density remains constant
  - At $t >$few t_R ($R \approx R_S$), IF velocity drops below c_i

- **Intermediate stages of HII region evolution**
  - Inside ionized sphere
    $\approx$ 200 times greater pressure than outside:
    $T_e \approx 10^4$ K, $T_n \approx 100$ K; twice number of particles
    $\rightarrow$ gas reacts to pressure increase by expanding
    $\rightarrow$ density drops $\rightarrow$ recombination rate drops
    $\rightarrow$ ionized region can become larger than R_S
  - sudden pressure increase = perturbation
    $\rightarrow$ propagates with velocity c_i into neutral gas
  - $c \propto \sqrt{T} \rightarrow c_i/c_n \approx 10 \rightarrow$ supersonic
    $\rightarrow$ shock wave forms ahead of IF

- **Final stage of HII region evolution**
  - Pressure equilibrium

$$2n_f k T_e = n_0 k T_n \rightarrow \boldsymbol{n_f} = \frac{T_n}{2T_e} n_0 \boldsymbol{\approx 0.005 \cdot n_0}$$

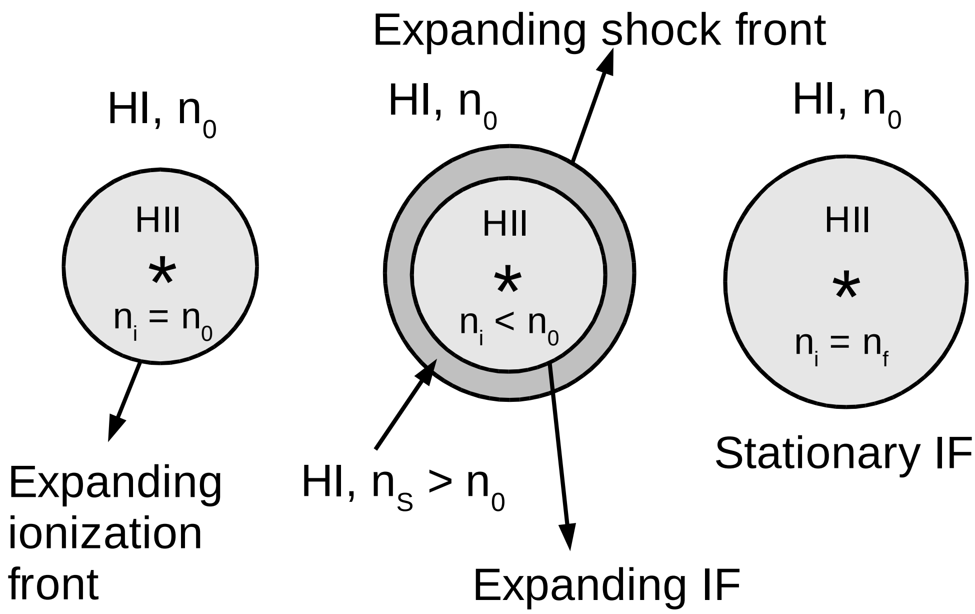
  - Final radius from ionization balance of whole region

