Review of radiative transfer and thermodynamics

- Radiative transfer
  - Process
  - Quantities ($I_\nu, j_\nu, \alpha_\nu, S_\nu, \tau_\nu$)
  - Transport equation
  - Formal solution
  - Quantities derived from I_ν
- Local thermodynamical equilibrium (LTE)
  - Planck function
  - RT equation
  - Radio astronomy:
    R-J approximation, brightness temperature
- Measurement of interstellar lines
  - Brightness temp. $\rightarrow$ cloud temperature
  - Equivalent width $\rightarrow$ column density
- State of the gas
  - Speed distribution
  - Equation of state

Interstellar spectral lines

- Atoms and ions
  - Hydrogen 21 cm
  - Recombination lines
  - IR fine structure lines
  - Absorption lines
- Molecules
  - Classification
  - Diatomic molecules:
    - Energy levels
    - Spectra

Spectra of diatomic molecules

- Electronic transitions
  far-UV: H_2, CO, OH
  near-UV: CH, CH^+, CN
  near-IR: C_2, CN

- Vibrational transitions
  Energy levels: $E_v = (v + \frac{1}{2})h\nu_e$ (approximation)
  $v = 0, 1, 2, \ldots$ vibrational quantum number;
  $\nu_e \ldots$ vibrational frequency
  Selection rules: $\Delta v = \pm 1$ (fundamental transition)
  $\rightarrow \Delta E = h\nu_e \rightarrow$ one line at $\nu_e \approx 10\mu$m $\rightarrow$ infrared
  $\Delta v = \pm 2, \pm 3, \ldots$ overtones

- Rotational transitions
  Energy levels: $E_J = BJ(J + 1)$
  $J = 0, 1, 2, \ldots$ rotational quantum number
  $B = \frac{h^2}{8\pi^2 \overline{m} r^2}$; $\overline{m} = \frac{m_A m_B}{m_A + m_B}$ reduced mass
  Selection rules: $\Delta J = \pm 1$,
  no lines for symmetric molecules (e.g. H_2, O_2, CH_4)
  $\Delta E = h\nu = 2B(J + 1)$
  $\rightarrow$ equidistant lines with separation $2B$
  $B \propto \frac{1}{\overline{m}}$
  $\rightarrow$ lines from lighter molecules at higher frequencies (shorter wavelengths) with larger separation between lines
  lines are at mm wavelengths (microwave)
  Example: CO $\lambda_{1\rightarrow 0} = 2.6$ mm,
  $\lambda_{4\rightarrow 3} = \lambda_{1\rightarrow 0}/4 = 0.65$ mm, $\lambda_{10\rightarrow 9} = 0.26$ mm

- Vibration-rotation transitions (within one electronic state)
  Combined energy levels and selection rules:
  $E_{vJ} = (v + \frac{1}{2})h\nu_e + B_v J(J+1)$
  $\Delta v = \pm 1$, $\Delta J = \pm 1$
  $v = 0 \rightarrow v = 1$: $B_1 < B_0$ because $r_1 > r_0$ and $B \propto \frac{1}{r^2}$
  - $J \rightarrow J+1$: $\Delta E = h\nu_e + 2B_1 + (3B_1 - B_0)J + (B_1 - B_0)J^2$
    $\rightarrow$ for each J a line at $\nu > \nu_e \rightarrow$ "R branch"
    frequency intervals $\Delta\nu$ decrease with J until they become 0 at a maximum ν, then negative with $|\Delta\nu|$ increasing ($\rightarrow$ "band head" at large J, e.g. $J = 108$ for CO, $\rightarrow$ usually not visible)
  - $J \rightarrow J-1$: $\Delta E = h\nu_e - (B_1 + B_0)J + (B_1 - B_0)J^2$
    $\rightarrow$ for each J a line at $\nu < \nu_e \rightarrow$ "P branch"
    frequency intervals $|\Delta\nu|$ increase with J
- Intensity of rotational transitions determined by
  1. Transition dipole moment
  2. Populations of levels
     calculated from Boltzmann statistics in LTE:
     $P_J = \frac{N_J}{N} = (2J+1)\exp(-BJ(J+1)/\mathrm{k}T)/(\mathrm{k}T/(sB))$
     $s \ldots$ 2 or 1 for symmetric or nonsymmetric molecules
     (assumption: only ground vibrational and electronic states are populated)
     J at maximum population ($\frac{\mathrm{d}N_J}{\mathrm{d}J} = 0$): $J_{\max} \propto \sqrt{T}$
     $\rightarrow$ cloud temperatures can be measured from relative intensities of rotational lines

Cooling in the ISM

- Criteria for efficient cooling

  1. High abundances of colliding particles
     Atoms: H, C, N, O; e^-;
     Molecules: H_2 (electric quadrupole transitions), CO at $\approx$ 10 K
  2. $E_{kin} \approx E_{exc}$