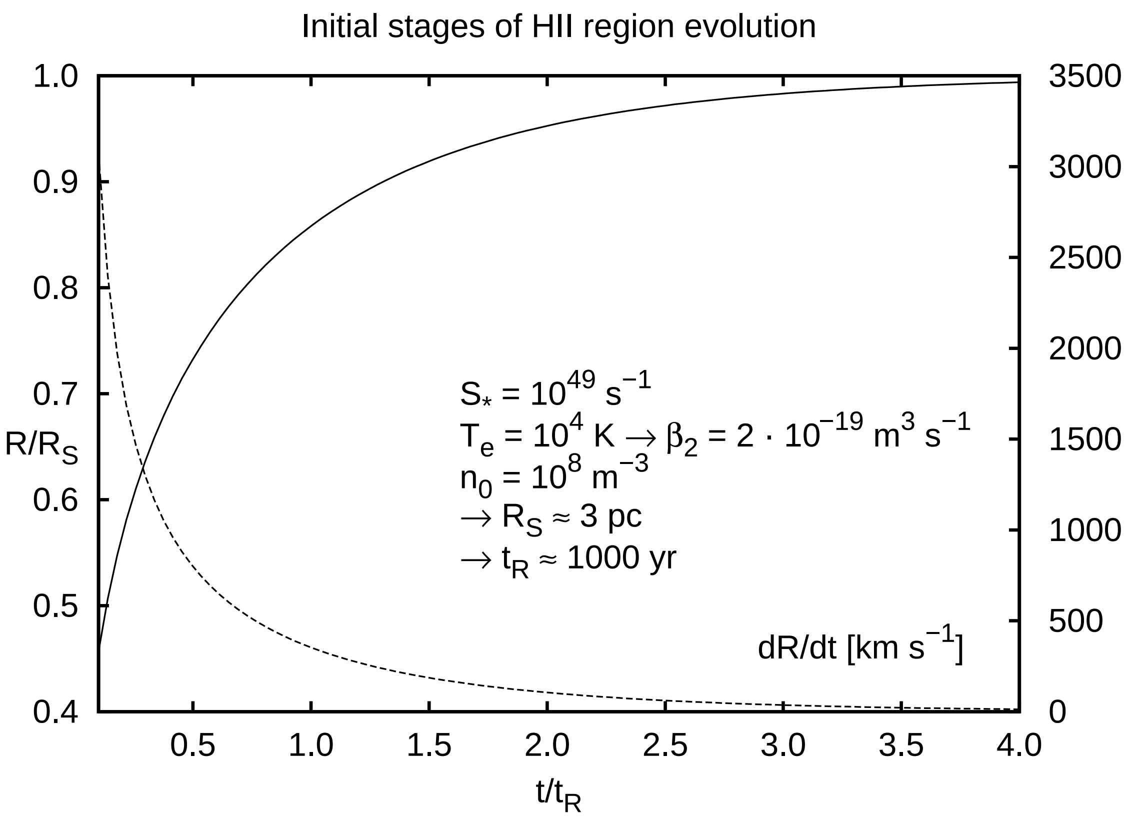
$$\frac{4}{3}\pi R_f^3 n_f^2 \beta_2 = S_* = \frac{4}{3}\pi R_S^3 n_0^2 \beta_2 \rightarrow \boldsymbol{R_f} = \left(\frac{2T_e}{T_n}\right)^{\frac{2}{3}} R_S \boldsymbol{\approx 34 \cdot R_S}$$

  - Final mass

$$\frac{\boldsymbol{M_f}}{\boldsymbol{M_S}} = \frac{R_f^3 n_f}{R_S 3 n_0} = \frac{2T_e}{T_n} \boldsymbol{\approx 200}$$

Evolution of an HII region





- **Intermediate stages – expansion phase**
  Assumptions for simple model:
  - Thin layer of shocked gas
    $\rightarrow$ one radius R for IF and shock
    IF "follows" shock
    – expansion velocity of ionized sphere $\approx$ shock velocity
    $$\frac{\mathrm{d}R}{\mathrm{d}t} = V_\mathrm{S}$$
  - Strong shock (high Mach number)
    and efficient cooling ("isothermal" shock)
    $$P_\mathrm{S} = \rho_0 V_\mathrm{S}^2 = n_0 m_\mathrm{H} V_\mathrm{S}^2$$
  - Pressure behind shock (P_S) = pressure in ionized gas (P_i)
    $$P_\mathrm{i} = 2n_\mathrm{i} k T_\mathrm{e} = n_\mathrm{i} m_\mathrm{H} c_\mathrm{i}^2 \qquad \left(c_\mathrm{i}^2 = \frac{kT_\mathrm{e}}{1/2 \cdot m_\mathrm{H}}\right)$$
    $$\rightarrow V_\mathrm{S}^2 = \frac{n_\mathrm{i}}{n_0} c_\mathrm{i}^2$$
  - Ionization balance for region within R
    $$S_* = \frac{4}{3}\pi R^3 n_\mathrm{i}^2 \beta_2$$
    $$R^3 = \frac{3S_*}{4\pi n_\mathrm{i}^2 \beta_2}$$
    $\rightarrow$ take square root + multiply by V_S^2
    $$\boldsymbol{R^{\frac{3}{2}} V_\mathrm{S}^2} = \left(\frac{3S_*}{4\pi n_\mathrm{i}^2 \beta_2}\right)^{\frac{1}{2}} \frac{n_\mathrm{i}}{n_0} c_\mathrm{i}^2 = \left(\frac{3S_*}{4\pi n_0^2 \beta_2}\right)^{\frac{1}{2}} c_\mathrm{i}^2 \boldsymbol{= R_\mathrm{S}^{\frac{3}{2}} c_\mathrm{i}^2}$$
    $\rightarrow$ Differential equation for R as a function of t

    $\rightarrow$ Calculate $\frac{R}{R_\mathrm{S}}$ as a function of $\frac{c_\mathrm{i}}{R_\mathrm{S}} \cdot t$

    $\rightarrow$ Result: expansion velocity $V_\mathrm{S} < c_\mathrm{i}$ and decreasing

- **Timescales for typical HII region**
  - O7V star with $S_* = 10^{49}\ \mathrm{s}^{-1}$
    $n_0 = 10^8\ \mathrm{m}^{-3}$
    $T_\mathrm{e} = 10^4\ \mathrm{K}$
  - Initial stages: recombination timescale $t_\mathrm{R} = (n_0\beta_2)^{-1}$

    R almost R_S at $t = t_\mathrm{R}$

    $R_\mathrm{S} \approx 3$ pc ($\approx 10^{17}$ m)

    $\boxed{t_\mathrm{R} \approx 1000\ \mathrm{yr}}$
  - Expansion phase: expansion timescale $t_\mathrm{e} = \frac{R_\mathrm{S}}{c_\mathrm{i}}$

    expansion velocity $V_\mathrm{S} \leq 0.65 \cdot c_\mathrm{i}$ at $t \geq t_\mathrm{e}$

    $c_\mathrm{i} \approx 10\ \mathrm{km\ s}^{-1} \rightarrow \boxed{t_\mathrm{e} \approx 3 \cdot 10^5\ \mathrm{yr}} \rightarrow t_\mathrm{e} \approx 200 \cdot t_\mathrm{R}$
  - Final stage: equilibrium timescale t_eq

    $R = R_\mathrm{f}$ at $t = t_\mathrm{eq}$

    $R_\mathrm{f}/R_\mathrm{S} \approx 34$
    $\rightarrow$ from expansion phase model: $\frac{c_\mathrm{i}}{R_\mathrm{S}} \cdot t_\mathrm{eq} \approx 273$

    $\rightarrow t_\mathrm{eq} \approx 300 \cdot t_\mathrm{e} \rightarrow \boxed{t_\mathrm{eq} \approx 10^8\ \mathrm{yr}}$
  - Main-sequence lifetime of ionizing star
    $t_\mathrm{MS} \approx 10^{10} \left(\frac{M}{M_\odot}\right)^{-2}$ yr

    $M(\mathrm{O7}) \approx 30 M_\odot \rightarrow \boxed{t_\mathrm{MS} \approx 10^7\ \mathrm{yr}}$

    $\rightarrow$ Final stage of HII region not reached during lifetime of star

- **Conversion of stellar UV energy to gas energy during the expansion phase**
  - Energy absorbed by gas at time t: $E_* = S_* \langle h\nu \rangle t$
  - Gas kinetic energy $\propto V_S^2$
    ionized gas thermal energy $\propto c_i^2$
    $V_S < c_i \rightarrow$ most of energy in ionized gas is thermal energy:

$$E_{th}^{i} \approx \frac{4}{3}\pi R^3 \cdot 2n_i \cdot \frac{3}{2} kT_e \approx 2\pi R^3 n_i m_H c_i^2 \qquad \left(P_i = 2n_i kT_e = n_i m_H c_i^2\right)$$