Transition	$\Delta E/k$
C II ($^2P_{1/2} \rightarrow {}^2P_{3/2}$)	92 K
Si II ($^2P_{1/2} \rightarrow {}^2P_{3/2}$)	413 K
O I ($^3P_2 \rightarrow {}^3P_{1,0}$)	228 K
	326 K
H_2 ($\Delta J = \pm 2$)	510 K
CO (J = 0 $\rightarrow$ 1)	6 K

  3. Large collisional excitation cross-section $\rightarrow$ ions + e^-
  4. $A_{ul} > C_{ul}$ (strong transitions)
  5. $\tau < 1 \rightarrow$ balance between 1. and 4.
     not satisfied for CO in dense clouds
     $\rightarrow$ OH, H_2O, ^{13}CO, $C^{18}O$ important coolants

- Cooling rate Λ
  loss of thermal energy per unit volume and per second
  $\Lambda_{C^+} \propto n(C^+) \cdot n(e^-) \cdot \frac{1}{\sqrt{T}}$

- Cooling time t_c
  time necessary for a significant change of temperature: ratio between internal energy of the gas per unit volume and cooling rate

Heating processes

1. Starlight
   - Photoionization of C, Si, Fe
     mean ΔQ $\approx$ **2.1** eV per ionization
   - Photodissociation of H_2
     excitation into state with larger internuclear separation than ground state
     mean ΔQ $\approx$ **0.4** eV per dissociation
2. Cosmic rays (p + e^- with E $\approx$ MeV)
   and soft X-rays (E $\approx$ 1 keV)
   - Ionization of H by cosmic rays
     mean ΔQ $\approx$ **3.4** eV per ionization
   - Ionization of He by X-rays
     mean ΔQ $\approx$ **6** eV per ionization
   - Ionization of H_2 by CRs and X-rays
     ΔQ $\approx$ **15** eV per ionization for 2 MeV proton
3. Grains
   photoelectric effect (UV photons)
   mean E_{kin} of liberated electron $\approx$ **5** eV
   important in diffuse neutral clouds
4. (Magneto)hydrodynamic heating
   - Stellar winds
   - Supernova explosions
   - Expansion of HII regions
   - Gravitational collapse of a cloud
   - Magnetic fields

Molecule formation

- Problems for direct formation A + B $\rightarrow$ AB + E
  - Short collision contact time
  - Low transition probability
  - Very low third body collision probability
- Solutions
  - Involvement of ions
  - Dust grain catalysis (e.g. H_2)
  - Indirect formation by chemical reactions A + B $\rightarrow$ C + D
- Ion-molecule reactions
  - Very rapid
  - Rates independent of temperature
  - Rates similar for different molecules
- Neutral exchange reactions
  - Lower rates than ion-molecule reactions
  - Rates depend on temperature
  - Rates similar for different molecules
- Reaction networks
  - Molecules formed and destroyed by above two types of reactions occuring successively, starting from H_2 + destroyed by UV radiation
  - Set of differential equations for the change of the number densities of all molecules with time
  - Equilibrium abundances for $\frac{\mathrm{d}n(\mathrm{M})}{\mathrm{d}t} = 0$
  - Examples: Ionization level and number density of atomic hydrogen in dense clouds, CO formation

Interstellar dust

- Evidence for dust grains
  - Element abundances in the ISM
  - Extinction of starlight
  - Polarization of starlight
  - Scattered light
  - Spectral features from dust (absorption + emission)
  - Photoluminescence
- Formation of grains (condensation)
  - IS clouds (slow)
  - Atmospheres of cool giants (fast)
  - Envelopes of protostars
- Destruction of grains
  - Evaporation
  - Sputtering in shocks (e.g. SN blast wave)
- Grain properties
  - Temperature (10 ... 50 K, time dependent for small grains)
  - Charge
    - 1 ... 100 e, depending on temperature, from collisions
    - positive charge from photoelectric effect
- H_2 formation on grains (DW 4.6 and essay topic)

Radiatively excited regions

- Regions of high temperatures and low densities:
  Clouds of atomic H near hot stars
  - T up to 10000 K, $n \approx 10^8 - 10^9$ m^{-3}
  - HII regions: clouds around stars
  - "diffuse ionized medium": lower densities, not clearly associated with stars
  - Observed spectra show emission lines:
    H Balmer, forbidden lines from O, N, S ions
- Hydrogen nebulae
  - Ionization of H by UV radiation from star
  - Recombination of p + e$^-$
  - Ionization equilibrium
  - Degree of ionization (x): $x \approx 1$
  - Variation of x with distance from star: well defined edge
  - Strömgren radius R_s
  - Temperature T_e from energy balance: $T_\mathrm{e} \approx T_\mathrm{eff}$
- Heavier elements in HII regions
  - Oxygen-line cooling
  - Ionization stratification
- Radio continuum spectrum
- Radio line spectrum (DW 5.4.2)
- Determination of electron temperature and density
  from observations