  - Neutral gas sound speed is lower (lower temperature)
    $\rightarrow$ most of energy in neutral gas is kinetic energy:

$$E_k^n = \frac{1}{2}\left(\frac{4}{3}\pi R^3 n_0 m_H - \frac{4}{3}\pi R^3 n_i m_H\right) V_S^2$$

$$E_k^n \approx \frac{2}{3}\pi R^3 n_0 m_H V_S^2 \qquad (n_i < n_0)$$

$$\rightarrow \frac{E_k^n}{E_{th}^i} \approx \frac{1}{3}\frac{n_0}{n_i}\frac{V_S^2}{c_i^2} \approx \frac{1}{3} \qquad \left(V_S^2 = \frac{n_i}{n_0} c_i^2\right)$$

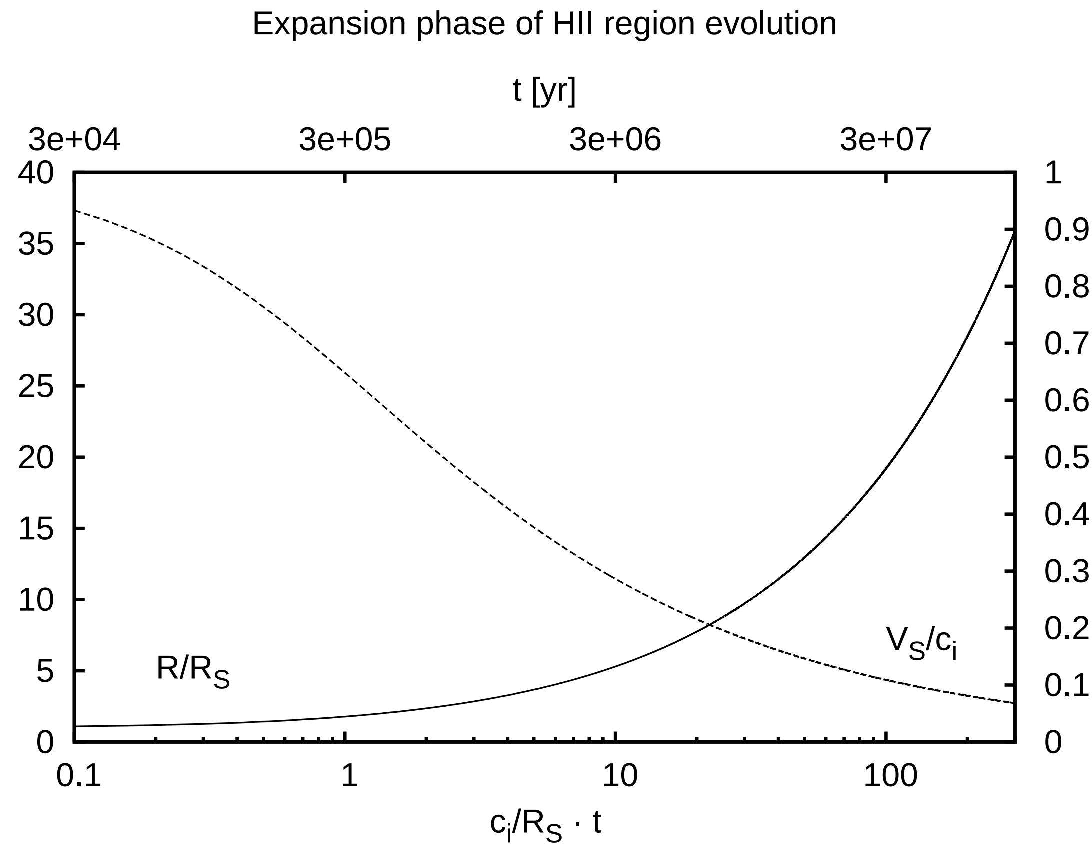
$\rightarrow$ E_k^n and E_{th}^i are same order of magnitude

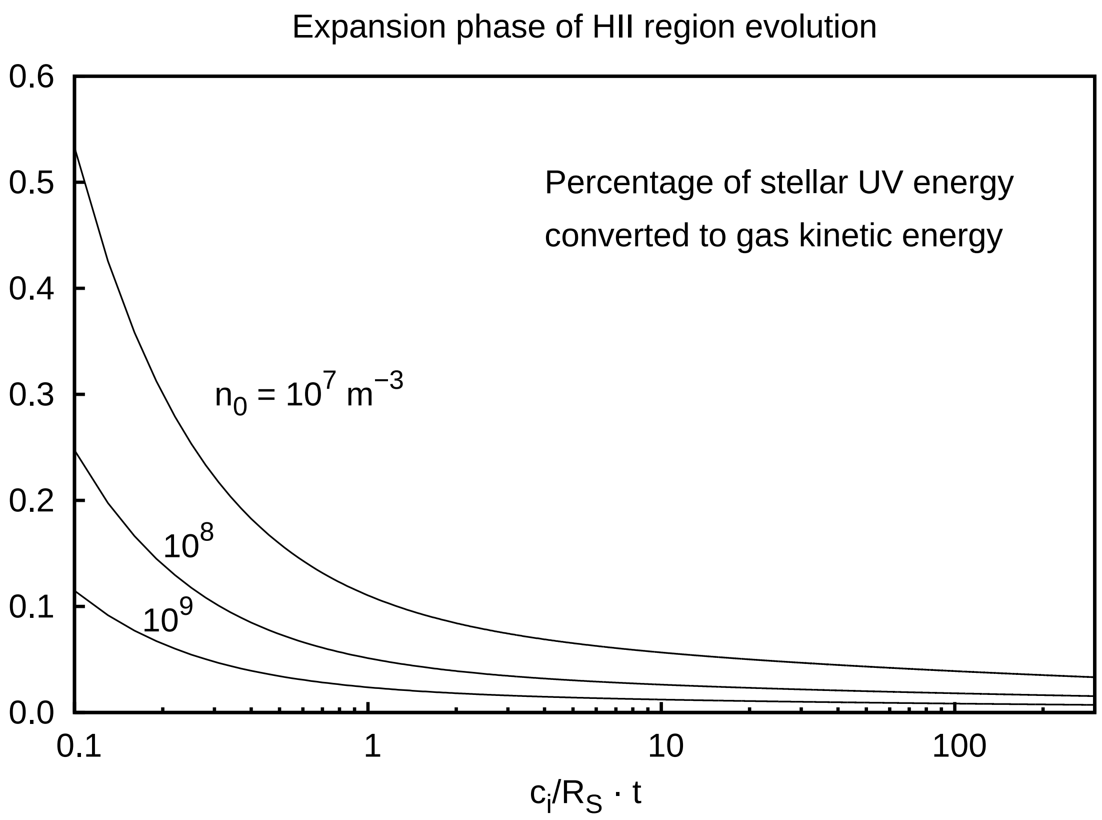
$\rightarrow$ Calculate, for example,
the ratio of stellar energy to gas kinetic energy

$$f \approx \frac{E_k^n}{E_*} \approx \frac{2}{3}\pi R^3 \frac{n_0 m_H V_S^2}{S_* \langle h\nu \rangle t}$$

$\rightarrow$ With $S_* = \frac{4}{3}\pi R_S^3 n_0^2 \beta_2$, $\langle h\nu \rangle = 2 \cdot 10^{-18}$ J,
and $R(t)$, $V_S(t)$ from expansion model,
we can calculate f as a function of $\frac{c_i}{R_S} \cdot t$

$\rightarrow$ Result: less than 1% of stellar UV radiation
converted to gas energy,
most is radiated away in forbidden lines


Expansion phase of HII region evolution
t [yr]
3e+04
3e+05
3e+06
3e+07
40
35
30
25
20
15
10
5
0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R/RS
VS/ci
0.1
1
10
100
ci/RS · t


Expansion phase of HII region evolution
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Percentage of stellar UV energy
converted to gas kinetic energy
n0 = 10^7 m^−3
10^8
10^9
0.1
1
10
100
ci/RS · t